JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS

Gravitational Field

Learning Outcomes

Candidates should be able to:

Gravitational field	(a)	show an understanding of the concept of a gravitational field as an example of field of force and define the gravitational field strength at a point as the gravitational force exerted per unit mass placed at that point.
	(b)	recognise the analogy between certain qualitative and quantitative aspects of gravitational and electric fields (*To be taught in Electric Fields)
Gravitational force between point masses	(c)	recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$
Gravitational field of a point mass	(d)	derive, from Newton's law of gravitation and the definition of gravitational field strength, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass
	(e)	recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass to new situations or to solve related problems
Gravitational field near to the surface of the Earth	(f)	show an understanding that near the surface of the Earth gravitational field strength is approximately constant and is equal to the acceleration of free fall
Gravitational potential	(g)	define the gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to that point
	(h)	solve problems using the equation $\phi = -\frac{GM}{r}$ for the gravitational potential in the field of a point mass
Circular orbits	(i)	analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes
	(j)	show an understanding of geostationary orbits and their application.

Introduction

- Isaac Newton (1642 –1726), published *Philosophiæ Naturalis Principia Mathematica ("Mathematical Principles of Natural Philosophy")* in 1687. In it, he formulated the Newton's laws of motion (Dynamics), and the Newton's law of gravitation (Gravitational Field).

- Newton's studies in gravitation started when he observed an apple falling to the ground and he began to ask himself questions. Why does a falling apple always descend perpendicularly to the ground? Did the force that pulled an apple to the Earth also extend further and pull bodies like the Moon towards the Earth? Would this force affect the orbit of the moon? We will answer these questions and more in this chapter.

1 GRAVITATIONAL FORCE

(c) Candidates should be able to recall and use Newton's law of gravitation in the form $F = \frac{Gm_1m_2}{r^2}$.

1.1 Newton's law of gravitation

Newton's law of gravitation states that the force of attraction between two point masses is directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

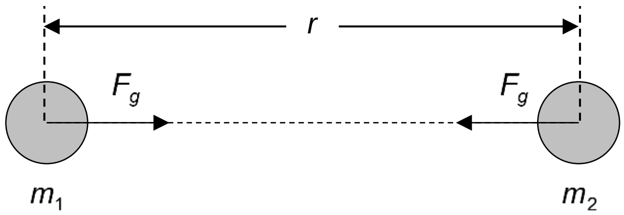


Fig. 1 Gravitational force between two bodies

- Mathematically, it is written as $\boxed{F_g = G\frac{m_1m_2}{r^2}}$ where

  where G is the gravitational constant with a value of 6.67×10^{-11} N m^2 kg^{-2},
  m_1 and m_2 are the masses of the two bodies respectively,
  r is the distance between their centres of masses, and
  F_g is the magnitude of the gravitational force acting between them.

- This is known as an inverse square law, as the magnitude of the force varies with the inverse square of the separation of the particles.

- The gravitational forces of attraction between two bodies form an action-reaction pair, consistent with Newton's third law.

- For spherical masses, the distance r is measured between their respective centres of mass (assume mass concentrated at centre).

- In vector form, the equation is written as $\vec{F} = -G\dfrac{m_1 m_2}{\vec{r}^{\,2}}$ to indicate the attractive nature of the force. The negative sign is normally ignored when only magnitude is required.

Example 1

A man of mass 85.0 kg is standing on the surface of the Earth. Given that the mass of the Earth is 5.98×10^{24} kg with a radius of 6.37×10^{6} m, calculate the gravitational force the Earth exerts on the man.

Hence, state the gravitational force the man exerts on the Earth.

Solution:

$$\begin{aligned} F_g &= \frac{G m_1 m_2}{r^2} \\ &= \frac{(6.67 \times 10^{11})(5.98 \times 10^{24})(85.0)}{(6.37 \times 10^{6})^2} \\ &= 835.539\text{... N} \\ &\approx 840 \text{ N} \end{aligned}$$

The gravitational force that the Earth exerts on the man is 840 N towards Earth.

The gravitational force that the man exerts on the Earth is 840 N towards the man.

Example 2

Two stationary objects of masses 5.0×10^{3} kg and 1.0×10^{3} kg are placed at a distance of 10 m apart. A third object of mass 1.0 kg, placed between the two masses, experiences no resultant force.

(a) On Fig. 1.1, draw the forces acting on the 1.0 kg object.

(b) Calculate the distance *x* between the 1.0 kg and the 1000 kg objects.

Solution:

(a)

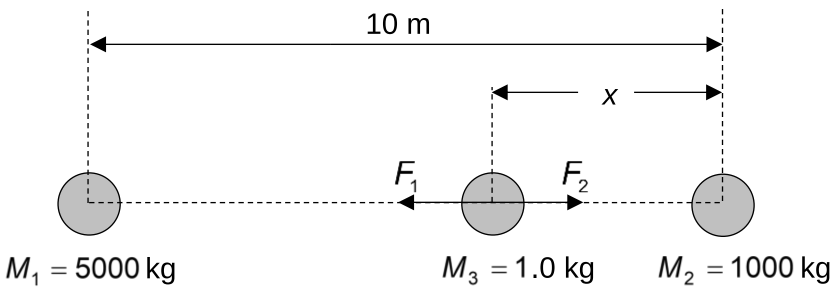


Fig. 1.1

(b) Since the 1.0 kg object experiences zero resultant force,

$$G\frac{M_1M_3}{(10-x)^2} = G\frac{M_2M_3}{x^2}$$

$$\frac{5000}{(10-x)^2} = \frac{1000}{x^2}$$

$$\frac{x^2}{(10-x)^2} = \frac{1000}{5000}$$

Taking square root on both sides of the equation,

$$\frac{x}{(10-x)} = \sqrt{\frac{1}{5}}$$

$$10 - x = \sqrt{5}x$$

$$x \approx 3.1 \text{ m}$$

Therefore, the distance x between the 1.0 kg and 1000 kg objects is 3.1 m.

2 GRAVITATIONAL FIELD

(a) Candidates should be able to show an understanding of the concept of a gravitational field as an example of field of force and define gravitational field strength as force per unit mass.

2.1 Concept of gravitational field

- A field is a model used by physicists to help them understand how objects not in direct contact with each other can affect each other's behaviour. Common types of field are gravitational, electric and magnetic fields.

- A gravitational field is a region of space in which a mass experiences a force due to the presence of another mass.

- The gravitational field of a mass can be represented by field lines. The field is directed towards the centre of the mass and gets stronger as one gets closer to the surface.

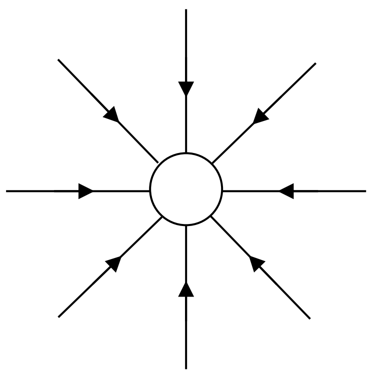


Fig. 2 Gravitational field lines of a mass

(d) ***Candidates should be able to derive, from Newton's law of gravitation and the definition of gravitational field strength, the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass***

2.2 Gravitational field strength

Gravitational field strength at a point is defined as the gravitational force per unit mass acting at that point.

$$\boxed{g = \frac{F_g}{m} = \frac{GM}{r^2}}$$

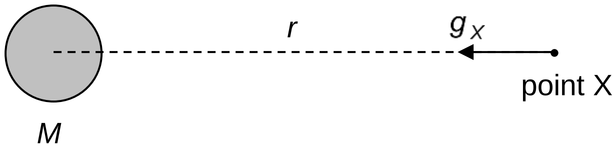


Fig. 3 Gravitational field strength at point X due to an object of mass M is $g_X = \frac{GM}{r^2}$

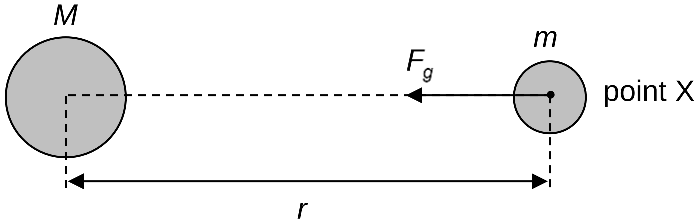
- It is a vector having both direction and magnitude, therefore addition of gravitational field strengths must be done by vector addition. The direction is the same as the gravitational force. Units for g is N kg^{-1}.
- Note that the gravitational field at point X in Fig. 3 is independent of mass m. The value of g only depends on the distance r from the object causing the gravitational field.

Example 3

Using Newton's law of gravitation and the definition of gravitational field strength, derive the equation $g = \frac{GM}{r^2}$.

Solution:

Consider a mass M and another mass m placed at point X, which is at a distance r away from mass M.



By Newton's law of gravitation, the force F_g experienced by mass m is $\frac{GMm}{r^2}$.

From the definition of gravitational field strength at point X,

$$g \text{ at point X} = \frac{F_g \text{ experienced by mass } m}{\text{mass } m}$$

$$g = \frac{G\frac{Mm}{r^2}}{m}$$

$$g = \frac{GM}{r^2}$$

Hence, the gravitational field strength at point X due to mass M is $\frac{GM}{r^2}$.

(e) Candidates should be able to recall and apply the equation $g = \frac{GM}{r^2}$ for the gravitational field strength of a point mass to new situations or to solve related problems

Example 4

The mass of the Earth is approximately 80 times the mass of the Moon and the Earth's radius is 3.7 times that of the Moon. If the gravitational field strength at the Earth's surface is 10 N kg^{-1}, estimate its value on the Moon's surface.

Solution:

Using the expression for gravitational field strength at a distance r,

$$g_E = \frac{GM_E}{r_E^{\ 2}} \quad \text{at the Earth's surface} \quad \text{--------------- (1)}$$

$$g_M = \frac{GM_M}{r_M^{\ 2}} \quad \text{at the Moon's surface} \quad \text{--------------- (2)}$$

Dividing the two equations, we have

$$\frac{g_M}{g_E} = \frac{M_M r_E^{\ 2}}{M_E r_M^{\ 2}}$$

$$= \frac{1}{80}(3.7)^2$$

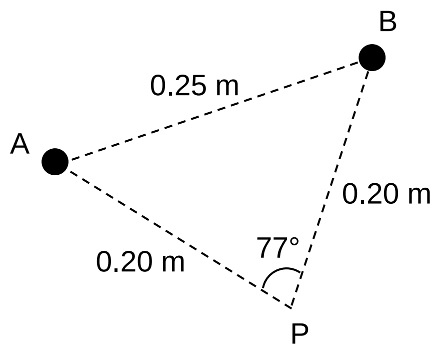
$$g_M = \frac{1}{80}(3.7)^2(10)$$

$$g_M \approx 1.7 \text{ N kg}^{-1}$$

Therefore, the gravitational field strength on the Moon's surface is 1.7 N kg^{-1}.

Example 5

The distance between an object A of mass 8000 kg and another object B of mass 8000 kg is 0.25 m. Calculate the resultant gravitational field strength due to both the objects at a point P 0.20 m away from both A and B.



Solution:

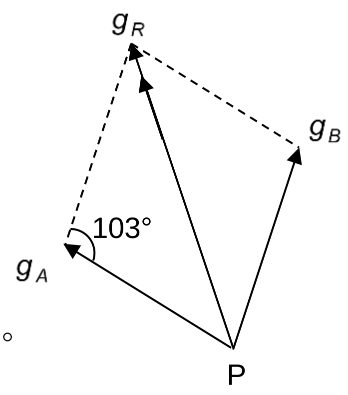
Let the gravitational field strengths due to objects A and B at P be g_A and g_B respectively.

Since masses A and B are the same and P is equidistant from A and B,

$$g_A = g_B = G\frac{M_A}{{r_A}^2}$$

$$= 6.67 \times 10^{-11} \times \frac{8000}{0.20^2}$$

$$\approx 1.33 \times 10^{-5} \text{ N kg}^{-1}$$



Using cosine rule,

$${g_R}^2 = {g_A}^2 + {g_B}^2 - 2g_A g_B \cos 103°$$

$${g_R}^2 = \left(1.33 \times 10^5\right)^2 + \left(1.33 \times 10^5\right)^2 - 2\left(1.33 \times 10^2\right)^2 \cos 103°$$

$$g_R \approx 2.09 \times 10^{-5} \text{ N kg}^{-1}$$

Therefore, the resultant gravitational field strength is 2.09×10^{-5} N kg^{-1} and its direction is towards the mid point of AB.

(f) Candidates should be able to show an appreciation that on the surface of the Earth, gravitational field strength is approximately constant and is equal to the acceleration of free fall.

2.3 Variation of Earth's gravitational field strength

- From the surface of the Earth upwards, the value of g follows an inverse square law ($\propto \frac{1}{r^2}$).
- Within the Earth (assuming that the Earth is uniform in density), g decreases uniformly ($\propto r$) with the radius r until it is zero at the centre of the Earth.
- The value of g can vary at some places due to the fact that the Earth is not a perfect sphere. Also, the rotation of the Earth causes deviations from the standard value.

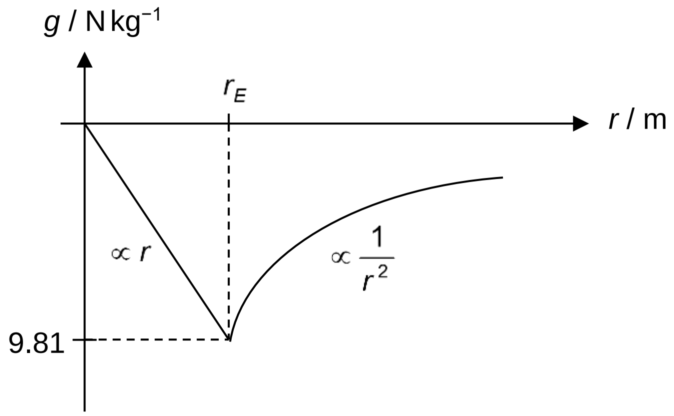


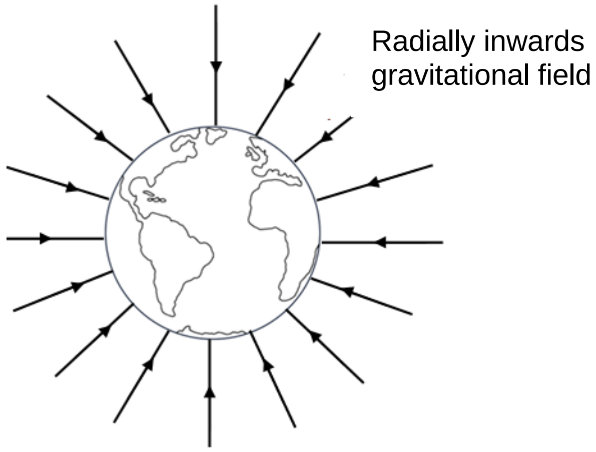
Fig. 4 Variation of Earth's gravitational field strength with distance from centre of the Earth

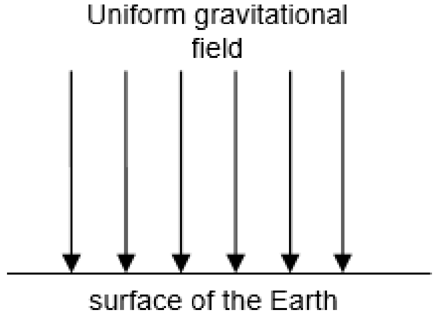
2.4 Earth's gravitational field strength at the surface

- The gravitational field strength g near the surface of the Earth is approximately constant at 9.81 N kg^{-1}, also known as acceleration of free fall, with the value of 9.81 m s^{-2}.
- The approximate constancy should be appreciated by considering the value of g at height h above the surface, where h is small compared to the radius R of the Earth of mass M.

$$g = \frac{GM}{(h+R)^2} \approx \frac{GM}{R^2} \quad \text{since } h << R$$

- Near the surface of the Earth, the gravitational field lines are nearly uniform.





3 GRAVITATIONAL POTENTIAL AND POTENTIAL ENERGY

(g) Candidates should be able to define gravitational potential at a point as the work done per unit mass in bringing a small test mass from infinity to that point.

3.1 Gravitational potential

Gravitational potential ϕ at a point is the work done per unit mass in bringing a small test mass from infinity to that point.

$$\boxed{\phi = -\frac{GM}{r}}$$

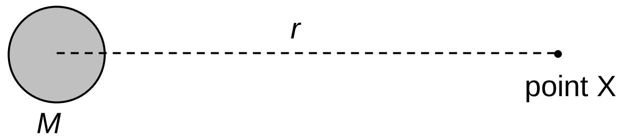


Fig. 5 Gravitational potential at point X due to mass M is $\phi_X = -\frac{GM}{r}$

- The gravitational potential at infinity is taken to be zero.
- The negative sign indicates that the gravitational force is an attractive force.
- ϕ is a scalar and a property of a point in the field. It does not depend on the mass at that point. The units for ϕ is $\mathrm{J\,kg^{-1}}$.

3.2 Equipotential surface

- All points at the same distance from the centre of the Earth have the same gravitational potential.

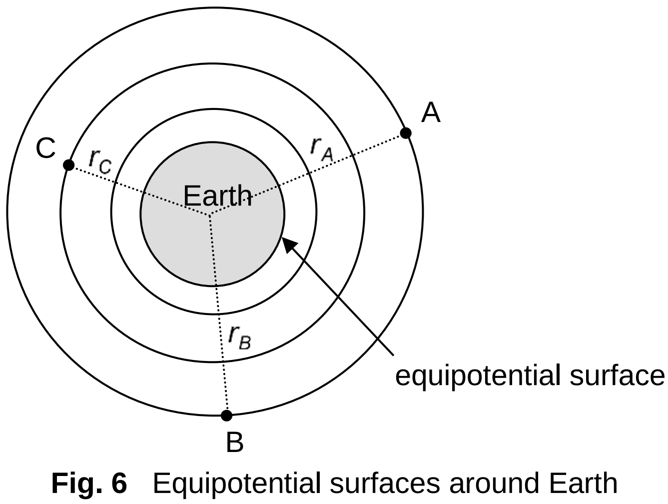


Fig. 6 Equipotential surfaces around Earth

- The surface where all points on it have the same gravitational potential is known as an equipotential surface. The gravitational field lines are always normal to equipotential surfaces.

- From Fig. 6, since $r_A = r_B > r_C$, $\phi_A = \phi_B > \phi_C$.

- The negative of the gradient of ϕ-r graph gives the gravitational field strength at that point because $g = -\dfrac{d\phi}{dr}$.

3.3 Gravitational potential energy

> **Gravitational potential energy** *U* of a body of mass *m* at a point is the work done in moving the body from infinity to that point.

$$\boxed{U = -\frac{GMm}{r}}$$

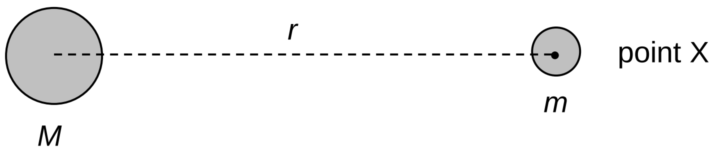


Fig. 7 Gravitational potential energy of mass *m* placed at point X

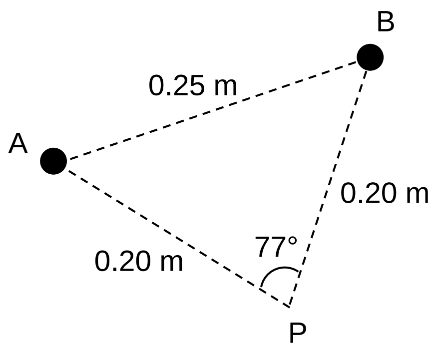
- The negative of the gradient of *U*-*r* graph gives the gravitational force because $F = -\dfrac{dU}{dr}$.

- *U* is negative for any definite distance and indicates the body *m* is always bonded to the main body *M*. *U* at infinity is taken to be zero.

(h) Candidates should be able to solve problems using the equation $\varphi = -\dfrac{GM}{r}$ for the potential in the field of a point mass

Example 6

The distance between an object of mass 8000 kg and another object B of mass 8000 kg is 0.25 m. What is the gravitational potential at P due to the two objects?



Solution:
Let the gravitational potential at point P due to object A be ϕ_A and that due to object B be ϕ_B. Since gravitational potential is a scalar quantity, we can simply add the potentials due to objects A and B.

$$\phi_P = \phi_A + \phi_B$$
$$\phi_P = -\frac{GM_A}{r} + \left(-\frac{GM_B}{r}\right)$$
$$\phi_P = -\frac{6.67\times10^{-11}\times8000}{0.20} + \left(-\frac{6.67\times10^{-11}\times8000}{0.20}\right)$$
$$\phi_P \approx -5.3\times10^{-6}\ \mathrm{J\,kg^{-1}}$$

Example 7
Calculate the energy required to move a 3000 kg satellite to a height of 200 km above the Earth's surface.
(Given that radius of the Earth $R_E = 6.4\times10^{6}$ m; mass of the Earth $M_E = 6.0\times10^{24}$ kg.)

Solution:
Energy required = change in *G.P.E.*

$$= -\frac{GMm}{R+h} - \left(-\frac{GMm}{R}\right)$$
$$= -GMm\left(\frac{1}{R+h} - \frac{1}{R}\right)$$
$$= -\left(6.67\times10^{-11}\right)\left(6.0\times10^{24}\right)\times3000\times\left[\frac{1}{\left[\left(6.4\times10^{6}\right)+\left(200\times10^{3}\right)\right]} - \frac{1}{6.4\times10^{6}}\right]$$
$$\approx 5.7\times10^{9}\ \mathrm{J}$$

Exercise: Can you prove that the gain in gravitational potential energy by an object of mass m moving a distance h from the Earth's surface is mgh?

3.4 Escape velocity

- **Escape velocity (*v_{esc}*)** from a point on the surface of a planet is the minimum velocity required to project a mass *m* to infinity

- To escape from the Earth's surface, an object needs to have sufficient kinetic energy in order to overcome the gravitational pull of the Earth.

- The minimum kinetic energy required is equivalent to the gain in gravitational potential energy by the object of mass *m* in leaving Earth to escape to infinity.

  loss in *K.E.* = gain in *G.P.E.*

  $$\frac{1}{2}mv_{esc}{}^{2} - 0 = U_{\infty} - U_{Earth's\ surface}$$ (assuming that *K.E.* at infinity is zero)

  $$\frac{1}{2}mv_{esc}{}^{2} = \left[0 - \left(-\frac{GM_{E}m}{R_{E}}\right)\right]$$ since $U_{\infty} = 0$.

  $$\boxed{v_{esc} = \sqrt{\frac{2GM_{E}}{R_{E}}}}$$

  Since $g = \frac{GM_{E}}{R_{E}{}^{2}}$, v_{esc} can also be written as $\boxed{v_{esc} = \sqrt{2gR_{E}}}$, where *g* is the gravitational field strength at the Earth's surface.

- Therefore, from the Earth's surface, the escape velocity is

  $$v = \sqrt{2 \times 9.81 \times 6.4 \times 10^{6}} \approx 11200 \text{ m s}^{-1}$$

  This is equivalent to travelling at about 34 times the speed of sound!

  Note: If the launch of the satellite is not from the surface of the Earth, we cannot use g = 9.81 N kg^{-1}.

4 GEOSTATIONARY ORBITS

(i) Candidates should be able to analyse circular orbits in inverse square law fields by relating the gravitational force to the centripetal acceleration it causes

4.1 Circular orbits

- A satellite is kept in orbit by the gravitational attraction of the body about which it is orbiting.

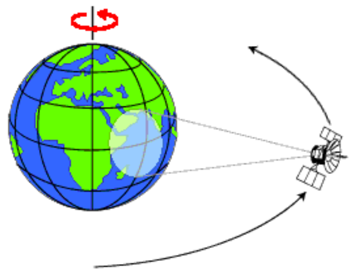


Fig. 8 A satellite orbiting around Earth

- When a satellite of mass *m* orbits around the Earth, the centripetal force is provided by the Earth's gravitational attraction.

$$F_g = F_c$$

$$\frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$\boxed{v = \sqrt{\frac{GM_E}{r}}}$$

- The speed v at which the satellite is orbiting around the Earth is therefore $\sqrt{\frac{GM_E}{r}}$. Note that this expression is different from the expression for escape speed! (Compare $v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$.)

Example 8

Given that a satellite orbits around the Earth at a height of 850 km above its surface, determine

(a) the speed of the satellite,
(b) its period of revolution T.
(Given that radius of the Earth $R_E = 6.4 \times 10^6$ m; mass of the Earth $M_E = 6.0 \times 10^{24}$ kg.)

Solution:

(a)
$$F_g = F_c \Rightarrow \frac{GM_E m}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM_E}{r}}$$

$$\therefore v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.4 \times 10^6 + 850 \times 10^3}} \approx 7400 \text{ m s}^{-1}$$

(b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi r}{v} = \dfrac{2\pi \times \left[\left(6.4\times10^{6}\right)+\left(850\times10^{3}\right)\right]}{7430} \approx 6131\ \text{s} \approx 1.7\ \text{hours}$

(j) Candidates should be able to show an understanding of geostationary orbits and their application.

4.2 Geostationary satellite

- The term geostationary implies a satellite is fixed about a position above the Earth.
- This is possible because
  - the satellite has the same period as the Earth's rotation (T = 24 hours),
  - the satellite revolves from West to East, in the same direction as the Earth's rotation,
  - the plane of revolution of the satellite is in the plane of the Equator.
- Examples of geostationary satellites are GMS1, GEOS, METEOSAT, etc. Most of these satellites are used for commercial and military purposes.

Example 9

For a geostationary satellite, determine the radius of its orbit and its height above the Earth's surface.

Solution:
By Newton's second law,

$$F_g = F_c$$

$$\frac{GM_E m}{r^2} = mr\omega^2$$

$$\frac{GM_E}{r^2} = r\left(\frac{2\pi}{T}\right)^2$$

$$T^2 = \frac{4\pi^2 r^3}{GM_E}$$ (Kepler's 3rd law, where $T^2 \propto r^3$)

For geostationary satellite, T = 24 hours, hence

$$\left(24\times3600\right)^2 = \frac{4\pi^2 r^3}{6.67\times10^{-11}\times\left(6.0\times10^{24}\right)}$$

$$r \approx 4.2\times10^7\,\text{m}$$

Hence, the radius of the satellite's orbit is 4.2×10^7 m.

Height of satellite above the Earth's surface

$$= \left(4.2\times10^7\right) - \left(6.4\times10^6\right)$$

$$\approx 3.6\times10^7\,\text{m}$$

4.3 Energy of satellite in orbit

- For a satellite of mass m in a circular orbit of radius r about a large mass M, the gravitational potential energy E_P it possesses is given by

$$\boxed{E_P = -\frac{GMm}{r}}$$

- The kinetic energy E_K of an orbiting satellite can be found by applying Newton's second law on the satellite.

$$F_g = F_c$$
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$
$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$
$$\boxed{E_K = \frac{GMm}{2r}}$$

- The total energy E_T is the sum of the gravitational potential and kinetic energies.

$$E_T = E_P + E_K$$
$$E_T = -\frac{GMm}{r} + \frac{GMm}{2r}$$
$$\boxed{E_T = -\frac{GMm}{2r}}$$

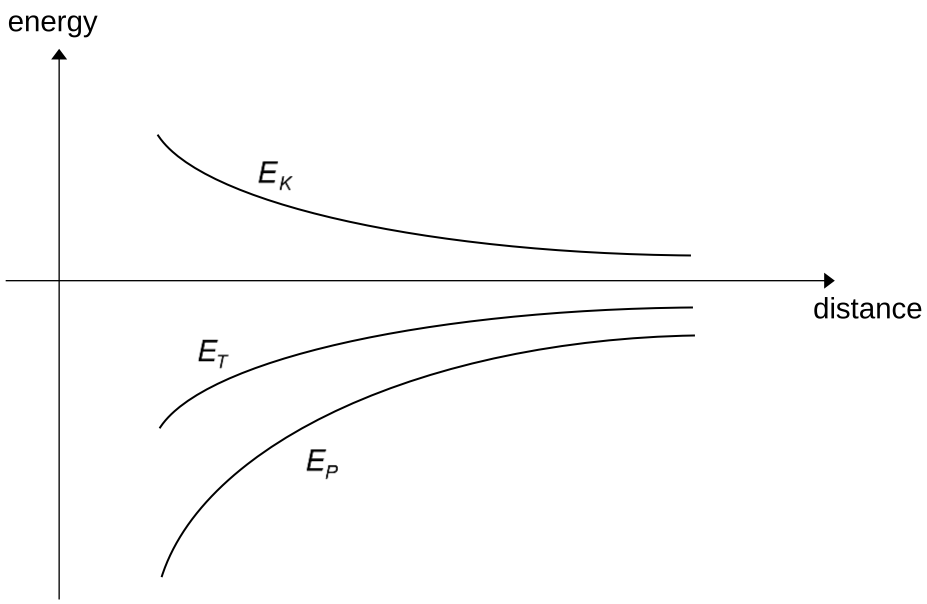


Fig. 9 Variation of energies for a satellite in orbit

Example 10
A satellite, of mass 120 kg, is orbiting Earth with an orbital radius R of 6610 km at a speed of 7780 m s^{-1}. It is boosted into higher orbit of radius 6890 km.

(a) Show that the speed of the satellite in the new orbit is 7620 m s^{-1}.
(b) Calculate the change in
  (i) kinetic energy,
  (ii) gravitational potential energy,
  (iii) total energy.
(c) State whether this change in total energy is an increase or decrease.

Solution:

(a) $F_g = F_c \Rightarrow \dfrac{GM_E m}{r^2} = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\dfrac{GM_E}{r}}$

$7780 = \sqrt{\dfrac{GM_E}{6610 \times 10^3}}$

$GM_E = 4.001 \times 10^{14}$

Therefore, speed of satellite in new orbit,

$v' = \sqrt{\dfrac{GM_E}{r'}}$

$= \sqrt{\dfrac{4.001 \times 10^{14}}{6890 \times 10^3}}$

≈ 7620 m s^{-1}

(b) (i) $\Delta K.E. = \dfrac{1}{2} m\ v_2{}^2 - \dfrac{1}{2} m\ v_1{}^2$

$= \dfrac{1}{2}(120)(7620^2 - 7780^2)$

$\approx -1.48 \times 10^8$ J

(ii) $\Delta G.P.E. = U_2 - U_1 = \left(\dfrac{-GM_E m}{R_2}\right) - \left(\dfrac{-GM_E m}{R_1}\right) = GM_E m\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$

$= (4.001 \times 10^{14}) \times 120 \times \left[\dfrac{1}{(6610 \times 10^3)} - \dfrac{1}{(6890 \times 10^3)}\right]$

$\approx 2.95 \times 10^8$ J

(iii) Total change in energy

$= \Delta K.E. + \Delta G.P.E.$

$= (-1.48 \times 10^8) + (2.95 \times 10^8)$

$= 1.47 \times 10^8$ J

(c) The total energy has increased by 1.47×10^8 J.

5 COMPARISON BETWEEN GRAVITATIONAL FIELD AND ELECTRIC FIELD

(b) Candidates should be able to recognise the analogy between certain qualitative and quantitative aspects of gravitational and electric fields.

- The table below shows the analogy between gravitational and electric fields.

	Gravitational field (due to point masses)	Electric field (due to point charges)
Nature of force	Force is attractive in nature. All masses attract each other.	Force can be attractive or repulsive. Unlike charges attract, like charges repel.
Force (Inverse square law)	Newton's law of gravitation states that the magnitude of the gravitational force between two point masses is directly proportional to the product of their masses and inversely proportional to the square of their distance apart. $$F_g = \frac{GMm}{r^2}$$	Coulomb's law states that the magnitude of the electrostatic force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their distance apart. $$F_E = \frac{Qq}{4\pi\varepsilon_o r^2}$$
Field strength	The gravitational field strength g at a point is defined as the gravitational force per unit mass m placed at that point. $$g = \frac{F}{m}$$	The electric field strength E at a point is defined as the electrostatic force per unit positive charge q placed at that point. $$E = \frac{F}{q}$$
	The gravitational field strength outside a point mass M is $$g = \frac{GM}{r^2}$$	The electric field strength outside a point charge Q is $$E = \frac{Q}{4\pi\varepsilon_o r^2}$$
Potential energy	The gravitational potential energy U_g of a mass m at a point is defined as the work done W in bringing it from infinity to that point.	The electric potential energy U_E of a charge q at a point is defined as the work done W in bringing it from infinity to that point.
	$$U_g = -\frac{GMm}{r}$$	$$U_E = \frac{Qq}{4\pi\varepsilon_o r}$$ (value can be positive or negative depending on the polarity of the charges)

Potential	The gravitational potential ϕ at a point is defined as the work done per unit mass in bringing a small test mass from infinity to that point. $$\phi = \frac{W}{m}$$	The electric potential V at a point is defined as the work done per unit positive charge in bringing a small test charge from infinity to that point. $$V = \frac{W}{q}$$
	The gravitational potential outside a point mass is $$\phi = -\frac{GM}{r}$$	The electric potential outside a point charge is $$V = \frac{Q}{4\pi\varepsilon_o r}$$ (value can be positive or negative depending on the polarity of the charges)
Relationship between field and potential	$$g = -\frac{d\phi}{dr}$$	$$E = -\frac{dV}{dr}$$
Relationship between force and potential energy	$$F_g = -\frac{dU_g}{dr}$$	$$F_E = -\frac{dU_E}{dr}$$

Table 1 Analogy between certain qualitative and quantitative aspects of electric field and gravitational field

References

1 Robert Hutchings, *Bath Advanced Science Physics (2nd Edition)*, Nelson Thornes

2 Young & Geller, *College Physics (8th Edition)*, Pearson

3 Poh Liong Yong, *Physics A Level Volume 1*, Pan Pacific

4 Douglas C. Giancoli, *Physics - Principles with Applications (6th edition)*, Prentice Hall