REGENT SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2020
SECONDARY FOUR EXPRESS

NAME: ______________________________ INDEX NUMBER: ________

CLASS: ________ SETTER: MS SU RY

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **15 September 2020**

2 hours 30 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

	TARGET
100	
PARENT'S SIGNATURE	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!\,r!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The roots of the quadratic equation $2x^2 - 4x + 3 = 0$ are α and β.

(i) Show that $\alpha^2 + \beta^2 = 1$. [4]

(ii) Find the quadratic equation whose roots are $\alpha^2 + 3$ and $\beta^2 + 3$. [3]

2 **(i)** In the binomial expansion of $\left(x+\frac{k}{x}\right)^7$, where k is a positive constant, the coefficients of x and x^3 are the same. Find the value of k. [6]

(ii) Using the value of k found in part (i), find the coefficient of x^7 in the expansion of $\left(1-5x^2\right)\left(x+\frac{k}{x}\right)^7$. [4]

3 Recorded values of the mass, *m* grams, of a radioactive substance, *t* hours after observations began, are shown in the table below.

t (hours)	2	4	6	8	10
m (mg)	48.2	41.5	35.7	30.8	26.5

It is known that *m* and *t* are related by the equation $m = m_0 e^{-kt}$, where m_0 and *k* are constants.

(i) Plot $\ln m$ against *t* and draw a straight line graph. [3]

Use your graph to estimate

(ii) mass of the substance when the observations began, [2]

(iii) the value of k, [2]

(iv) the time taken for the substance to lose half of its original mass. [2]

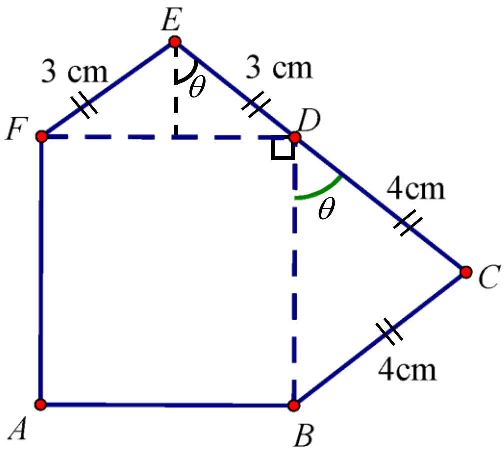
3 (i)

4 The equation of a curve is $y = 2x^2 + kx + 6 - k$, where k is a constant.

(i) Find the range of values of k for which the curve lies completely above the x-axis. [3]

(ii) In the case where $k = 2$, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

5 The diagram shows a pentagon *ABCEF* formed by a rectangle *ABDF* and two isosceles triangles *DEF* and *BCD*, where *CDE* is a straight line.
The angle *BDC* is θ, *BC* = *CD* = 4 cm, *DE* = *EF* = 3 cm.



(i) Show that *P* cm, the perimeter of the pentagon, can be expressed in the form $14+6\sin\theta+8\cos\theta$. [3]

(ii) Express *P* in the form $14+R\sin(\theta+\alpha)$ where $R>0$ and $0° < \alpha < 90°$. [3]

(iii) Find the value of θ for which $P = 23$ cm. [3]

6 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by the expression $v = 2t^2 - 5t + 2$. Find

(i) the range of time when the particle is decelerating, [3]

(ii) the time when the particle is instantaneously at rest, [2]

(iii) an expression for the displacement, s metres, in terms of t, [2]

(iv) the total distance travelled in the first 3 seconds. [5]

7 It is given that $\mathrm{f}(x)=2x^3+ax^2+bx+6$ has a factor of $(x+2)$ and leaves a reminder of 15 when divided by $(x-3)$.

(i) Find the value of a and of b. [4]

(ii) Solve $\mathrm{f}(x)=0$, leaving your answers in exact value. [4]

(iii) Hence, solve the equation $16y^3-8y^2-18y+6=0$, leaving your answers in exact value. [4]

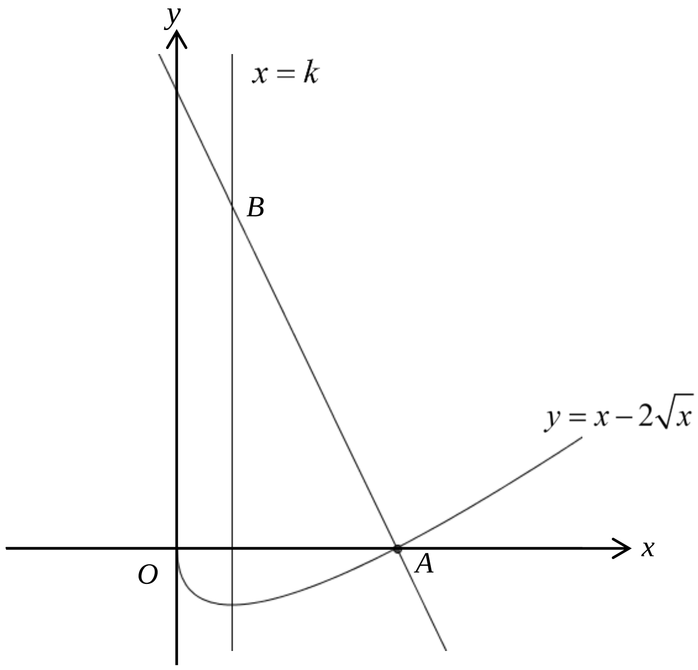
8 **(i)** Prove the identity $\tan A + \cot A = 2\operatorname{cosec} 2A$. [4]

(ii) Find all the angles between 0° and 360° which satisfy the equation $\tan A + \cot A = 5$. [5]

9 **(i)** Show that $\frac{d}{dx}(\cos^3 x - 3\cos x) = 3\sin^3 x$. [2]

(ii) Hence find $\int_{0.5}^{1} (3\sin^3 x - 2\sin x)dx$. [4]

10 The diagram below shows a curve $y = x - 2\sqrt{x}$. The curve passes through O and A. The line AB, the normal to the curve at point A, cuts the line $x = k$ at point B.



(i) Find the equation of the line AB. [4]

(ii) Given that $\frac{dy}{dx}=0$ where $x>0$, find the coordinates of *B*. [3]

(iii) Calculate the area bounded by the line $x=k$, the normal line *AB* and the curve $y=x-2\sqrt{x}$. [3]

11 A circle, centre O, has a diameter PQ where P is the point $(2, 3)$ and Q is the point $(8, 11)$.

(i) Find the coordinates of centre O and the radius of the circle. [3]

(ii) Find the equation of the circle. [1]

(iii) Show that the y-axis is a tangent to the circle. [2]

(iv) Find the equation of tangent at point Q. [3]

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