# Anglo - Chinese School

(Independent)

## FINAL EXAMINATION 2018

## YEAR 3 INTEGRATED PROGRAMME

## CORE MATHEMATICS
## PAPER 2

**MONDAY** **9th October 2017** **1 hour 30 minutes**

### INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

### INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.*

*Answer all the questions on the answer sheets provided. Begin each question on a new page.*

**1** *[Maximum mark: 6]*

**(i)** Express $\frac{y-x}{x^2-xy-2y^2}+\frac{2}{3(x+y)}$ as a single fraction in its simplest form.

[3]

$$\frac{y-x}{(x-2y)(x+y)}+\frac{2}{3(x+y)}$$

Students failed to factorize the denominator. They should **OBSERVE** that the other fraction has $x+y$ as denominator. Hence one of the factor must be $x+y$

$$=\frac{3y-3x+2(x-2y)}{3(x-2y)(x+y)}$$

$$=\frac{3y-3x+2x-4y}{3(x-2y)(x+y)}$$

$$=\frac{-y-x}{3(x-2y)(x+y)}$$

$$=\frac{1}{3(2y-x)}$$

**(ii)** Hence or otherwise, find the value of $x$ when $w = 1.25$ and $y = 0.03$ if

$$\frac{y-x}{x^2-xy-2y^2}+\frac{2}{3(x+y)}=w.$$

[3]

$$1=3w(2y-x)$$

$$\frac{1}{3w}=2y-x$$

$$x=2y-\frac{1}{3w}$$

$$x=2(0.03)-\frac{1}{3(1.25)}$$

$$x=-0.207$$

**2** *[Maximum mark: 13]*

(a) Expand and simplify $3(y-x)+4[2x-y-2(3x-4y)]$.

[3]

$3y-3x+4[2x-y-6x+8y]$

$=3y-3x+4(-4x+7y)$

$=3y-3x-16x+28y$

$=31y-19x$

**(a)** Solve the equation $\sqrt{11x^2+45}=4x$.

[3]

$11x^2+45=16x^2$

$5x^2=45$

$x^2=9$

$x=\pm 3$ (reject $-3$)

Students need to learn how to check which answer should be rejected

**(b)** The solution of $x\sqrt{5}=x\sqrt{3}+\sqrt{48}$ is $a+b\sqrt{15}$. Find the values of the integers $a$ and $b$.

[4]

$x\sqrt{5}-x\sqrt{3}=4\sqrt{3}$

$x(\sqrt{5}-\sqrt{3})=4\sqrt{3}$

$x=\frac{4\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$x=\frac{4\sqrt{3}}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$x=\frac{4\sqrt{15}+12}{2}$

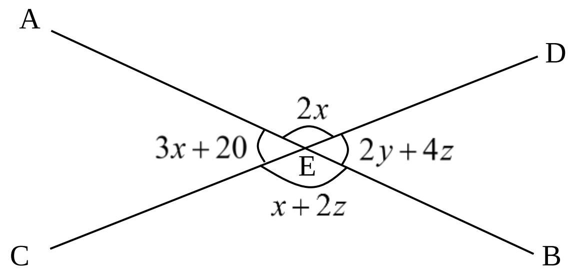
$x=6+2\sqrt{15}$

Very poorly attempted.

- Students need to realise that $\sqrt{48}$ can be simplified to a simpler surd first - $4\sqrt{3}$
- Common practise to shift all $x$ terms to one side of the equation

**3** *[Maximum mark: 9]*

(a) AB and CD are straight lines intersecting at the point E. The angles between the lines are shown in the diagram below.




Find the value of $x+y-z$. [4]

$3x+20+2x=180$

$x=32$

$3x+20+x+2z=180$

$z=16$

$x+2z+2y+4z=180$

$y=26$

$x+y-z=32+26-16=42$

Poorly attempted by students too.
Many cannot see that angle on a straight line is 180 degree.
They form simultaneous equations using "opposite angles are equal" and get stuck along the way.

(b) Two squares have a combined area of 300 m$^2$. The sum of the perimeter of the squares is 80 m. Find the dimensions of the squares by forming a pair of simultaneous equations and solving them.

[5]

$x^2 + y^2 = 300$

$x + y = 20$

$x = 20 - y$

Sub $x = 2 - y$ into (1)

$(20 - y)^2 + y^2 = 300$

$400 - 40y + y^2 + y^2 = 300$

$2y^2 - 40y + 100 = 0$

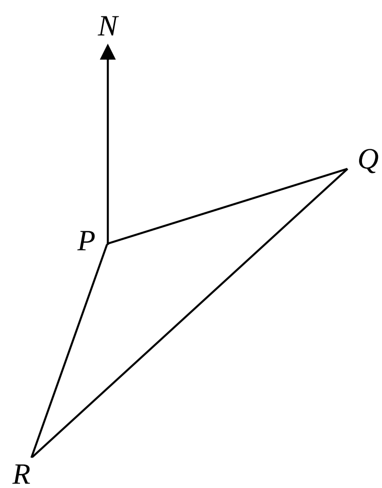
$y = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(2)(100)}}{2(2)}$

$y = 2.93$ or $y = 17.1$

$x = 17.1$ or $x = 2.93$

**4** *[Maximum mark: 10]*

In the diagram, the bearing of $Q$ and $R$ from $P$ are 058° and 208° respectively. Given that $QR$ = 125 m and the bearing of $R$ from $Q$ is 225°,



**(a)** find the bearing of P from R,

[2]

$$360 - 208 = 152$$

$$180 - 152 = 28$$

**(b)** find the length of PR,

[3]

$$360 - 122 - 225 = 13$$

$$\frac{PR}{\sin 13} = \frac{125}{\sin 150}$$

$$PR = 56.2$$

**(c)** calculate the shortest distance from P to QR.

[3]

$$180 - 150 - 13 = 17$$

$$\sin 17 = \frac{x}{56.2}$$

$$x = 16.4$$

**(d)** A vertical tower of 30 m is built at $P$. Find the largest angle of elevation of the top of the tower when a person walks from R to Q.

[2]

$$\tan \theta = \frac{30}{16.4}$$

$$\theta = 61.3^{o}$$

**5** *[Maximum mark: 13]*

(a) Let $y = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$. Given that $y$ can be written in the form $y = \frac{\ln ax}{\ln b}$, write down the value of $a$ and of $b$.

[3]

$$y = \log_3 \left(\frac{x}{2}\right)(16)\left(\frac{1}{4}\right)$$

$$y = \log_3(2x)$$

$$y = \frac{\ln 2x}{\ln 3}$$

(b) Given that $p = \lg x$, $q = \lg y$, $r = \lg z$. Write $\lg\left(\frac{y^2\sqrt{x}}{z^4}\right)$ in terms of $p$, $q$ and $r$.

[3]

$$\lg y^2 + \lg \sqrt{x} - \lg z^4$$

$$= 2\lg y + \frac{1}{2}\lg x - 4\lg z$$

$$= 2q + \frac{1}{2}p - 4r$$

(c) Solve the following equations:

(i) $13^{2p-3} = 6$

[3]

$$\lg 13^{2p-3} = \lg 6$$

$$2p - 3 = \frac{\lg 6}{\lg 13}$$

$$2p = \frac{\lg 6}{\lg 13} + 3$$

$$p = 1.85$$

(ii)

$\ln(2e^x + 3) = 2x$

[4]

$2e^x + 3 = e^{2x}$

Let $y = e^x$

$2y + 3 = y^2$

$y^2 - 2y - 3 = 0$

$(y - 3)(y + 1) = 0$

$y = 3$

$e^x = 3$

$x = \ln 3$

$x = 1.10$

**6** *[Maximum mark: 12]*

(a) Given that the equation $x^2 - px + p + 3 = 0$ has real and distinct roots and the equation $(p + 1)x^2 + 4px = 8x - 2p$ has no real roots, find the possible value(s) of $p$ if $p$ is an integer.

[7]

$(-p)^2 - 4(1)(p + 3) > 0$

$p^2 - 4p - 12 > 0$

$(p - 6)(p + 2) > 0$

$p < -2$ or $p > 6$

Students have difficulties solving quadratic inequality.
Many solved it like an equation, which leads to a wrong final inequality.

$(4p - 8)^2 - 4(p + 1)(2p) < 0$

$p^2 - 9p + 8 < 0$

$(p - 8)(p - 1) < 0$

$1 < p < 8$

Hence,

$6 < p < 8$

(b) The equation $kx^2 - k^2x = x + k - 4$, where $k$ is a constant, has roots which are reciprocal of each other. Find the value of $k$.

[5]

$kx^2 - k^2x - x + 4 - k = 0$

$kx^2 - (k^2 + 1)x + 4 - k = 0$

Let one of the root be $\alpha$

The other root will be $\frac{1}{\alpha}$

$(\alpha)\left(\frac{1}{\alpha}\right) = \frac{4-k}{k}$

$1 = \frac{4-k}{k}$

$k = 4 - k$

$2k = 4$

$k = 2$

**7** *[Maximum mark: 15]*

**Answer the whole of this question on a sheet of graph paper.**

The variables $x$ and $y$ are connected by the equation $y = 2x + \frac{5}{x}$. Some corresponding values of $x$ and $y$ are given in the following table.

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 4 | 5 | 6 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 7.0 | 6.3 | $a$ | 7.0 | 7.7 | 9.3 | $B$ | 12.8 | 16.6 |

**(a)** Calculate the value of $a$ and of $b$.

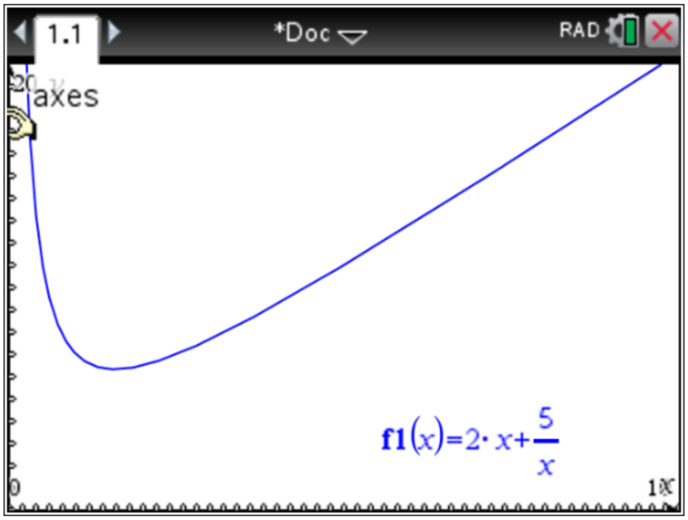
[2]

$a = 6.5$

$b = 11$

**(b)** Taking 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical axis, draw the graph of $y = 2x + \frac{5}{x}$ for $1 \le x \le 8$.

[4]




**(c)** By drawing a suitable line on the graph, solve the equation $3x + \frac{5}{x} - 10 = 0$.

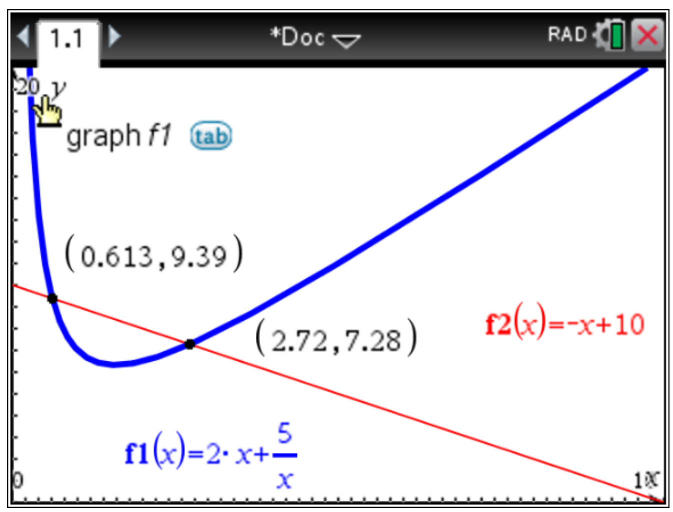
[3]

$$2x + \frac{5}{x} = -x + 10$$

$$y = -x + 10$$

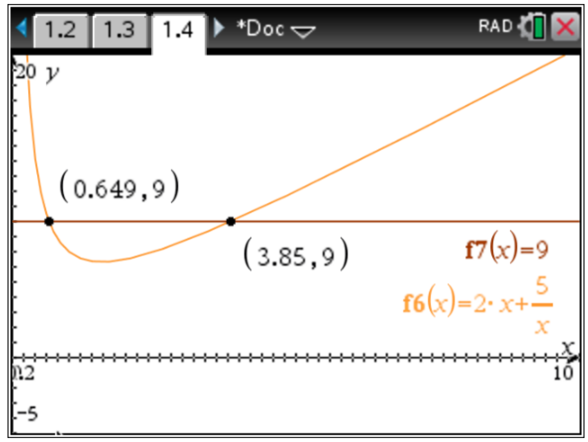
$$x = 2.72$$

A $y = -x + 10$ line must pass through 10 at the y-axis. Many attempted to draw the line but did not even make it pass through 10 as the y-intercept.



**(d)** When y = 9, find the value of x.

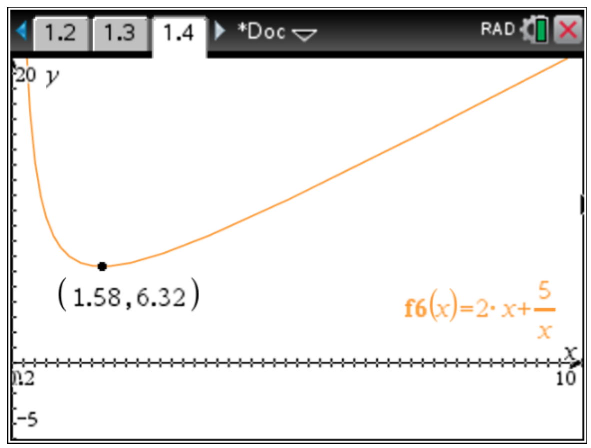
x = 3.85




**(e)** Minimum value of y and corresponding value of x

Minimum value of y = 6.32

When x = 1.58




**(f)** The point $P$ on the curve has the same gradient as the line $2y - x - 5 = 0$. Find the $x$ −coordinate of $P$.

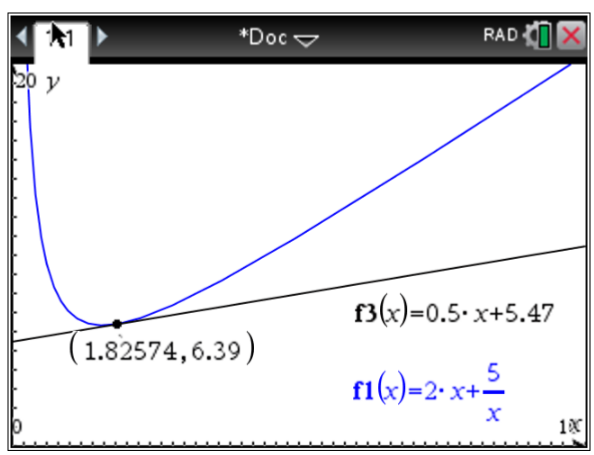
[2]

$2y = x + 5$

$y = \frac{1}{2}x + 2.5$

$x = 1.83$

Students need to show parallel lines, to indicate they have moved the line upwards.



**8** *[Maximum mark: 5]*

Triangle PQR has $p = 8.1$ cm, $q = 12.3$ cm and area 15 cm$^2$. Find the largest possible perimeter of triangle PQR.

[5]

$\frac{1}{2}(8.1)(12.3)\sin\theta = 15$

$\sin\theta = 0.301$

$\theta = 162$

$r^2 = 8.1^2 + 12.3^2 - 2(8.1)(12.3)\cos 162$

$r = 20.2$

Perimeter = $20.2 + 8.1 + 12.3$

Perimeter = $40.6$

Many students took the first answer they obtain when they take inverse sine.

They should have considered CAST and use the answer in the second quadrant as this will result in a longer length, hence largest possible perimeter.