# Swiss Cottage Secondary School 2024 WA2 Practise Paper

**MATHEMATICAL FORMULAE**

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## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots\ldots + \binom{n}{r}a^{n-r}b^r + \ldots\ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

**1** Given that $\mathrm{f}(x) = \dfrac{3(3-4x)^2}{2\sqrt{3-4x}}$, find $\mathrm{f}'(x)$. [3]

**2** The total surface area of a spherical ball of ice is decreasing at a rate of $2\,\text{cm}^2/\text{s}$. Find the rate of change of the volume when its radius is 0.5 cm.

[Volume of sphere = $\frac{4}{3}\pi r^3$; total surface area of sphere = $4\pi r^2$] [5]

**3** It is given that $y = (3x+1)\sqrt{2x-1}$.

**(i)** Show that $\frac{dy}{dx}$ can be written in the form $\frac{mx+n}{\sqrt{2x-1}}$ where $m$ and $n$ are integers. [3]

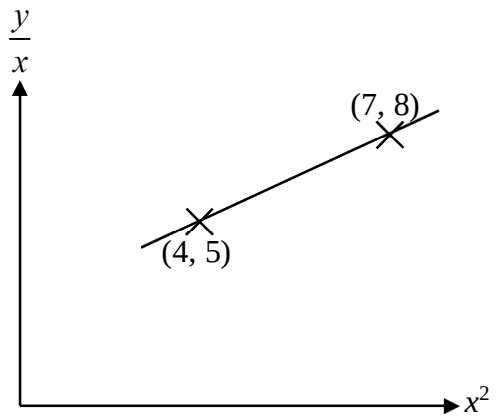
**(ii)** Show that $y$ is increasing for $x > \frac{1}{2}$. [2]

**4** A curve has the equation $y = \dfrac{2-x}{3x-4}$, $x \neq \dfrac{4}{3}$.

**(i)** Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

**(ii)** Find the coordinates of the points on the curve where the normal is parallel to the line $2y - 16x = 3$. [3]

**5** The diagram shows part of a straight line graph drawn to represent the equation $y = cx^3 + dx$.




Find the value of $c$ and of $d$. [4]

**6** The following table shows the population density, $d$, of a colony of bacteria, taken at various time, $t$, in minutes, during an experiment.

| $t$ | 16 | 32 | 48 | 64 | 80 |
|---|---|---|---|---|---|
| $d$ | 0.0364 | 0.0603 | 0.0998 | 0.165 | 0.273 |

It is believed that $d$, can under certain circumstances be modelled by $d = Ae^{kt}$, where $A$ and $k$ are constants.

**(i)** On a graph paper, plot ln $d$ against $t$ and draw a straight line graph. The vertical ln $d$-axis should have a scale of 2 cm to 0.5 unit, and the horizontal $t$-axis should start at 0 and have a scale 2 cm to 10 minutes. [3]

**(ii)** Use the graph to estimate the value of $A$ and of $k$. [3]

**(iii)** Estimate the population density at the $50^{th}$ minute. [2]

**End of Paper**