

Candidate Index Number

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Anglo - Chinese School (Independent)



FINAL EXAMINATIONS 2019 YEAR 3 INTEGRATED PROGRAMME CORE MATHEMATICS PAPER 1

WEDNESDAY

2nd OCTOBER 2019

1 h 30 min

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

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This paper consists of 17 printed pages.

[Turn over

Answer **all** the questions in the spaces provided.

(b) If $a^2 + b^2 = 2$ and $c^2 + d^2 = 3$, find the value of $(ac + bd)^2 + (ad - bc)^2$. [4 marks]

.....[Working may be continued next page]

[illegible]

2

[Maximum mark: 10]

(a) Simplify the following:

(i) $\frac{1}{1+a^n} + \frac{1}{1+a^{-n}}$, [3 marks]

(ii) $\frac{243^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}.$ [3 marks]

(b) Given that $x = \sqrt{5} + 2$, express $\frac{4}{x^2 + 1}$ in the form of $a + b\sqrt{5}$. [4 marks]

.....[Working may be continued next page]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or additional markings.

3

[Maximum mark: 4]

- (a) Solve the simultaneous equations $y = x^3$ and $y = \frac{32}{x^2}$.

[2 marks]

(b) On the same axes, draw the graphs of $y = x^3$ and $y = \frac{32}{x^2}$, labelling the point of intersection clearly on your sketch. [2 marks]

[2 marks]

[illegible]

4 [Maximum mark: 8]

(a) Given that $-8 \leq a \leq 2$ and $-1 \leq b \leq 8$, and a and b are integers.
Find

(i) the smallest possible value of $a^2 - b$, [1 mark]

(ii) the greatest possible value of $\frac{b}{(a+4)^2 + 6}$. [2 marks]

(b) Solve the inequality $12 - 3x < 3x - 4 \leq 6x - 14$ and hence state the smallest even number that satisfies the inequality. [5 marks]

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[Maximum mark: 12]

(a) Find the values of y when $\left(\frac{1}{2}\right)^{3-y^2} = 8^{\frac{2}{3}} \div 4^{2y}$. *[4 marks]*

(b) Find the value of x given that $\frac{3^{14} - 3^{11}}{26} = \sqrt{3^{x-1}}$. [3 marks]

(c) Solve the equation $4^{\frac{2}{x}} + 4 = 5 \times 4^{\frac{1}{x}}$. [5 marks]

.....[Working may be continued next page]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6

[Maximum mark: 7]

A company uses the function $P = -n^2 + 8n - 12$ to model its profits, in thousands of dollars, when n hundred units of its products are manufactured.

- (a) Factorise $P = -n^2 + 8n - 12$ completely. [1 mark]
- (b) Hence, sketch the graph of $P = -n^2 + 8n - 12$, clearly labelling the coordinates of the axes-intercepts and turning point. [4 marks]
- (c) State which of the following statements is true. Justify your answer. [2 marks]
- (A) The company's profit is minimized when 200 units are produced.
- (B) The company's profit is maximized when 600 units are produced.
- (C) For the company to make a profit, it must produce more than 100 units but less than 700 units.
- (D) For the company to make a profit, it must produce more than 200 units but less than 600 units.

.....[Working may be continued next page]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

[illegible]

9 [Maximum mark: 8]

Solve the following equations:

(a) $\log_4(x+12) \times \log_x 2 = 1$ [4 marks]

(b) $3^{1-\log_3 x} = 9\left(3^{\log_3 x - 1}\right)$ [4 marks]

[illegible]

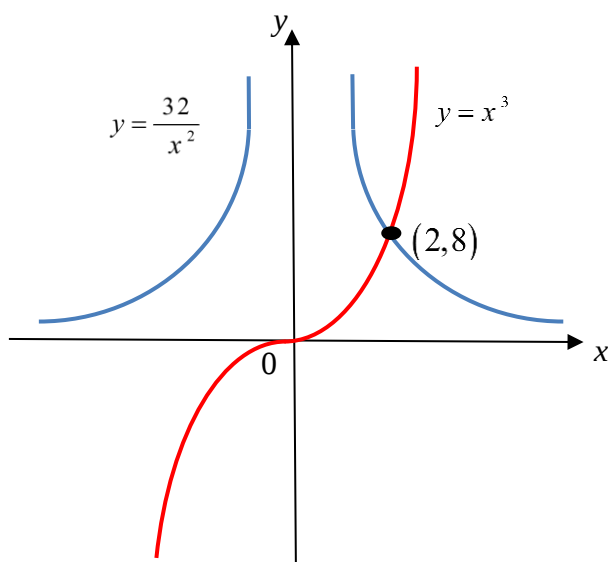
..[Working may be continued next page]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

End of Paper 1

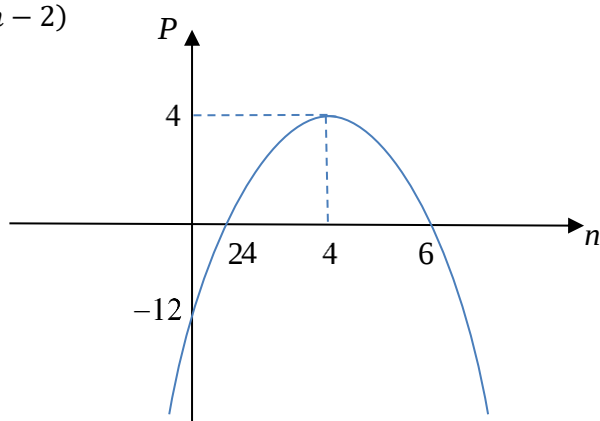
Answers

- 1 (ai) $4n^2$ (aii) 16 (b) 6
 2 (ai) 1 (aii) 9 (b) $2 - \frac{4}{5}\sqrt{5}$
 3 (a) $x = 2$ and $y = 8$ $\therefore$ Point of intersection is (2,8)
 (b)



- 4 (ai) -8 (aii) $\frac{4}{3}$ (b) $x \geq \frac{10}{3} \Rightarrow$ smallest even number is 4.
 5 (a) $y = -5$ or 1 (b) $x = 23$ (c) $x = 1$

- 6 (a) $-(n-6)(n-2)$
 (b)



- (c) [D] The curve is above the n -axis (ie $P > 0$) between $n = 2$ and $n = 6$, which means profit is made in this interval of n .
 7 (a) $y = -\frac{3}{7}x + \frac{29}{7}$ (b) $y = -x + 13$
 (c) $X\left(\frac{31}{2}, -\frac{5}{2}\right)$ (d) $AX: DX = 5:3$
 8 (ai) $(x-2)^2 + 6$ (aii) $-3 < x < \frac{1}{2}$
 (bi) $\alpha + \beta = \frac{7}{10}$ $\alpha \beta = \frac{1}{10}$ (bii) $10x^2 - 7x + 1 = 0$
 9 (a) $x = 4$ or -3 (NA) (b) $x = 1$