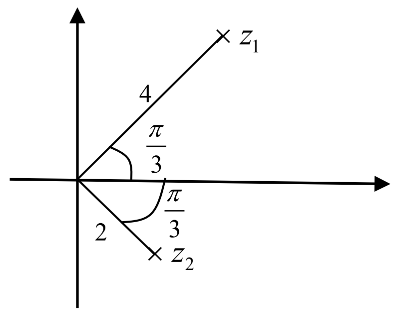
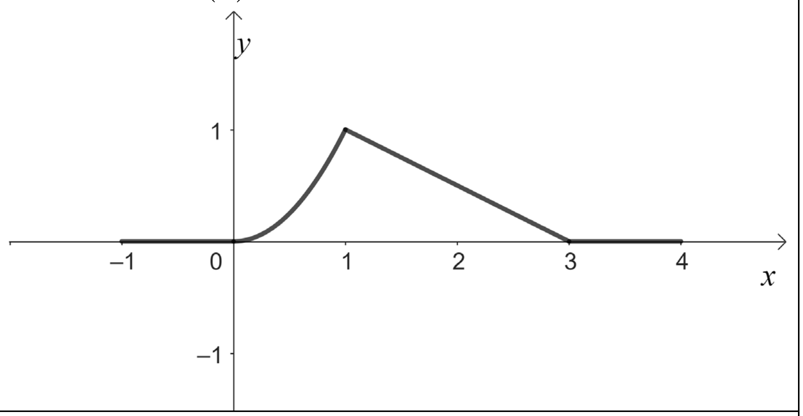
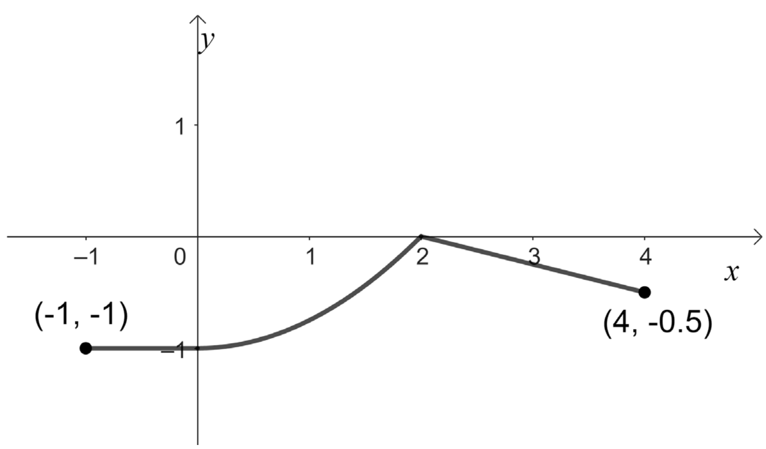
2023 H2MA Prelim Paper 2

1	Solution [5] Complex Numbers P2 Q1
(i)	 $\lvert z_1 - z_2 \rvert$ = Distance between z_1 and z_2 $=\sqrt{4^2+2^2-2(4)(2)\cos\left(\frac{\pi}{3}+\frac{\pi}{3}\right)}$ - Using Cosine Rule $=\sqrt{28}$
(ii)	$z_1 = az_2 \Rightarrow a = \frac{z_1}{z_2}$ $a = \frac{4e^{i\frac{\pi}{3}}}{2e^{-i\frac{\pi}{3}}} = 2e^{i\frac{\pi}{3}-\left(-i\frac{\pi}{3}\right)}$ $= 2e^{i\frac{2\pi}{3}}$ Either: z_1 is the **scaling of** z_2 **by factor of 2** and **rotating** z_2 $\frac{2\pi}{3}$ **anti-clockwise** about the Origin, Or: z_2 is the **scaling of** z_1 **by factor of** $\frac{1}{2}$ and **rotating** z_1 $\frac{2\pi}{3}$ **clockwise** about the Origin,

2	Solution [6] P2 Sequence
(i)	When $n=0$, $u_1-u_0=-\frac{4}{3}\left(\frac{1}{3}\right)^0+Q\left(\frac{2}{3}\right)^0$ $\frac{4}{3}-3=-\frac{4}{3}\left(\frac{1}{3}\right)^0+Q\left(\frac{2}{3}\right)^0$ $Q=\frac{8}{3}-3=-\frac{1}{3}$ and $u_2-u_1=-\frac{4}{3}\left(\frac{1}{3}\right)^1-\frac{1}{3}\left(\frac{2}{3}\right)^1$ $u_2=-\frac{4}{3}\left(\frac{1}{3}\right)-\frac{1}{3}\left(\frac{2}{3}\right)+\frac{4}{3}=\frac{2}{3}$
(ii)	$\left\|u_{n+1}-u_n\right\|=\left\|-\frac{4}{3}\left(\frac{1}{3}\right)^n-\frac{1}{3}\left(\frac{2}{3}\right)^n\right\|$ $\left\|u_{n+1}-u_n\right\|=\frac{1}{3}\left\|4\left(\frac{1}{3}\right)^n+\left(\frac{2}{3}\right)^n\right\|\le\frac{1}{3}\left\|4\left(\frac{2}{3}\right)^n+\left(\frac{2}{3}\right)^n\right\|$ $\left\|u_{n+1}-u_n\right\|\le\frac{5}{3}\left(\frac{2}{3}\right)^n$ Thus $\left\|u_{n+1}-u_n\right\|\le\varepsilon$ when $\frac{5}{3}\left(\frac{2}{3}\right)^n\le\varepsilon$ $\left(\frac{2}{3}\right)^n\le\frac{3\varepsilon}{5}$ $n\ge\frac{\ln\left(\frac{3\varepsilon}{5}\right)}{\ln\left(\frac{2}{3}\right)}$ n_0 can be $\left\lceil\frac{\ln\left(\frac{3\varepsilon}{5}\right)}{\ln\left(\frac{2}{3}\right)}\right\rceil$ or celling of $\frac{\ln\left(\frac{3\varepsilon}{5}\right)}{\ln\left(\frac{2}{3}\right)}$.

	When $\varepsilon = 0.001$, $n \geq \dfrac{\ln\left(\dfrac{3\varepsilon}{5}\right)}{\ln\left(\dfrac{2}{3}\right)} = 18.29$ n_0 can be 19.

<table>
<tr><td>3</td><td>Solution [7] Transformation of graphs</td></tr>
<tr><td>(i)</td><td>Graph of $y = \mathrm{f}(x)$ for $-1 \le x \le 4$
</td></tr>
<tr><td>(ii)</td><td>Graph of $y = \mathrm{f}\left(\frac{1}{2}x\right) - 1$, for $-1 \le x \le 4$

The end-points are at $(-1, -1)$ and $(4, -0.5)$

Note: When $x = 4$
$y = \mathrm{f}\left(\frac{1}{2}x\right) - 1 = \mathrm{f}(2) - 1 = -\frac{1}{2}(2) + \frac{3}{2} - 1 = -\frac{1}{2}$</td></tr>
<tr><td>(b)</td><td>Scale parallel to y-axis by factor 3.

Translate 2 units in the negative x-direction.
Reflect about the y-axis.
OR
Reflect about the y-axis.
Translate 2 units in the positive x-direction.</td></tr>
</table>

4	Solution [10] P2 3D Vectors
(a)	$l_1 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$ Let $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$. Given that $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}$ $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$ Let $\mathbf{n}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ $\pi_1 : \mathrm{r} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} = 3$ $\pi_1 : x - y - z = -3$
(ii)	Q is the foot of perpendicular of P on l_1 $\overrightarrow{OQ} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ for some λ $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 5 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -4 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\overrightarrow{PQ}$ perpendicular to the line l_1 $\overrightarrow{PQ} \cdot \mathbf{d} = 0$

	$\left[\begin{pmatrix}3\\-4\\-2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}\right]\cdot\begin{pmatrix}1\\1\\0\end{pmatrix}=0$ $-1+2\lambda=0$ $\lambda=\dfrac{1}{2}$ $\overrightarrow{OQ}=\begin{pmatrix}0\\0\\3\end{pmatrix}+\dfrac{1}{2}\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}0.5\\0.5\\3\end{pmatrix}$
(iii)	l_2 parallel to π_1 $\Rightarrow \mathbf{d}_2$ is perpendicular to $\mathbf{n}_1$ $\Rightarrow \mathbf{d}_2\cdot\mathbf{n}_1=0$ $\Rightarrow\begin{pmatrix}3\\1\\m\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=0$ $\Rightarrow -3+1+m=0$ $\Rightarrow m=2$ --- $h=$ distance between l_2 and π_1 $h=$ distance between $P(-3,4,5)$ and $\pi_1:\mathrm{r}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=3$ Let N be the foot of perpendicular of P on π_1. $l_{PN}:\mathbf{r}=\begin{pmatrix}-3\\4\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix},\ \lambda\in\mathbb{R}$ --- (1) $\pi_1:\mathrm{r}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=3$ --- (2) To find N, sub (1) into (2): $\left[\begin{pmatrix}-3\\4\\5\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}\right]\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=3$

$$\begin{pmatrix}-3\\4\\5\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\1\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=3$$

$12+3\lambda=3$

$\lambda=-3$

$$\overrightarrow{ON}=\begin{pmatrix}-3\\4\\5\end{pmatrix}-3\begin{pmatrix}-1\\1\\1\end{pmatrix}$$

$$h=\overrightarrow{PN}=\begin{pmatrix}-3\\4\\5\end{pmatrix}-3\begin{pmatrix}-1\\1\\1\end{pmatrix}-\begin{pmatrix}-3\\4\\5\end{pmatrix}=-3\begin{pmatrix}-1\\1\\1\end{pmatrix}$$

$$PN=\left|-3\begin{pmatrix}-1\\1\\1\end{pmatrix}\right|=3\left|\begin{pmatrix}-1\\1\\1\end{pmatrix}\right|=3\sqrt{3}$$

Alternatively

Note that $A(1,0,4)$ is a point on the plane

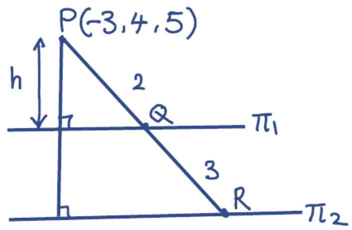
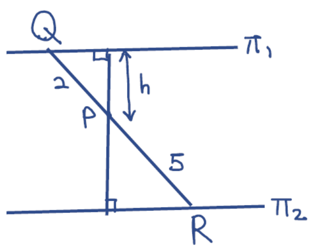
Dist of $P(-3,4,5)$ from $\pi_1:\mathrm{r}\cdot\begin{pmatrix}-1\\1\\1\end{pmatrix}=3$ is h.

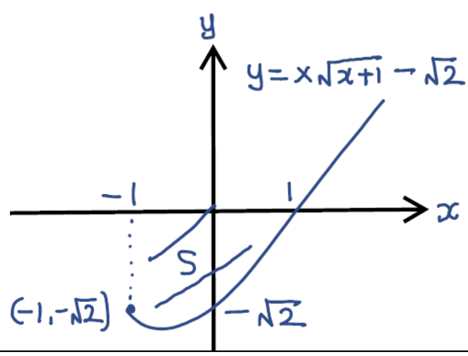
$$h=\left|\overrightarrow{AP}\cdot\hat{\mathbf{n}}\right| \text{ where } \overrightarrow{AP}=\begin{pmatrix}-3\\4\\5\end{pmatrix}-\begin{pmatrix}1\\0\\4\end{pmatrix}=\begin{pmatrix}-4\\4\\1\end{pmatrix}$$

$$h=\left|\begin{pmatrix}-4\\4\\1\end{pmatrix}\cdot\frac{1}{\sqrt{3}}\begin{pmatrix}-1\\1\\1\end{pmatrix}\right|$$

$$=\frac{9}{\sqrt{3}}$$

$$=3\sqrt{3}$$

(iv)	$PQ : PR = 2 : 5$ Case 1: Q is between P and R Distance between π_1 and $\pi_2 = \dfrac{3h}{2}$  Case 2: P is between Q and R Distance between π_1 and $\pi_2 = \dfrac{7h}{2}$ 

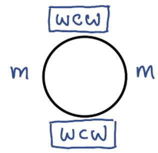
5	Solution [12] Integration
(i)	$u^2 = x+1$ when $x=-1, u=0$ $2u\dfrac{\mathrm{d}u}{\mathrm{d}x}=1$ when $x=a-1, u=\sqrt{a}$ $\dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{1}{2u}$ $\int_{-1}^{a-1} x\sqrt{x+1}\,\mathrm{d}x$ $=\int_0^{\sqrt{a}}(u^2-1)(u)(2u)\,\mathrm{d}u$ $=2\int_0^{\sqrt{a}}u^4-u^2\,\mathrm{d}u$ $=2\left[\dfrac{1}{5}u^5-\dfrac{1}{3}u^3\right]_0^{\sqrt{a}}$ $=2\left[\left(\dfrac{1}{5}\sqrt{a}^5-\dfrac{1}{3}\sqrt{a}^3\right)-\left(\dfrac{1}{5}(0)^5-\dfrac{1}{3}(0)^3\right)\right]$ $=\dfrac{2}{5}a^{\frac{5}{2}}-\dfrac{2}{3}a^{\frac{3}{2}}$
(ii)	C: $y=x\sqrt{x+1}$ Translate curve C, $\sqrt{2}$ units in the negative y direction, to obtain curve D. D: $y=x\sqrt{x+1}-\sqrt{2}$ Let S denote the region bounded by curve D, x-axis and $x=-1$. Volume obtained by revolving region R, 2π radians about the line $y=\sqrt{2}$, is the same as the volume obtained by revolving region S, 2π radians about the x-axis. 

	From GC, x-intercept happens at $x=1$. Volume of solid $=\pi\int_{-1}^{1}\left(x\sqrt{x+1}-\sqrt{2}\right)^2\ \mathrm{d}x$ $=\pi\int_{-1}^{1}x^2\left(x+1\right)-2\sqrt{2}\left(x\sqrt{x+1}\right)+2\ \mathrm{d}x$ $=\pi\int_{-1}^{1}x^3+x^2+2\ \mathrm{d}x-2\sqrt{2}\pi\int_{-1}^{1}x\sqrt{x+1}\ \mathrm{d}x$ $=\pi\left[\frac{x^4}{4}+\frac{x^3}{3}+2x\right]_{-1}^{1}-2\sqrt{2}\pi\left[\frac{2}{5}(2)^{\frac{5}{2}}-\frac{2}{3}(2)^{\frac{3}{2}}\right]$ $=\pi\left[\left(\frac{1}{4}+\frac{1}{3}+2\right)-\left(\frac{1}{4}-\frac{1}{3}-2\right)\right]-2\sqrt{2}\pi\left[\frac{8}{5}\sqrt{2}-\frac{4}{3}\sqrt{2}\right]$ $=\frac{14}{3}\pi-\frac{16}{15}\pi$ $=\frac{18}{5}\pi\text{ units}^3$

6 Two fair six-sided dice are thrown and the highest score X is recorded.
(i) State the probability distribution for X. [1]
(ii) Find the expected value and variance for X. [3]

(i)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$\mathrm{P}(X=x)$</td><td>$\frac{1}{36}$</td><td>$\frac{3}{36}$</td><td>$\frac{5}{36}$</td><td>$\frac{7}{36}$</td><td>$\frac{9}{36}$</td><td>$\frac{11}{36}$</td></tr></table>	
(ii)	$\mathrm{E}(X)$ $=(1)\left(\frac{1}{36}\right)+(2)\left(\frac{3}{36}\right)+(3)\left(\frac{5}{36}\right)+(4)\left(\frac{7}{36}\right)$ $+(5)\left(\frac{9}{36}\right)+(6)\left(\frac{11}{36}\right)$ $=\frac{161}{36}\approx 4.472$ $\mathrm{E}(X^2)=(1)^2\left(\frac{1}{36}\right)+(2)^2\left(\frac{3}{36}\right)+(3)^2\left(\frac{5}{36}\right)+(4)^2\left(\frac{7}{36}\right)$ $+(5)^2\left(\frac{9}{36}\right)+(6)^2\left(\frac{11}{36}\right)$ $=\frac{791}{36}$ $\mathrm{Var}(X)=\mathrm{E}(X^2)-\mathrm{E}(X)^2$ $=\frac{791}{36}-\left(\frac{161}{36}\right)^2$ $=\frac{2555}{1296}\approx 1.97$	

7	Solution [7] P&C P2 Q7
(i)	Required number of ways to have WCW, with M all separated = All W separated $=(4-1)!\times 4!$ $=144$ Alternatively **Case 1**: 2 groups of WCW.



Number of ways to choose first group of
WCW $=\binom{4}{2}\binom{2}{1}(2!)$

Number of ways to permute 2 men amongst the 2 groups of WCW $= 2!$

When seating the 4 units around a circle
- Let a WCW unit be seated first
- This set the position of the second WCW unit
- The 2 men are then slotted in between the WCW units in 2! Ways.

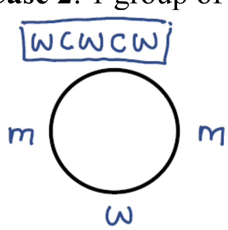
Number of ways to arrange 4 units: 2 men and 2 groups of WCW in a circle $= 2!$

Note there are 3 groups of CWC, so divide by 2! So as not to overcount

Number of ways

$$=\frac{\binom{4}{2}\binom{2}{1}(2!)(2!)}{(2!)}(2)=48$$

Case 2: 1 group of WCWCW



Number of ways $=\binom{4}{3}(3!)(2!)(2!)=96$

Total number of ways = 48 + 96 = 144

Alternatively

Case 1: 2 groups of WCW.

	There are $(4!)(2!)$ ways to get 2 groups of WCW. Number of ways $=\frac{2!}{2!}(4!)(2!)=48$ **Case 2**: 1 group of WCWCW There are $(4!)(2!)$ ways to get 1 group of WCWCW and a W. Number of ways $=\frac{2!}{2!}(4!)(2!)(2!)=96$ Total number of ways = 48 + 96 = 144
(ii)	Number of ways = $\frac{8!}{4!}=1680$ since 4! Ways to arrange women if there is no restriction on them.
(iii)	Number of ways = $\frac{\binom{2}{1}\binom{4}{2}\binom{2}{1}}{2!}=\frac{24}{2}=12$

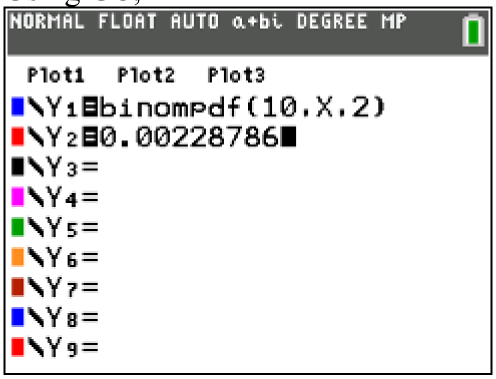
8	Solution [7] Probability
(i)	P(Get all Bots $\cap$ Get all number sets) $=\mathrm{P}(\text{box contain } \alpha \text{ and } \mathbb{R})\times\mathrm{P}(\text{box contain } \beta \text{ and } \mathbb{Z})$ $\times\mathrm{P}(\text{box contain } \gamma \text{ and } \mathbb{Q})\times\mathrm{P}(\text{box contain } \omega \text{ and } \mathbb{N})$ $+\mathrm{P}(\text{box contain } \alpha \text{ and } \mathbb{Z})\times\mathrm{P}(\text{box contain } \beta \text{ and } \mathbb{Q})$ $\times\mathrm{P}(\text{box contain } \gamma \text{ and } \mathbb{R})\times\mathrm{P}(\text{box contain } \omega \text{ and } \mathbb{N})$ $=\left(\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}\times\frac{1}{10}\right)(4!)+\left(\frac{1}{8}\times\frac{1}{8}\times\frac{1}{8}\times\frac{1}{10}\right)(4!)$ $=\frac{24}{2560}$ $=\frac{3}{320}\ (\text{shown})$
(ii)	$\mathrm{P}(\text{Get all bots} \mid \text{get all number sets})$ $=\frac{\mathrm{P}(\text{Get all Bots } \cap \text{ Get all number sets})}{\mathrm{P}(\text{Get all number sets})}$ $=\frac{\frac{3}{320}}{\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times 4!}$ $=\frac{1}{10}$
(c) (iii)	$\mathrm{P}(\text{get all bots})=0.3\times0.3\times0.3\times0.1\times4!$ $=0.0648$ $\neq 0.1$ $=\mathrm{P}(\text{get all bots} \mid \text{get all number sets})$ The event that he gets all the bots and the event that he get all the number sets are not independent. OR $\mathrm{P}(\text{Get all bots} \cap \text{Get all number sets})=\frac{3}{320}$ $\mathrm{P}(\text{Get all bots})\times\mathrm{P}(\text{Get all number sets})$ $=\left(0.3^3\times0.1\times(4!)\right)\left(\frac{1}{4^4}\times(4!)\right)=0.006048$

	$\mathrm{P}(\text{Get all bots} \cap \text{Get all number sets})$ $\neq \mathrm{P}(\text{Get all bots}) \times \mathrm{P}(\text{Get all number sets})$ The event that he gets all the bots and the event that he get all the number sets are not independent

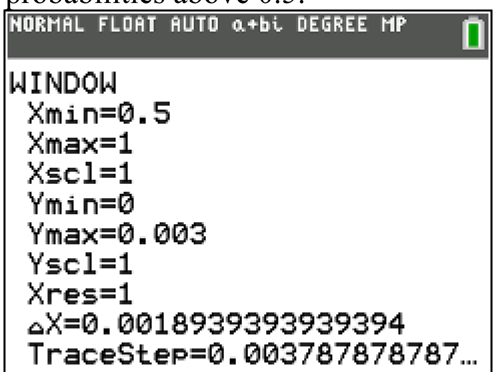
9	Solution [8] Binomial Dist
(i)	The probability of a student achieving distinction in the course is a constant OR The event that a student achieves distinction is independent of the event that another student achieves distinction.
(ii)	$X \sim B\left(10, \frac{p}{100}\right)$ Mode of X is 7. $\mathrm{P}(X=6) < \mathrm{P}(X=7)$ $\binom{10}{6}\left(\frac{p}{100}\right)^6\left(1-\frac{p}{100}\right)^4 < \binom{10}{7}\left(\frac{p}{100}\right)^7\left(1-\frac{p}{100}\right)^3$ $\frac{10!}{6!4!}\left(\frac{p}{100}\right)^6\left(1-\frac{p}{100}\right)^4 < \frac{10!}{7!3!}\left(\frac{p}{100}\right)^7\left(1-\frac{p}{100}\right)^3$ $7\left(1-\frac{p}{100}\right) < 4\left(\frac{p}{100}\right)$ $p > \frac{700}{11}$ and $\mathrm{P}(X=7) > \mathrm{P}(X=8)$ $\frac{10!}{7!3!}\left(\frac{p}{100}\right)^7\left(1-\frac{p}{100}\right)^3 > \frac{10!}{8!2!}\left(\frac{p}{100}\right)^8\left(1-\frac{p}{100}\right)^2$ $8\left(1-\frac{p}{100}\right) > 3\left(\frac{p}{100}\right)$ $p < \frac{800}{11}$ $\Rightarrow \frac{700}{11} < p < \frac{800}{11}$ $\Rightarrow 63.6 < p < 72.7\ (3sf)$

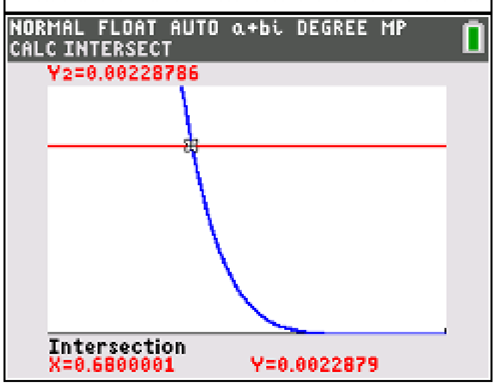
(iii) $\mathrm{P}(X=2)=0.00228786$

Using GC,



Note that since $63.6 < p < 72.7$, we should look for probabilities above 0.5.



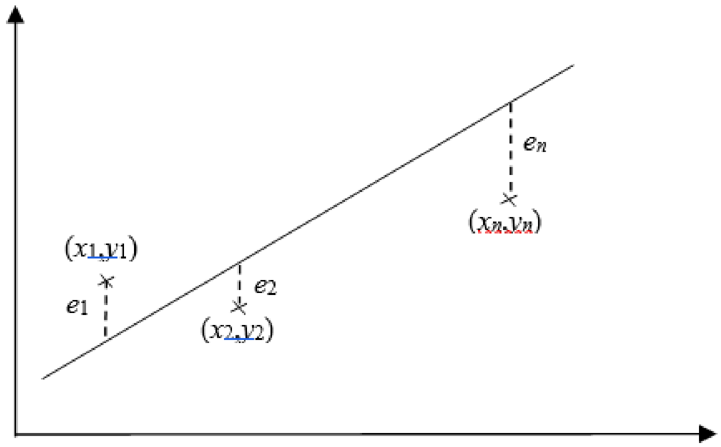


$$\therefore \frac{p}{100}=0.68$$

$$\Rightarrow p=68$$

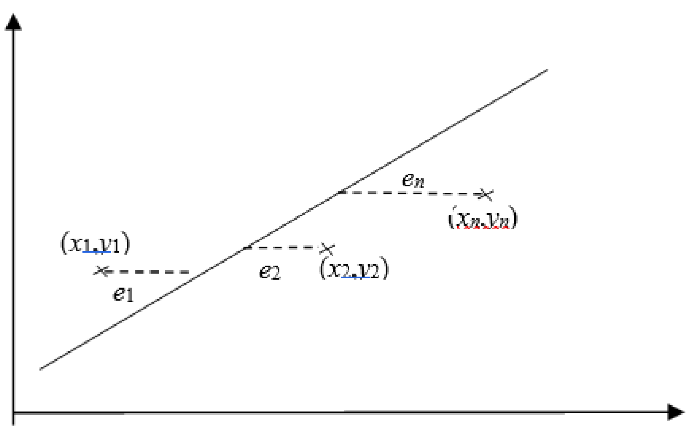
(iv)	$X \sim B(10, 0.68)$ $\mathrm{P}(X > 6) = 1 - \mathrm{P}(X \le 6) = 0.595637$ Let Y be the number of concluded courses out of 8 courses, with more than 6 students achieving distinction $Y \sim B(8, 0.595637)$ $P(Y \le 7 \mid Y \ge 5) = \dfrac{P(5 \le Y \le 7)}{P(Y \ge 5)}$ $= \dfrac{P(Y \le 7) - P(Y \le 4)}{1 - P(Y \le 4)}$ $= \dfrac{0.984156 - 0.416069}{1 - 0.416069}$ $\approx 0.973\ (3sf)$

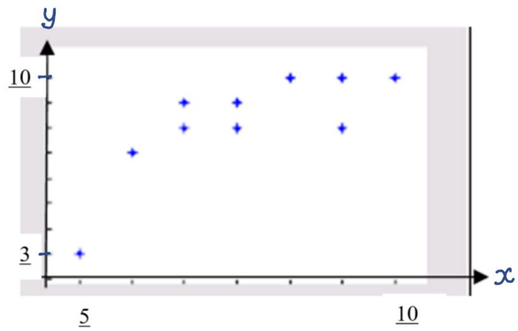
10	Solution [10] C&R P2 Q10
(a)	



For regression line of y on x, the values of x are considered as accurate and the sum of squares of deviation in the y-direction is minimized.

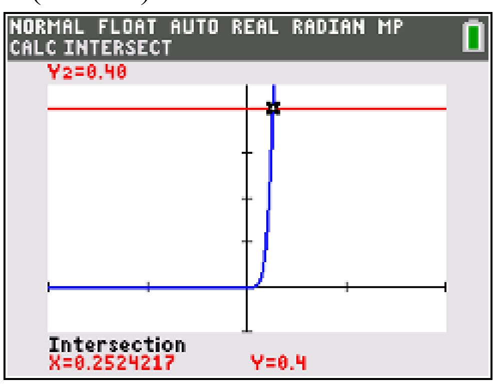
For regression line of x on y, the values of y are considered as accurate and the sum of squares of deviation in the x-direction is minimized.



(b) (i)	 Not appropriate as the scatter diagram indicates that as x increases, y increases at a decreasing rate.
(ii)	$y = -3.506924046 + 6.082293948 \ln x$ $y = -3.51 + 6.08 \ln x$ When y = 12, $12 = -3.506924046 + 6.082293948 \ln x$ $x = 12.80095 \approx 12.8$ ml
(iii)	Not reliable as y = 12 is outside the input data range, $3 \le y \le 10$.
(iv)	No change in the product-moment correlation coefficient. y in cm. y' in mm Then $y = \frac{y'}{10}$. $\frac{y'}{10} = -3.506924046 + 6.082293948 \ln x$ $y' = -35.1 + 60.8 \ln x$

<table>
<tr><td>11</td><td>Solution [12] Normal Dist P2 Q11</td></tr>
<tr><td>(i)</td><td>Mass of a completed ornament Y = 1.05(0.9X) = 0.945X
$Y \sim N(283.5, 357.21)$

$P(290 < Y < 350) = 0.365238 \approx 0.365$</td></tr>
<tr><td>(ii)</td><td>Required prob
$= \binom{9}{2} 0.365^2 (1-0.365)^7\, 0.365 = 0.072878121 \approx 0.0729$</td></tr>
<tr><td>(iii)</td><td>Let B be the mass of a box. $B = \alpha Y \sim N(283.5\alpha, 357.21\alpha^2)$
Let S be the total mass of an ornament and its box.
$S = Y + B \sim N(283.5\alpha + 283.5, 357.21\alpha^2 + 357.21)$
$S \sim N(283.5(\alpha+1), 357.21(\alpha^2+1))$

$P(S > 360) = 0.4$


By GC, $\alpha = 0.252$</td></tr>
<tr><td></td><td>If $W \sim N(130, 80^2)$, $P(W < 0) \approx 0.0521$.
That is, Approx 5.21% of the blocks are of negative masses. Thus, this distribution is not appropriate.

<u>Alternatively</u>
99.7% of the population should be within the range of $\mu \pm 3\sigma$ i.e. (-110, 370). However, this range contains negative values which are not possible. Thus the distribution is not appropriate.</td></tr>
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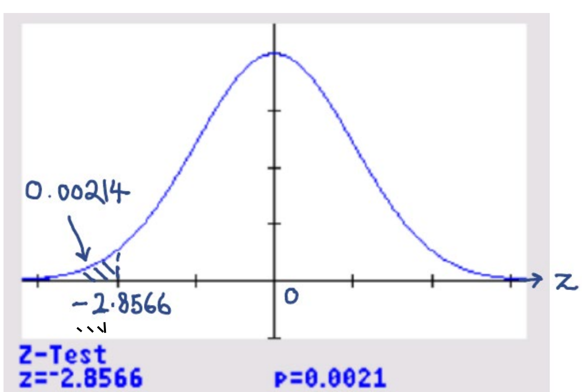
12	Solution [12] Hypothesis Testing P2 Q12
(i)	$\bar{x}=\dfrac{7644}{52}=147$ $\Sigma\left(x-20\right)\ =\Sigma x-20(52)=6604$ $s^2=\dfrac{1}{51}\left[840008-\dfrac{6604^2}{52}\right]=\dfrac{16900}{663}\approx 25.5$
(ii)	Test H₀ : $\mu=149$ against H₁ : $\mu<149$ at 5% level of significance. Test statistic: Under H_0, $Z=\dfrac{\bar{X}-149}{s/\sqrt{52}}\sim N\left(0,1\right)$ approximately $z_{\text{calculated}}=-2.86$ p-value = $P\left(Z<z_{\text{calculated}}\right)=$ 0.00214 <0.05, we reject H_0 There is sufficient evidence at 5% level of significance that the newly developed filter is more effective.
(iii)	p-value = 0.00214 = $P\left(Z<-2.86\right)=P\left(\bar{X}<147\right)$ p-value of 0.00214 refers to a probability of 0.00214 of obtaining a sample mean as extreme as the observed sample mean of 147 value given that the true population mean is 149.  Alternatively

	p-value of 0.00214 means that the least significance level the test concluding that the new filter is more effective is 0.214%. 
	Let Y denotes the amount of impurities in water of another town. Test $\quad$ H_0 : $\mu = 150$ against $\quad$ H_1 : $\mu \neq 150$ at 5% level of significance. $s^2 = \frac{100}{99}(29.85)^2$ $s \approx 30$ Test statistic: Under H_0, $Z = \frac{\bar{Y} - 150}{30/\sqrt{100}} \sim N(0,1)$. To reject Ho, $\lvert z_{cal} \rvert > z_{0.975} = 1.959964$ $\left\lvert \frac{\bar{y} - 150}{3} \right\rvert > 1.959964$ $\frac{\bar{y} - 150}{3} < -1.959964 \quad or \quad \frac{\bar{y} - 150}{3} > 1.959964$ $0 < \bar{y} < 144 \quad or \quad \bar{y} > 156$