Anglo - Chinese School
(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATIONS 2011

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

Monday	**10 October 2011**	**1 h 30 min**

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

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This question paper consists of 07 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer ***all*** *the questions on the answer sheets provided. Please start each question on a new page.*

1 *[Maximum mark: 7]*

(a) Solve $\frac{23}{x}+3=\frac{500}{x^2}$, giving your answer correct to 2 decimal places. [4 marks]

(b) Find the range of values of x for which $x\,(1-x) < x^2 - 6$. [3 marks]

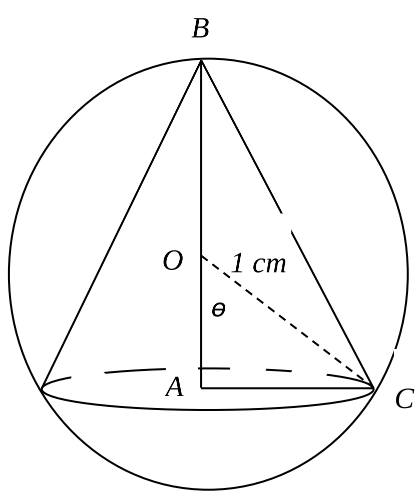
2 *[Maximum mark: 4]*

A cuboid has a square base of side $(1+\sqrt{5})$ m and a volume of $(\sqrt{5}+2)(2\sqrt{5}-1)\,\text{m}^3$. Find the height of the cuboid in the form $(a+b\sqrt{5})$.

3 *[Maximum mark: 4]*

The figure shows a right circular cone inscribed in a sphere with centre O and radius of 1 cm. The cone has a height AB and a base radius AC. Given that $\angle AOC = \theta$ radians, express the volume of the cone in terms of θ.

[Volume of a Sphere$=\frac{4}{3}\pi r^3$, *Volume of a cone*$=\frac{1}{3}\pi r^2 h$]



4 *[Maximum mark: 7]*

Singapore Airlines has a fleet of aircraft consisting of 50 aircraft of type A, 80 of type B and 70 of type C. Each type has 3 classes of seat known as Economy, Business and First Class. The table below shows the number of these seats in each of the 3 types of aircraft.

Type of Aircraft	Economy	Business	First Class
A	350	80	50
B	200	50	30
C	120	25	10

(a) Write down the two matrices whose product shows the total number of seats in each class. [2 marks]

(b) Evaluate the product of the two matrices. [2 marks]

On a typical day, each aircraft made one flight. It was found that 5%, 10% and 20% of the Economy, Business and First Class seats respectively were empty.

(c) Write down a matrix whose product with the matrix found in part (b) will give the total number of empty seats on that typical day. [1 mark]

(d) Hence, find the total number of empty seats. [2 marks]

5 *[Maximum mark: 7]*

(a) Express $y = x^2 - 4x + 9$ in the form $y = (x - p)^2 + q$. [2 marks]

(b) Hence, sketch the curve of $y = x^2 - 4x + 9$. Indicate clearly, on your diagram the coordinates of the intercept(s) and the turning point. [3 marks]

(c) The line joining the origin O to the turning point makes an angle θ with the x-axis. Calculate the angle θ. [2 marks]

6 *[Maximum mark: 10]*

During an experiment, the number n , of bacteria after t hours is given by $n = 1400(1.55)^{\frac{t}{3}}$.

(i) Find the initial number of bacteria. [1 mark]

(ii) Find the number of bacteria after 6 hours. [1 mark]

(iii) Find the time taken, in hour and minute (correct to the nearest minute), for the number of bacteria to reach 25000. [4 marks]

In another experiment, the number of bacteria is given by $n = 1500e^{-\frac{t}{3}}$

(iv) Find the time taken for the bacteria to reach the same amount in both experiments. [4 marks]

7 *[Maximum mark: 12]*

(a) Solve each of the following equations:

(i) $5^{3x}\left(25^{x-1}\right) = \frac{1}{125}$; [3 marks]

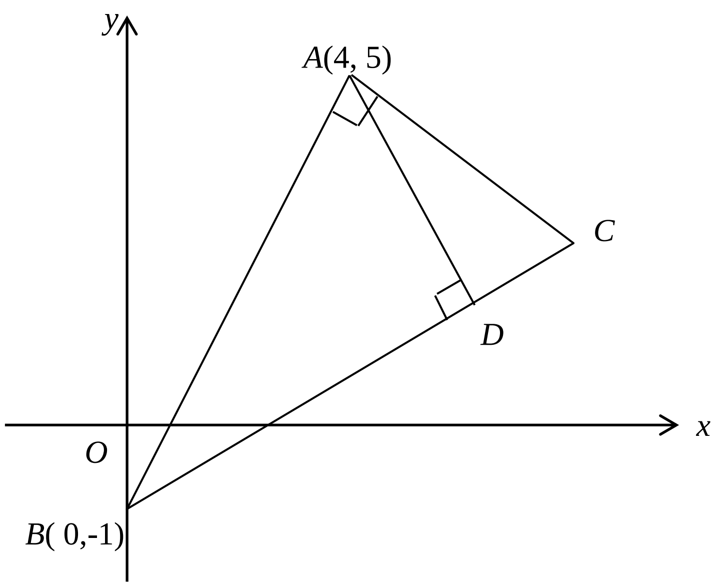
(ii) $\log_{\sqrt{2}}(2-x) = \log_2(5-4x)$. [5 marks]

(b) By using an appropriate substitution, solve $9^{x+1} + 3^{x+1} = 2$. [4 marks]

8 *[Maximum mark: 9]*

Solutions to this question by accurate drawing will not be accepted.

The diagram, which is not drawn to scale, shows a right-angled triangle *ABC* in which $\angle BAC = 90^{\circ}$. The coordinates of *A* and *B* are (4, 5) and $(0, -1)$ respectively.



Given that the gradient of *BC* is $\frac{1}{2}$ and D is the foot of the perpendicular from *A* to *BC*,

find

(i) the equation of *BC* , [1 mark]

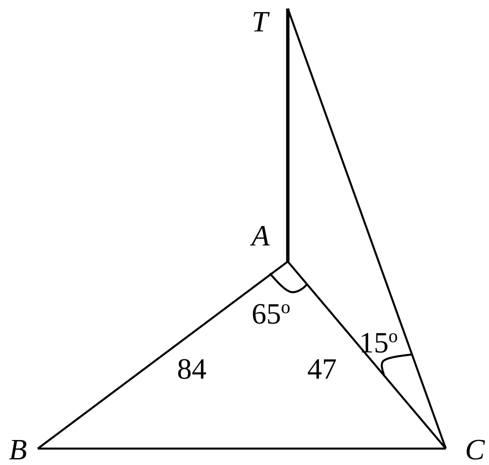
(ii) the equation of *AC* , [3 marks]

(iii) the coordinates of *C*, [2 marks]

(iv) the coordinates of *D*. [3 marks]

9 *[Maximum mark: 12]*

In the following diagram, *ABC* represents a horizontal triangular garden and *AT* represents a vertical pole . A walking path runs along the edge of the garden *BC*. It is also given that *AB* = 84 m, *AC* = 47 m and angle *BAC* = 65°.



(a) The angle of elevation of the top of the pole when viewed from *C* is 15°. Calculate the height of the pole. [2 marks]

(b) Calculate the length of the walking path *BC* [3 marks]

(c) Calculate the area of the garden *ABC*. [2 marks]

(d) Calculate the shortest distance from *A* to the walking path *BC*. [2 marks]

(e) Calculate the greatest angle of elevation of the top of the pole when viewed from any point on the walking path. [3 marks]

10 *[Maximum mark: 8]*

Answer the whole of this question on a sheet of graph paper.

The table below gives some values of x and the corresponding values of y, correct to one decimal place, where $y = 5 - \frac{x^2}{10} - \frac{4}{x}$.

x	0.5	1	2	3	4	5	7	9
y	-3.0	0.9	2.6	2.8	p	1.7	-0.5	-3.5

(a) Find the value of p. [1 mark]

(b) Using a scale of 2cm to represent 1 unit on each axis, draw a horizontal x-axis for $0.5 \le x \le 9$ and a vertical y-axis for $-5 \le y \le 3$. On your axes, plot the points given in the table and join them with a smooth curve. [3 marks]

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 2$. [2 marks]

(d) By drawing a suitable straight line, find the range of values of x in the interval $0.5 \le x \le 9$ for which $-\frac{x^2}{10} - \frac{4}{x} \ge -\frac{5}{2} - \frac{x}{2}$. [2 marks]

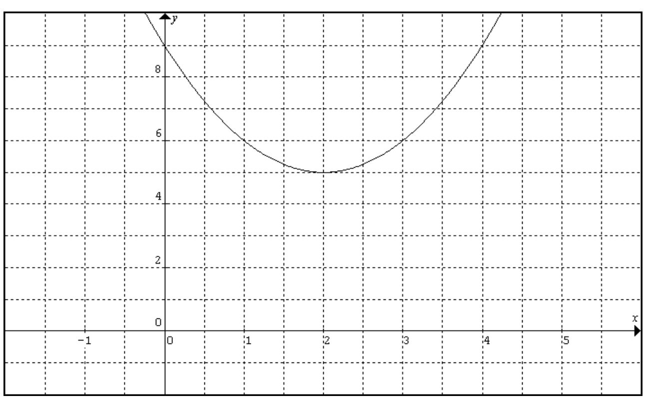
End of Paper

Answers

1a	9.63, -17.30
1b	$x < \frac{-3}{2}, x > 2$
2	$\frac{9}{8} + \frac{1}{8}\sqrt{5}$
4a	$(50\ 80\ \ 70)\begin{pmatrix} 350 & 80 & 50 \\ 200 & 50 & 30 \\ 120 & 25 & 10 \end{pmatrix}$
4b	(41900 9750 5600)
4c	$(41900\ 9750\ \ 5600)\begin{pmatrix} 0.05 \\ 0.10 \\ 0.20 \end{pmatrix}$
4d	(4190)
5a	$y = (x-2)^2 + 5$
5b	Diagram on right side
5c	68.2^o
6a	1400
6b	3363 or 3360 –3sf
6c	19 hr 44 min
6d	0.144 hr
7ai	$x = \frac{-1}{5}$
7aii	1, -1
7b	-1
8i	$y = \frac{1}{2}x - 1......LineBC....$
8ii	$so\ \ \mathrm{y} = \frac{-2}{3}x + \frac{23}{3}..$
8iii	$C(\frac{52}{7}, \frac{19}{7})$
8iv	$D(\frac{28}{5}, \frac{9}{5}) or (5.6, 1.8)$
9a	12.6 m
9b	77.0 m
9c	1790 m^2
9d	46.5 m
9e	15.2^o
10a	2.4
10c	0.60
10d	$Range: 1.34 \le \mathrm{x} \le 7.60$

Solutions for 2011 3IP Core Maths Paper 2 Final Exam

Q1a	$\frac{23}{x}+3=\frac{500}{x^2}$ $3x^2+23x-500=0$ $x=\frac{-23\pm\sqrt{23^2-4(3)(-500)}}{2\times 3}$ $=\frac{-23\pm\sqrt{6529}}{6}$ $=9.63;-17.30$
Q1b	$x(1-x)<x^2-6.$ $0<2x^2-x-6$ $0<(2x+3)(x-2)$ $x<\frac{-3}{2}\ or\ \ x>2$
2	$(\sqrt{5}+2)(2\sqrt{5}-1)=(1+\sqrt{5})^2h$ $2(5)-\sqrt{5}+4\sqrt{5}-2=(1+2\sqrt{5}+5)h$ $h=\frac{8+3\sqrt{5}}{6+2\sqrt{5}}$ $h=\frac{8+3\sqrt{5}}{6+2\sqrt{5}}\times\frac{6-2\sqrt{5}}{6-2\sqrt{5}}$ $=\frac{48-16\sqrt{5}+18\sqrt{5}-6(5)}{36-4(5)}$ $=\frac{18+2\sqrt{5}}{16}=\frac{9}{8}+\frac{1}{8}\sqrt{5}$
3	$\sin\theta=\frac{AC}{1}$ $AC=\sin\theta$ $\cos\theta=\frac{OA}{1}$ $OA=\cos\theta$ $Vol\text{ of cone}=\frac{1}{3}\pi(AC)^2(1+OA)$ $=\frac{\pi}{3}(\sin^2\theta)(1+\cos\theta)$
4a	

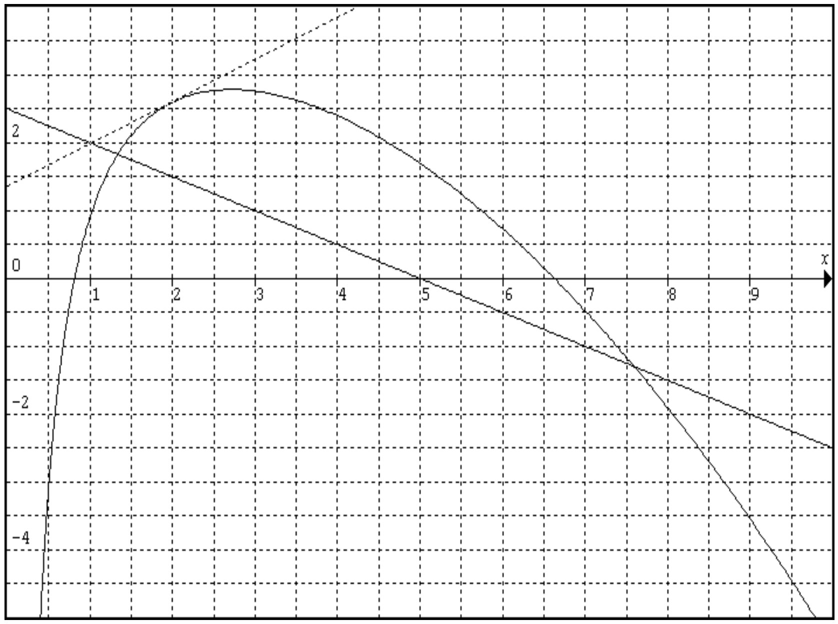
	A B C Eco Busi First $(50\ 80\ 70)\begin{pmatrix}350 & 80 & 50\\ 200 & 50 & 30\\ 120 & 25 & 10\end{pmatrix}$ OR $\begin{pmatrix}Eco\\ Bus\\ First\end{pmatrix} - \begin{pmatrix}350 & 200 & 120\\ 80 & 50 & 25\\ 50 & 30 & 10\end{pmatrix}\begin{pmatrix}50\\ 80\\ 70\end{pmatrix} - \begin{pmatrix}A\\ B\\ C\end{pmatrix}$
4b	**1 x 3 and 3 x 3 yields 1x3 matrix---matrix multiplication** (41900 9750 5600) OR **3 x 3 and 3 x 1 yields 3x1 matrix---matrix multiplication** $\begin{pmatrix}41900\\ 9750\\ 5600\end{pmatrix}$
4c	$(41900\ 9750\ \ 5600)\begin{pmatrix}0.05\\ 0.10\\ 0.20\end{pmatrix}$ OR $(0.05\ \ 0.10\ \ 0.20)\begin{pmatrix}41900\\ 9750\\ 5600\end{pmatrix}$
4d	Yield a 1 x 1 matrix for both methods (4190)
5a	$a)\ y = x^2 - 4x + 9$ $= x^2 - 4x + 2^2 + 9 - 2^2$ $y = (x-2)^2 + 5$
5b	
5c	$m = \frac{5}{2}$ $\tan\theta = \frac{5}{2}$ $\theta = 68.2°$

6	$i) n = 1400(1.55)^{\frac{t}{3}}$ $t = 0, \quad \mathrm{n} = 1400$
	$ii) n = 1400(1.55)^{\frac{6}{3}}$ $\mathrm{n} = 3363 \approx 3360$
	$iii) n = 1400(1.55)^{\frac{t}{3}}$ $25000 = 1400(1.55)^{\frac{t}{3}}$ $17.85714 = (1.55)^{\frac{t}{3}}$ $\ln(17.85714) = \ln(1.55)^{\frac{t}{3}}$ $\ln(17.85714) = \frac{t}{3}\ln(1.55)$ $t = 19.731 hr$ $t = 19 hr\, 44 \min$
	$iv) 1400(1.55)^{\frac{t}{3}} = \frac{1500}{e^{\frac{t}{3}}}$ $(1.55e)^{\frac{t}{3}} = \frac{15}{14}$ $4.213337^{\frac{t}{3}} = 1.0714286$ $\frac{t}{3}\lg(4.213337) = \lg(1.0714286)$ $t = 0.144 hr$

7a i)	$5^{3x}\left(25^{x-1}\right)=\frac{1}{125}$ $5^{3x}\times 5^{2(x-1)}=5^{-3}$ $5^{3x+2x-2}=5^{-3}$ $5x-2=-3$ $x=\frac{-1}{5}$
7a ii)	$(ii)\quad \log_{\sqrt{2}}(2-x)=\log_2(5-4x).$ $\frac{\log_2(2-x)}{\log_2 2^{\frac{1}{2}}}=\log_2(5-4x)$ $2\log_2(2-x)=\log_2(5-4x)$ $(2-x)(2-x)=5-4x$ $4-4x+x^2=5-4x$ $x^2=1$ $x=1.or\ x=-1$
7b	$9^{x+1}+3^{x+1}=2$ $(3^2)^{x+1}+3^x\times 3=2.....Let\ \ 3^x=m$ $9m^2+3m-2=0$ $(3m-1)(3m+2)=0$ $m=\frac{1}{3}or\ \ \mathrm{m}=\frac{-2}{3}$ $3^x=3^{-1}$ or $3^{\mathrm{x}}=\frac{-2}{3}(NA)$ $x=-1$

8(i) **8ii)**	$y=\frac{1}{2}x+c \Rightarrow y=\frac{1}{2}x-1......LineBC....$ $m_{AB}=\frac{5-(-1)}{4-0}=\frac{3}{2}$ $m_{AC}=\frac{-2}{3}$ $y=\frac{-2}{3}x+c$ $use(4,5).....5=\frac{-2}{3}(4)+c$ $c=\frac{23}{3}$ $so\ \ \mathrm{y}=\frac{-2}{3}x+\frac{23}{3}..........line\ AC$
iii)	$y=\frac{1}{2}x-1-----eqn1$ $y=\frac{-2}{3}x+\frac{23}{3}-----eqn2$ $solve\ \frac{-2}{3}x+\frac{23}{3}=\frac{1}{2}x-1$ $x=\frac{52}{7}, y=\frac{19}{7}$ $C(\frac{52}{7},\frac{19}{7})$
iv)	$m_{AD}=-2$ $use(4,5)......5=-2(4)+c$ $c=13$ $Line\ AD....y=-2x+13$ $\mathrm{P}revios\ \mathrm{line\ BC}...y=\frac{1}{2}x-1$ $Solve\,\mathrm{simult\ eqn:}\ \ \mathrm{D}(\frac{28}{5},\frac{9}{5})\ \ \mathrm{or}\,(5.6,1.8)$

9a	$\tan 15^\circ = \frac{AT}{47}$ $AT = 12.5936$ $AT = 12.6m$
9b	$BC^2 = 84^2 + 47^2 - 2(84)(47)\cos 65$ $BC^2 = 7056 + 2209 - 3336.994$ $BC = \sqrt{5928.006}$ $BC = 77.0m$
9c	$Area = \frac{1}{2}(84)(47)\sin 65$ $= 1789.05$ $\approx 1790m^2$
9d	$Area = \frac{1}{2}(BC)(d) = 1789.05$ $d = 46.5m$
9e	$\tan x = \frac{12.6}{46.469}$ $x = \tan^{-1}(0.271148)$ $= 15.2°$
10a)	$p = 5 - \frac{4^2}{10} - \frac{4}{4}$ $= 2.4$

b)	
c)	**Tangent drawn** $grad = \frac{y_2 - y_1}{x_2 - x_1} = 0.60$
d)	$-\frac{x^2}{10} - \frac{4}{x} \geq -\frac{5}{2} - \frac{1}{2}x$ *Plus* 5 to both sides $-\frac{x^2}{10} - \frac{4}{x} + 5 \geq -\frac{5}{2} - \frac{1}{2}x + 5$ $y = \frac{5}{2} - \frac{1}{2}x \ldots draw$ *Range*: $1.34 \leq x \leq 7.60$