JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS

TEMPERATURE AND IDEAL GASES

Content

- **Thermal equilibrium**
- **Temperature scales**
- **Equation of state**
- **Kinetic theory of gases**
- **Kinetic energy of a molecule**

Learning Outcomes

Candidates should be able to:

(a) show an understanding that regions of equal temperature are in thermal equilibrium.

(b) explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero).

(c) convert temperatures measured in degrees Celsius to kelvin: T / K = T / °C + 273.15.

(d) recall and use the equation of state for an ideal gas expressed as $pV = nRT$, where n is the amount of gas in moles.

(e) state that one mole of any substance contains 6.02×10^{23} particles and use the Avogadro number $N_A = 6.02 \times 10^{23}$ mol^{-1}.

(f) state the basic assumptions of the kinetic theory of gases.

(g) explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship $pV = \frac{1}{3}Nm\langle c^2 \rangle$, where N is the number of gas molecules (a simple model considering one-dimensional collisions and then extending to three dimensions using $\langle c_x{}^2 \rangle = \frac{1}{3}\langle c^2 \rangle$ is sufficient).

(h) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$) to new situations or to solve related problems.

Introduction

Heat and temperature are often used interchangeably in everyday language. However, these terms have different and specific meanings in physics. Macroscopically, temperature can be defined in terms of its measurement, while heat refers to the energy transferred between two systems at different temperatures. Linking these two ideas is the concept of thermal equilibrium.

The properties of matter depend on temperature, pressure, volume, etc. The condition in which a particular material exists is known as its state, and this could be described by such macroscopic physical quantities, including its mass, which is a measure of the amount of substance. Most gases at room temperature and atmospheric pressure behave approximately like ideal gases. The ideal gas equation expresses the relationship between state variables for an ideal gas.

The kinetic theory of gases is a simple example of a link between macroscopic properties (pressure, volume, temperature) with microscopic properties (mass and speed of randomly-moving individual molecules that make up a gas). To a good approximation, we can use Newtonian mechanics to model the microscopic motion of gas molecules, and we can apply concepts from kinematics and dynamics to analyse the average pressure exerted by the randomly-moving molecules.

1 Thermal equilibrium

1.1 Temperature

- Temperature is a property of a body or system. It measures the degree of hotness or coldness of a body as indicated on a calibrated scale. It determines the way heat is transferred from one body to another.

(a) Candidates should be able to show an understanding that regions of equal temperature are in thermal equilibrium.

1.2 Thermal equilibrium

- When two objects A and B are placed in thermal contact, heat flows from the hotter object to the colder object B, until they reached thermal equilibrium.

- At thermal equilibrium, both objects A and B are at the same temperature.

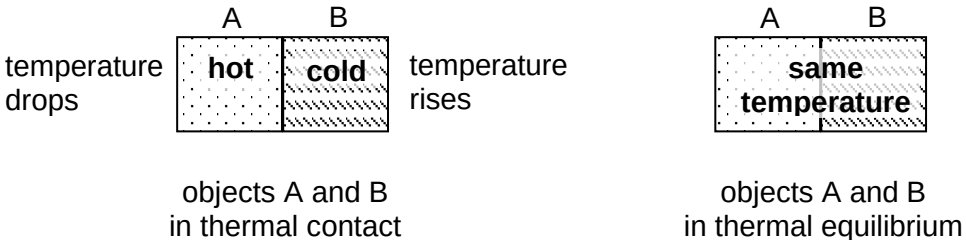


Fig. 1.1 Thermal equilibrium

- Hence, two objects are said to be in thermal equilibrium if there is no net heat exchange when they are placed in thermal contact.

- Note that thermal contact does not necessarily mean physical contact between two objects. Two systems are in thermal contact as long as there is a mechanism for the transfer of heat (thermal energy).

- The method for comparing temperatures is encapsulated in the **zeroth law of thermodynamics** (not in syllabus), which states that if two objects A and B are each in thermal equilibrium with a third object C, then A and B are also in thermal equilibrium with each other.

2 Temperature scales

(b) Candidates should be able to explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero).

2.1 Gas laws

2.1.1 Boyle's law

- For a given quantity of gas, it is found experimentally that the volume of a gas is inversely proportional to the pressure of the gas when the temperature is kept constant.

- This relation is known as Boyle's law, named after Robert Boyle.

- Mathematically, the law can be expressed as $V \propto \frac{1}{p}$, or

$$p_1V_1 = p_2V_2$$

where p_1 and V_1 are the initial pressure and volume of the gas, and p_2 and V_2 are the final values after a change of pressure and volume carried out at constant temperature.

- A graph of p against V shows a curve for a fixed temperature, where the curve is known as an isotherm. On the other hand, a graph of p against $\frac{1}{V}$ shows a straight line passing through the origin.

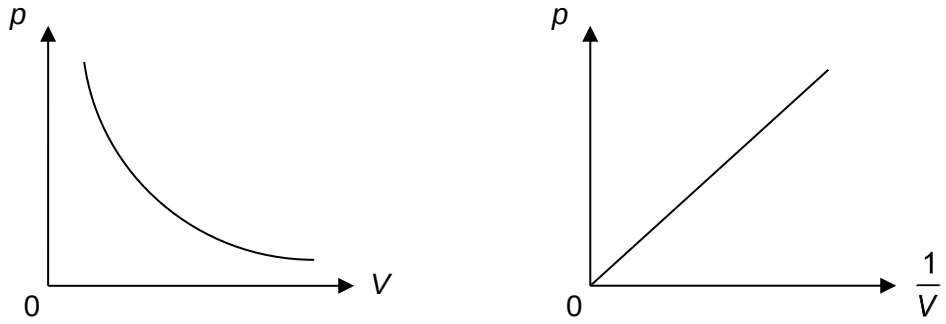


Fig. 2.1 Boyle's law

2.1.2 Charles' law

- For a given quantity of gas, it is found experimentally that when the pressure of a gas is kept constant, the volume of the gas increases with temperature at a nearly constant rate.

- When the straight line graph is projected to lower temperatures, it crosses the axis at about −273 °C. For any gas, a straight line is obtained and always project back to −273 °C at zero volume.

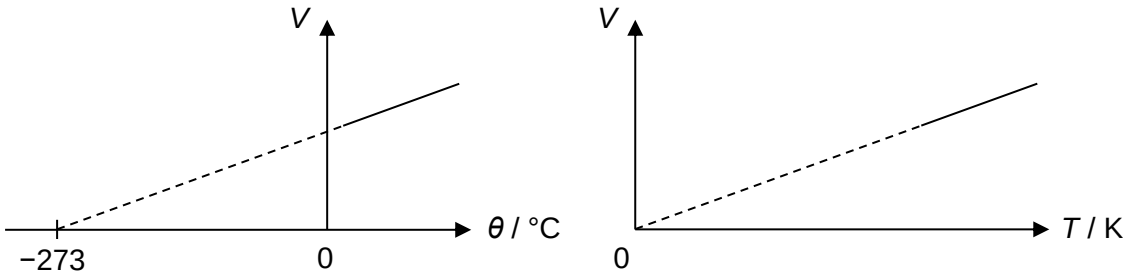


Fig. 2.2 Charles' law

- It could be argued that −273 °C is the lowest temperature possible for any gas. The exact value is determined to be −273.15 °C, which is equivalent to the absolute zero of temperature (0 K).

- If the Celsius temperatures are converted to thermodynamic temperatures, a straight line passing through the origin would be obtained. Hence, the volume of a gas is directly proportional to the temperature of the gas when the pressure is kept constant.

- This relation is known as Charles' law, named after Jacques Charles.

- Mathematically, the law can be expressed as $V \propto T$, or

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

  where V_1 and T_1 are the initial volume and temperature of the gas, and V_2 and T_2 are the final values after a change of volume and temperature carried out at constant pressure.

2.1.3 Gay-Lussac's law

- For a given quantity of gas, it is found experimentally that the pressure of a gas has a linear relationship with the temperature of the gas when the volume is kept constant.

- This relation is known as Gay-Lussac's law, named after Joseph Gay-Lussac.

- Mathematically, the law can be expressed as $p \propto T$, or

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

where p_1 and T_1 are the initial pressure and temperature of the gas, and p_2 and T_2 are the final values after a change of pressure and temperature carried out at constant volume.

- A graph of p against T shows a straight line passing through the origin, when projected at lower temperatures. For any gas, a straight line always extrapolates back to 0 K at zero pressure.

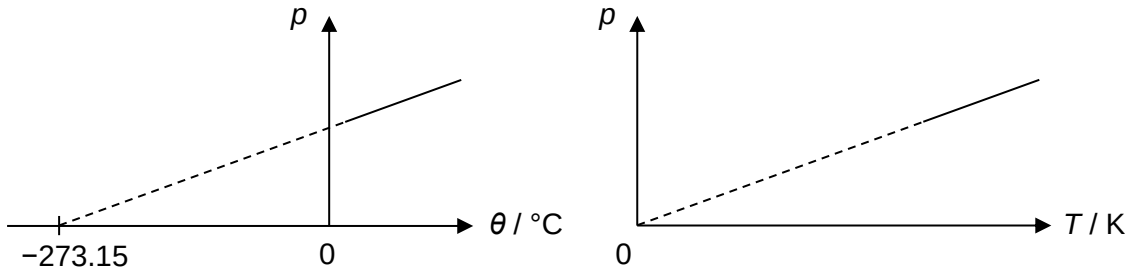


Fig. 2.3 Gay-Lussac's law

- Similar to the graph shown in Fig. 2.2, −273.15 °C is the lowest temperature possible for any gas.

2.2 Absolute scale of temperature

- The kelvin scale is an absolute scale of temperature and is also called the thermodynamic temperature scale. It is a theoretical scale that is independent of the thermometric properties of any substance, that is, it does not make reference to any property of any particular substance such as the volume of mercury or the resistance of a metal.

- Temperature is actually a measure of the average kinetic energy of the microscopic particles. It is much more logical to define "zero temperature" as the lowest possible temperature (when the object is so cold that the particles almost stop moving).

- Hence, the absolute thermodynamic temperature scale is defined using two fixed points, absolute zero and the triple point of water.

  - Lower fixed point: The absolute zero is the lowest theoretical possible temperature. It is given the value of 0 K.

  - Upper fixed point: The triple point of water corresponds to the single temperature and pressure at which water, water vapour and ice can co-exist in thermal equilibrium. The triple point of water occurs at a temperature of 273.16 K (0.01 °C) and a specific pressure of 611.66 Pa.

- The choice of the exact number 273.16 for the thermodynamic temperature of the triple point of water is to make the temperature interval of 1 K = 1 °C.

- The SI unit for temperature on the absolute scale of temperature is the kelvin (K). The kelvin is defined to be $\frac{1}{273.16}$ of the thermodynamic temperature of the triple point of water.

(c) Candidates should be able to convert temperatures measured in degrees Celsius to kelvin: T / K = T / °C + 273.15.

2.3 Conversion and relationship between temperature scales

- By choosing the temperature of the triple point of water as 273.16 K, the melting point of water is 273.15 K and the boiling point of water is 373.15 K. Both the kelvin scale and the Celsius scale have 100 units (divisions) between these two temperatures.

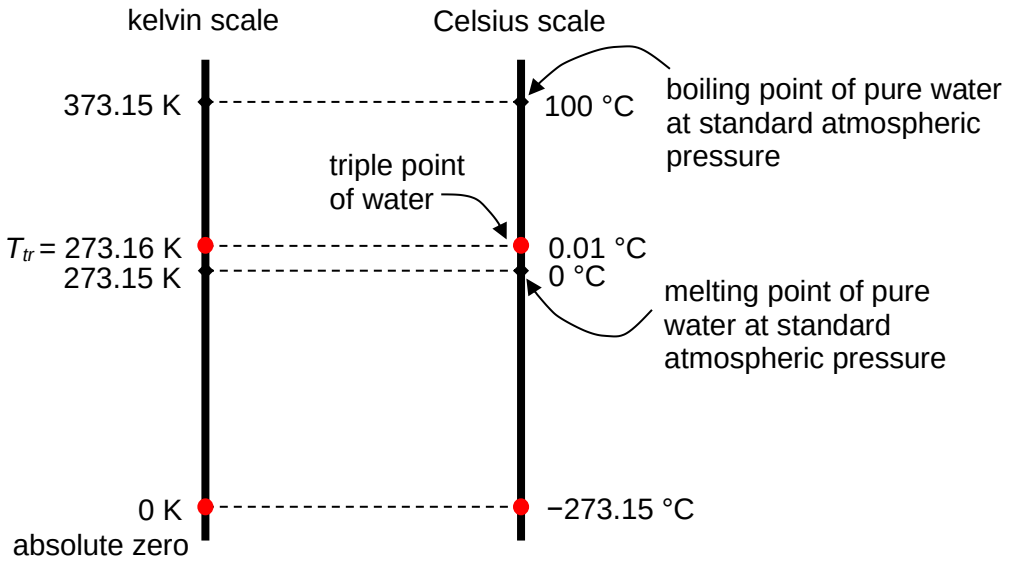


Fig. 2.4 Comparison between kelvin and Celsius scales

- A temperature difference of 1 K is precisely the same as 1 °C. It is thus a simple matter to convert between the two scales:

$$T \text{ / K} = T \text{ / °C} + 273.15$$

- Fig. 2.5 shows a few examples of temperatures on the kelvin and Celsius scales.

temperature	**kelvin scale / K**	**Celsius scale / °C**
absolute zero	0	−273.15
freezing point of water (standard pressure)	273.15	0
triple point of water	273.16	0.01
boiling point of water (standard pressure)	373.15	100
surface of the Sun	5800 (2 s.f.)	5500 (2 s.f.)

Fig. 2.5 Comparison between Kelvin and Celsius scales

3 Equation of state

3.1 State of a gas

- A system containing an amount of gas in moles n has a pressure p and is at a thermodynamic temperature T in a container of volume V. The equation that relates n, p, V and T is known as the equation of state of the gas.

- There are two approaches used in the understanding of gas behaviour; the macroscopic and the microscopic approach.

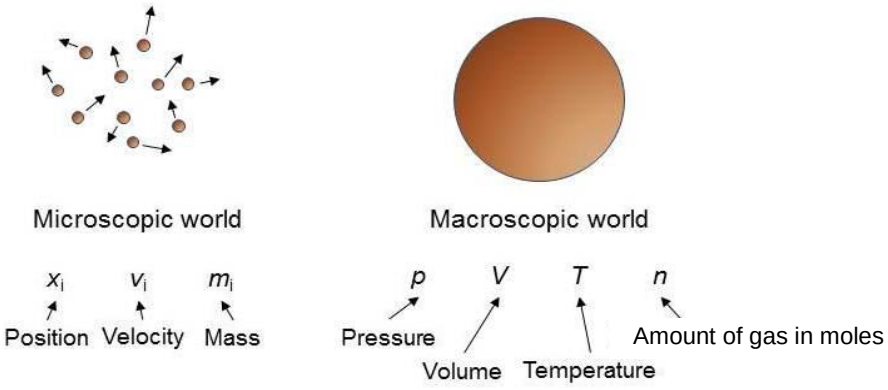


Fig. 3.1 Microscopic vs macroscopic approach

microscopic approach	**macroscopic approach**
Based on: The study of the behaviour of the gas and the interactions such as mass, velocity and position at molecular level. These interactions also help to explain the macroscopic properties of gases by considering their composition at a molecular level. Model: Kinetic theory of gases	Based on: The study of bulk properties of a gas such as its pressure, volume and temperature under different conditions. Gas laws: - Boyle's law - Charles' law - Gay-Lussac's law - Ideal gas law

Fig. 3.2 Comparison between microscopic and macroscopic approach

3.2 Standard notation

- The following notation will be used from this point on:

p	–	pressure of gas	(unit: Pa)
V	–	volume of gas	(unit: m^3)
T	–	thermodynamic temperature	(unit: K)
n	–	amount of gas in moles	(unit: mol) $\left(\text{Note that } n = \frac{N}{N_A} = \frac{M}{M_m}.\right)$
N	–	total number of molecules of gas	
N_A	–	the Avogadro constant	($N_A = 6.02 \times 10^{23}$ mol^{-1})
R	–	molar gas constant	(R = 8.31 J K^{-1} mol^{-1})
k	–	the Boltzmann constant	($k = 1.38 \times 10^{-23}$ J K^{-1})
M	–	total mass of gas	(unit: kg)
m	–	mass of one molecule of gas	(unit: kg)
M_m	–	molar mass of the gas	(unit: kg mol^{-1})

(e) Candidates should be able to state that one mole of any substance contains 6.02×10^{23} *particles and use the Avogadro number* $N_A = 6.02 \times 10^{23}$ *mol⁻¹.*

3.3 The Avogadro constant

- The Avogadro constant, N_A is the number of carbon atoms found in 0.012 kg of carbon−12. This is numerically equivalent to 6.02×10^{23} atoms.

- One mole of any substance is the amount of substance containing "Avogadro number" of elementary entities (particles). Hence, one mole of sulphuric acid (H_2SO_4) would contain 6.02×10^{23} H_2SO_4 molecules.

- As such, molar quantities refer to physical quantities associated with one mole of a substance. Therefore, the molar volume of CO_2 means the volume occupied by one mole of CO_2, the molar mass is the mass of one mole of the substance, and likewise for the molar gas constant.

3.4 Ideal gas

- An ideal gas is a hypothetical gas consisting of identical point particles that
  - exert no intermolecular forces on one another,
  - move randomly and undergo perfectly elastic collisions with the walls of the container,
  - occupy negligible fraction of the volume of their container.

- An ideal gas is one which obeys the equation of state $pV = nRT$ at all pressures, volumes and temperatures.

- In reality ideal gas does not exist, but since real gases behave almost ideally at low pressures, the concept is a useful approximation to reality.

(d) Candidates should be able to recall and use the equation of state for an ideal gas expressed as pV = nRT, where n is the amount of gas in moles.

3.5 Equation of state for an ideal gas

- An ideal gas obeys the gas laws exactly. The behaviour of such a gas can be accounted for by the equation:

$$pV = nRT$$

  Note that T must be in kelvin (K), and not in degrees Celsius (°C).

- Strictly speaking, the laws of Boyle, Charles and Gay-Lussac are not really laws, as their validity is restricted. They are really only approximations that are accurate for real gases at low pressure and density, and provided that the gas is not too close to liquefaction.

- For a fixed amount of an ideal gas,

$$\frac{pV}{T} = \text{ constant}$$

- The ideal gas equation can also be expressed in term of the number of molecules N.

$$pV = nRT$$
$$pV = \frac{N}{N_A}RT$$
$$pV = N\frac{R}{N_A}T$$
$$pV = NkT$$

  where $k = \frac{R}{N_A} = 1.38 \times 10^{-23}$ J K^{-1} and is known as the Boltzmann constant.

Example 1

Calculate the number of molecules in a flask of volume 5.0×10^{-4} m^3 containing oxygen at a pressure of 2.0×10^5 Pa and a temperature of 300 K.

Solution:
Using $pV = NkT$ for an ideal gas,

$$\left(2.0 \times 10^5\right)\left(5.0 \times 10^{-4}\right) = N \times \left(1.38 \times 10^{-23}\right) \times 300$$
$$N \approx 2.4 \times 10^{22}$$

The number of molecules in the flask is 2.4×10^{22}.

Example 2

A fixed mass of gas, in passing through a jet engine, has its pressure increased from 3.0×10^5 Pa to 1.3×10^6 Pa, while its temperature rises from 80 °C to 1500 °C. Determine the ratio of the final volume to the initial volume of the gas.

Solution:

Using $pV = nRT$ for an ideal gas, since the mass is constant, n is constant.

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

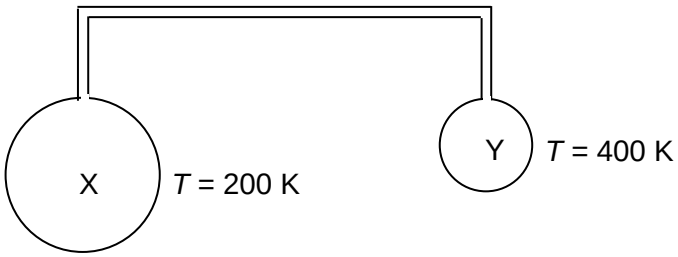
$$\frac{V_2}{V_1} = \frac{p_1T_2}{p_2T_1}$$

$$\frac{V_2}{V_1} = \frac{\left(3.0\times10^5\right)\left(273.15+1500\right)}{\left(1.3\times10^6\right)\left(273.15+80\right)}$$

$$\frac{V_2}{V_1} \approx 1.2$$

The ratio is 1.2.

Example 3



In the figure above, flasks X and Y are connected by a tube of negligible volume. The volume of flask X is twice that of flask Y. The system is filled with an ideal gas and a steady state is established with the flasks held at 200 K and 400 K. Given that the mass of gas in X is m, determine the mass of gas in Y in terms of m.

Solution:

When the gas stops flowing between the flasks, the pressure in flask X and Y are the same.

Let the volume of gas in Y be V and the final pressure be p.

Using the ideal gas equation $pV = nRT$,

For flask X, $p\times2V = \frac{m}{M_m}R\times200$ ---------- (1)

For flask Y, $p\times V = \frac{m'}{M_m}R\times400$ ---------- (2)

Solving, we have $2 = \frac{m}{m'}\times\frac{200}{400}$

$m' = 0.25m$

4 Kinetic theory of gases

4.1 Kinetic theory

- The concept that matter is made up of atoms which are in continuous random motion is called the kinetic theory.

- The kinetic theory of gases attempts to relate the macroscopic behaviour of an ideal gas (e.g. pressure, volume and temperature) to the microscopic properties of its atoms (e.g. speed, mass).

- It may be used to derive an expression relating the pressure of the gas to the microscopic properties such as the speed and mass of the gas.

(f) Candidates should be able to state the basic assumptions of the kinetic theory of gases.

4.2 Assumptions of the kinetic theory of gases

- The gas consists of a *very large number of molecules in continuous random motion*.

- There are *no intermolecular forces* between molecules except during collisions between molecules.

- The gas molecules undergo *elastic collisions* with one another and with the walls of the container.

- The *duration of collisions between molecules or between molecules and the walls of the container is negligible* compared to the time interval between collisions.

- The *volume of the gas molecules is negligible* compared to the volume of the container.

Example 4

The ideal gas equation may be written in the form

$$pV = nRT$$

where p is the pressure of the gas having volume V at temperature T. R is the gas constant.

a) State the meaning of the symbol n.

b) The radius of a helium-4 atom is 1.3×10^{-10} m.

A container of fixed volume contains 0.34 mol of helium-4 at a pressure of 1.0×10^5 Pa and a temperature of 21°C.

Calculate the volume of

i) the container,

ii) the atoms of helium-4.

c) Calculate the ratio

$$\frac{\text{volume of atoms of helium} - 4}{\text{volume of the container}}$$

Solution:

a) amount of substance / number of moles

(b) (i) Using $pV = nRT$

$(1.0 \times 10^5)V = (0.34)(8.31)(294)$

$\boldsymbol{V = 8.3 \times 10^{-3}\ m^3}$

(ii) volume of atoms = Number of molecules x Volume of 1 atom

$= (0.34)(6.02 \times 10^{23})\left(\frac{4}{3}\pi r^3\right)$

$= \mathbf{1.9 \times 10^{-6}\ m^3}$

c)

$$\text{ratio} = \frac{1.9 \times 10^{-6}}{8.3 \times 10^{-3}} = \frac{1}{4400} = \mathbf{2.3 \times 10^{-4}}$$

(g) *Candidates should be able to explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship* $pV = \frac{1}{3}Nm\langle c^2 \rangle$*, where N is the number of gas molecules (a simple model considering one-dimensional collisions and then extending to three dimensions using* $\langle c_x{}^2 \rangle = \frac{1}{3}\langle c^2 \rangle$ *is sufficient).*

4.3 Pressure of a gas

- Suppose a gas in a cubic container of side *L* contains *N* molecules, each of mass *m*. The motion of each molecule can be resolved into *x*−, *y*− and *z*−components.

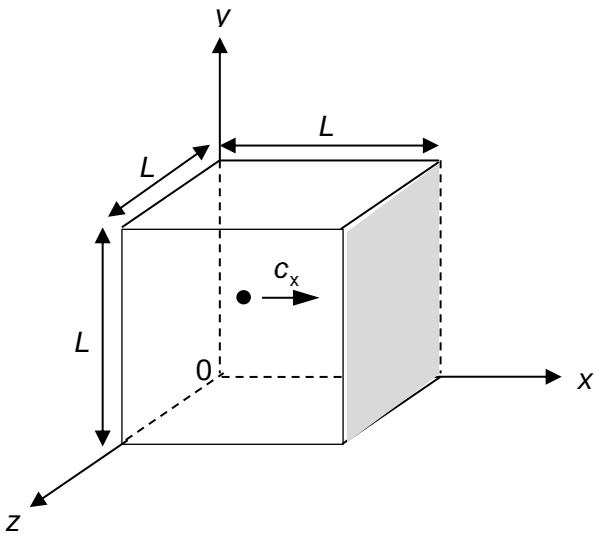


Fig. 4.1 A molecule moving with velocity c_x and colliding with the wall

- Consider one molecule of the gas with mass *m* collides elastically with the wall of the container perpendicular to the *x*−axis, with velocity c_x along the *x*−axis and rebounds.

  - Elastic collision with the wall results in a change of momentum $-2mc_x$.

  - The molecule has to travel a distance 2*L* before it next collides with the same wall. Hence, the time between collisions with the same wall is $\Delta t = \frac{2L}{c_x}$.

  - The rate of change of momentum of this molecule due to collision with the wall is

$$\frac{\Delta p}{\Delta t} = \frac{-2mc_x}{\frac{2L}{c_x}} = \frac{-mc_x{}^2}{L}.$$

- By Newton's second law of motion, the force that the wall exerts on the molecule is

$$F_{\text{on molecule}} = \frac{\Delta p}{\Delta t} = \frac{-mc_x^{\,2}}{L}.$$

The negative sign means that the force acting on the molecule is pointing away from the wall.

- By Newton's third law, the force acting on the wall by the molecule is

$$F_{\text{on wall}} = -F_{\text{on molecule}} = \frac{mc_x^{\,2}}{L}.$$

- Consider N molecules that collide elastically with the wall with velocities along the $x-$axis and rebound.

- The total force acting on the wall is $\frac{mc_{x1}^{\,2}}{L} + \frac{mc_{x2}^{\,2}}{L} + \frac{mc_{x3}^{\,2}}{L} + ... + \frac{mc_{xN}^{\,2}}{L}$, which can be written simply as $\frac{Nm\langle c_x^{\,2}\rangle}{L}$.

The notation $\langle c_x^{\,2}\rangle$ refers to the average value of $c_x^{\,2}$, which is also called the mean-square speed.

- The pressure on the wall will be

$$p = \frac{F_{\text{on wall}}}{A} = \frac{\frac{Nm\langle c_x^{\,2}\rangle}{L}}{L^2} = \frac{Nm\langle c_x^{\,2}\rangle}{V}.$$

Note that this expression relates only to the x-component of velocity of the molecules.

- For a molecule moving with velocity c, an extension of Pythagoras' theorem in three dimensions gives

$$c^2 = c_x^{\,2} + c_y^{\,2} + c_z^{\,2}.$$

Since there is a large number of molecules in random motion, the average value of the components in the $x-$direction will be the same as for those in the $y-$direction or the $z-$direction. Hence,

$$\langle c_x^{\,2}\rangle = \langle c_y^{\,2}\rangle = \langle c_z^{\,2}\rangle \text{ and } \langle c_x^{\,2}\rangle = \frac{1}{3}\langle c^2\rangle.$$

- The pressure of the gas for N molecules moving with velocity c is therefore

$$p = \frac{1}{3}\frac{Nm\langle c^2\rangle}{V},$$

where

$$pV = \frac{1}{3}Nm\langle c^2\rangle.$$

5 Kinetic energy of a molecule

(h) Candidates should be able to recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$) to new situations or to solve related problems.

5.1 Mean kinetic energy of a molecule of an ideal gas

- In a gas, the molecules are moving in all different directions with different speeds.
- Using the ideal gas equation $pV = NkT$ and the pressure of a gas $p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$, we can equate both equation such that

$$\frac{1}{3}\frac{Nm}{V}\langle c^2\rangle = \frac{NkT}{V}$$
$$\frac{1}{3}m\langle c^2\rangle = kT$$

Multiplying both sides of the equation by $\frac{3}{2}$, we have

$$\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT\ .$$

- Since the mean kinetic energy of a molecule is $\langle KE\rangle = \frac{1}{2}m\langle c^2\rangle$, we can conclude

$$\langle KE\rangle = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT\ .$$

The mean kinetic energy of a molecule of an ideal gas is therefore directly proportional to the thermodynamic temperature. Note that the expression is true only if the ideal gas is monatomic and that we only consider the translational kinetic energy of the molecules.

- The total kinetic energy of all the molecules in the monatomic ideal gas would be

$$\frac{1}{2}Nm\langle c^2\rangle = \frac{3}{2}NkT = \frac{3}{2}nRT\ .$$

5.2 Root-mean-square speed

- The mean kinetic energy of a molecule is related to the thermodynamic temperature by the expression $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$.

- The root-mean-square speed of the molecules can thus be written as

$$\sqrt{\langle c^2\rangle} = \sqrt{\frac{3kT}{m}},$$

where $\sqrt{\langle c^2\rangle}$ is sometimes written as $c_{\text{r.m.s.}}$, and referred to as the r.m.s. speed.

- Therefore, for a given mass of gas, the higher the temperature, the faster the molecules move because $c_{\text{r.m.s.}} \propto \sqrt{T}$.

- On the other hand, for a given temperature, the less massive molecules move faster than the more massive ones since $c_{\text{r.m.s.}} \propto \frac{1}{\sqrt{m}}$.

- The root-mean-square speed can also be expressed in terms of molar mass M_m.

Since $k = \frac{R}{N_A}$,

$$\sqrt{\langle c^2\rangle} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{N_A m}} = \sqrt{\frac{3RT}{M_m}}$$

Example 5

The speeds of nine particles are distributed as follows:

Speed / m s^{-1}:	1.0	2.0	3.0	4.0
Number of particles:	1	3	4	2

Determine the root-mean-square speed.

Solution:

$$c_{rms} = \sqrt{\frac{(1.0)^2 + 3(2.0)^2 + (4)(3.0)^2 + 2(4.0)^2}{1+3+4+2}} = 2.85 \text{ m s}^{-1}$$

Example 6

The molar mass of oxygen gas is 32 g mol^{-1}. For one mole of oxygen gas at a temperature of 300 K, calculate

(a) the average kinetic energy of a molecule,

(b) the total kinetic energy,

(c) the average value of the square of the speed,

(d) the root-mean-square speed.

Solution:

(a) Average kinetic energy of a molecule $= \frac{3}{2}kT$

$$= \frac{3}{2} \times \left(1.38 \times 10^{-23}\right) \times 300$$

$$\approx 6.2 \times 10^{-21} \text{ J}$$

(b) Total kinetic energy of the gas $= \frac{3}{2}NkT$

$$= \frac{3}{2} \times \left(1 \times 6.02 \times 10^{23}\right) \times \left(1.38 \times 10^{-23}\right) \times 300$$

$$\approx 3.7 \times 10^{3} \text{ J}$$

(c) Since $\frac{1}{2}m\left\langle c^2 \right\rangle = \frac{3}{2}kT$, $\left\langle c^2 \right\rangle = \frac{3kT}{m}$.

$$\left\langle c^2 \right\rangle = \frac{3kT}{\left(\frac{M_m}{N_A}\right)}$$

$$= \frac{3 \times \left(1.38 \times 10^{-23}\right) \times 300 \times \left(6.02 \times 10^{23}\right)}{\left(32 \times 10^{-3}\right)}$$

$$\approx 2.3 \times 10^{5} \text{ m}^2 \text{ s}^{-2}$$

(d) $c_{\text{r.m.s.}} = \sqrt{2.3365 \times 10^{5}}$

$$\approx 480 \text{ m s}^{-1}$$

Example 7

The r.m.s. speed of nitrogen molecules at 300 K is 500 m s^{-1}.

(a) Determine their r.m.s. speed at 600 K.

(b) Calculate the temperature at which the r.m.s. speed will be twice the value at 300 K.

Solution:

(a) Since $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$, for fixed mass of nitrogen, $c_{\text{r.m.s.}} \propto \sqrt{T}$.

$$\frac{c_{\text{r.m.s.}}}{\sqrt{600}} = \frac{500}{\sqrt{300}}$$

$$c_{\text{r.m.s.}} \approx 710 \text{ m s}^{-1}$$

Therefore, the r.m.s. speed at 600 K is 710 m s^{-1}.

(b) $$\frac{c_{\text{r.m.s.}}}{\sqrt{300}} = \frac{2c_{\text{r.m.s.}}}{\sqrt{T}}$$

$$T = 1200 \text{ K}$$

The temperature at which the r.m.s. speed will be doubled is 1200 K.

Example 8

Oxygen molecules in the Earth's atmosphere have a root-mean-square speed of about 600 m s^{-1}. Given that the relative molecular masses of oxygen and helium are 32 and 4 respectively, determine the r.m.s. speed of helium in the atmosphere.

Solution:

We assume that the helium and the oxygen are in thermal equilibrium, which means that they are at the same temperature and hence have the same mean kinetic energy.

Since $\frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$,

$$c_{\text{r.m.s.}} = \sqrt{\frac{3kT}{m}}$$

$$= \sqrt{\frac{3RT}{M_m}}$$

Since temperature is constant,

$$\frac{c_{\text{r.m.s. of oxygen}}}{c_{\text{r.m.s. of helium}}} = \frac{\sqrt{M_{\text{m of helium}}}}{\sqrt{M_{\text{m of oxygen}}}}$$

$$\frac{600}{c_{\text{r.m.s. of helium}}} = \frac{\sqrt{4}}{\sqrt{32}}$$

$$c_{\text{r.m.s. of helium}} \approx 1.7 \times 10^3 \text{ m s}^{-1}$$

Therefore, the r.m.s. speed of helium is 1.7×10^3 m s^{-1}.

5.3 Explanation of why cooling accompanies evaporation

- Evaporation is a process whereby particles in a liquid escape to become gas particles, and this takes place at the surface of the liquid and at any temperature.

- The particles that actually escape from the surface during evaporation are more energetic and move faster; otherwise they would be trapped in the liquid.

- The particles left behind in the liquid will thus be less energetic. Hence the mean kinetic energy of the remaining particles is reduced by evaporation, causing a decrease in temperature (cooling).

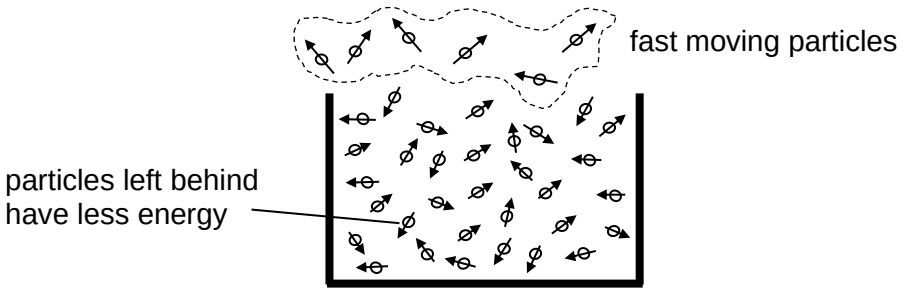


Fig. 5.1 Energetic particles escaping from the surface during evaporation

References

1 Robert Hutchings, *Bath Advanced Science Physics (2nd Edition)*, Nelson Thornes

2 Chris Mee, Mike Crundell, Brian Arnold, Wendy Brown, *International A/AS Level Physics*, Hodder Education

3 Douglas C. Giancoli, *Physics - Principles with Applications (6th Edition)*, Prentice Hall

4 Poh Liong Yong, *Physics A Level Volume 1*, Pan Pacific

5 Roger Muncaster, *A-Level Physics (4th Edition)*, Nelson Thornes

6 T Duncan, *Advanced Physics (5th Edition)*, Hodder Education