ANDERSON SERANGOON JUNIOR COLLEGE

Preliminary Examination **8865/01**

30 August 2022

H1 Mathematics Paper (100 marks) **3 hours**

Additional Material(s): List of Formulae (MF26)

CANDIDATE NAME

CLASS /

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
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11	
12	
13	
Total	

This document consists of 19 printed pages and 5 blank pages.

Section A: Pure Mathematics [40 marks]

1 **(i)** Solve the inequality $2x^2 - 3x \geq 2$. [2]

(ii) Hence by means of the substitution $y = e^x$, find the range of values which will satisfy the inequality $2e^{2x} - 3e^x < 2$. [2]

2 **(a)** Differentiate $\left(3x^2 - \frac{2}{\sqrt{x}}\right)^2$ with respect to x. [2]

(b) Find $\int \frac{2}{\sqrt[3]{5-4x}}\,dx$. [2]

3 The curve C has equation $y = 8 - 2e^{1-4x}$.

(a) Find the equation of the tangent to C at the point $x = 1$, giving your answer in the form $y = mx + c$, where m and c are exact. [4]

(b) The finite area, A, between the curve C, the x – axis and the lines $x = \frac{1}{4}$ and $x = 1$ is given by $A = p + \frac{q}{e^3}$ where p and q are constants. Use integration to find the exact values of p and q. [3]

4 The curve C has equation $y = x^4 - 10x^3 + 36x^2 - 54x + 17$ for $0 \leq x < 5$.

(i) Find $\frac{dy}{dx}$. Hence find the coordinates of the stationary points on the curve. [4]

(ii) Determine the nature of each of the stationary points. [2]

(iii) Sketch the graph of C, stating the coordinates of any point(s) where the curve crosses the axes. [3]

(iv) Determine the equation of a curve which, when used with the equation of C, will enable you to solve the equation $x^4 - 10x^3 + 37x^2 - 52x + 18 = 0$. Sketch this curve on your diagram. [2]

(v) Hence solve the equation $x^4 - 10x^3 + 37x^2 - 52x + 18 = 0$, giving your answers correct to 3 decimal places. [1]

5 A particular company sells three brands of laptop: Brand A, Brand B and Brand C. Last week, the company sold 16 Brand A, 23 Brand B and 20 Brand C laptops and the total income from these sales was \$109 741. The income from Brand C sales was \$23 596 more than the income from Brand A sales. This week, the company has sold 2 less of each of the three brands of laptop than last week and the total income has decreased by \$10 994. The prices of the laptops have not changed.

(a) Using this information, form three linear equations to find the total price of two Brand A laptops and three Brand C laptops. [4]

(b) The director of the company develops a model for predicting the annual profits. He believes that the annual profits, P million dollars, in the year up to time t years can be modelled by

$$P = 1.8 + 0.6\ln(3t - 2), \text{ for } t \geq 1.$$

(i) Sketch the graph of P against t. [2]

(ii) Find the value of P when $t = 3$ and explain in context of the question. [2]

(iii) Find $\frac{\mathrm{d}P}{\mathrm{d}t}$. [2]

(iv) Explain in the context what the expression for $\frac{\mathrm{d}P}{\mathrm{d}t}$ indicates about the annual profits in the future. [2]

(v) Hence find the rate at which P is increasing when $t = 5$. [1]

Section B: Probability and Statistics [60 marks]

6 An internet retailer has compiled a list of 10 000 regular customers and wishes to carry out a survey of customer opinions involving 10% of its customers.

(i) Describe how the customers could be chosen to do this survey using random sampling. [2]

(ii) Give one advantage and one disadvantage of random sampling in this context. [2]

7 A board of directors consist of Mr Koh, Mrs Ho and 7 other people. A committee consisting of 4 people is to be formed from this board of directors.

(i) How many committees can be formed if it must include both Mr Koh and Mrs Ho? [1]

(ii) Calculate the probability for the committee to include either Mr Koh or Mrs Ho but not both. [2]

The committee is formed and consists of Mr Koh, Mrs Ho and 2 other people. For a photograph, the 4 people need to stand in a line.

(iii) However, Mr Koh and Mrs Ho refused to stand next to each other. In how many ways can the committee members stand in a line? [2]

8 At a certain college, the number of male and female students studying Business, Engineering and Humanities are as shown in the following table. Each student studies exactly one of these three subjects.

	Business	Engineering	Humanities
Male	23	80	35
Female	40	57	65

A student is chosen at random from these 300 students. Let
B be the event that the student studies Business,
E be the event that the student studies Engineering,
H be the event that the student studies Humanities,
F be the event that the student is female,
M be the event that the student is male.

(i) Find $\mathrm{P}(F \cup B')$. [1]

(ii) Find $\mathrm{P}(B \mid M)$. [1]

(iii) Find $\mathrm{P}(M \cap E)$. [1]

(iv) Determine whether the events M and E are independent. [1]

On another occasion, 3 of these students are chosen at random without replacement.

(v) Find the probability that exactly 2 are studying Business, giving your answer correct to 3 decimal places. [2]

9 The events A and B are such that $\mathrm{P}(A) = \frac{1}{2}, \mathrm{P}(B \mid A') = \frac{1}{2}$ and $\mathrm{P}(A \mid B') = \frac{4}{9}$.

(i) Find the probability that at least one of A and B occurs, [2]

(ii) Show that the probability that both A and B occurs is 0.3. [3]

(iii) Draw a Venn diagram to represent this situation, showing the probability in each of the four regions. [2]

10 The probability that a pear tree, of one particular type, will produce pears in the first year after planting is 0.85. The owner of a garden centre sells 5 pear trees each time to his customers with a guarantee that if fewer than 3 trees produce pears in the first year, he will replace all the ones that fail.

(i) Find the probability that no replacement will be required for a customer. [2]

The owner has signed a deal to sell 5 trees to each of the 3000 customers.

(ii) How many customers will be expected to receive replacement(s)? [1]

(iii) Find the probability that more than 100 customers will require a replacement. [2]

(iv) Find the probability that the average number of trees per customer that will produce pears in the first year is more than 4.28. [2]

11 The total cards billing, y billion dollars, in the 1st Quarter of the year x, are given in the following table.

x	2015	2016	2017	2018	2019	2020	2021	2022
y	12.12	12.69	13.61	14.95	15.81	16.46	15.46	16.91

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [1]

(ii) Find the value of the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x, giving your answer in the form of $y = mx + c$, with the values of m and c correct to 4 decimal places. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line to calculate an estimate for the total cards billing in 1st Quarter of the year 2023. Comment on the reliability of your estimate. [2]

12 Tom is a mechanic and he services cars and vans. The times taken, in minutes, by Tom to service cars and vans have independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Car	90	5
Van	130	10

(i) Tom knows that 95% of vans take no more than t minutes to service. Write down an equation satisfied by t and hence find the value of t. [2]

(ii) Find the probability that the time taken by Tom to service a van will be 60 minutes within twice the time taken by Tom to service a car. [3]

At 8am on a randomly chosen morning, Tom will service 2 cars and 1 van. Between every vehicle, he will take a short break, so in total there will be 2 intervals. The durations of these breaks will be normally distributed with mean 5.5 minutes and standard deviation 1.4 minutes. The durations of these breaks are independent of one another.

(iii) Find the probability that he will finish servicing the vehicles before 1pm. [3]

Tom's labour charges are \$50 per hour to service a car and \$90 per hour to service a van.

(iv) Find the probability that the total labour charge for servicing 2 cars and 1 van is more than \$350. [3]

13 A website states that the mean crop weight per plant of a particular variety is 5 kg. A horticulturist claims that the true mean crop weight per plant is less than 5 kg. To test this claim, a random sample of 64 plants is taken. The crop weight, x, of each of the 64 plants is measured, with the following summarised results.

$$\sum x = 303.4 \qquad \sum x^2 = 1615.96 .$$

(i) Find unbiased estimates of the population mean and variance of crop weight per plant. [3]

A test is carried out, at 10% significance level, to determine if horticulturist's claim is supported by the data.

(ii) State appropriate hypotheses for the test and define any symbols you use. [2]

(iii) Find p-value of the test. [2]

(iv) Find smallest level of significance at which the test would result in rejection of null hypothesis. [1]

(v) Explain the meaning of "level of significance" in the context of the question. [1]

A new random sample of 20 plants of this particular species is taken from a different region, Region B. The mean crop weight in this sample is m kg and the population standard deviation is 1.5 kg. A test at the 1% significance level gives insufficient evidence that the population mean crop weight of plants of this type in Region B differs from 5 kg.

(vi) Find the range of possible values of m. [3]

(vii) State a necessary assumption for the test in **(vi)** to be valid. [1]

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