

Anglo - Chinese School

(Independent)



FINAL EXAMINATION 2023

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS

PAPER 2

MONDAY

9th October 2023

1 hour 30 minutes

ADDITIONAL MATERIALS:

Answer Paper (6 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.



This question paper consists of **5** printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. [Maximum mark: 8]

(a) Make y the subject of the formula $\frac{xy+2y}{x} = \frac{3}{5}x$. [2]

(b) Simplify $\left(\frac{1}{a+1} - \frac{2a}{a^2-1}\right)\left(\frac{1}{a} - 1\right)$, expressing your answer as a single fraction in its simplest form. [4]

(c) Evaluate $\frac{\lg[(6.82 \div 1.55 \times 10^{-2})]}{32.8}$, leaving your answer correct to 2 significant figures. [2]

2. [Maximum mark: 8]

Julian paid \$168 for x tickets to a theme park.

(a) Write down an expression, in terms of x , for the cost of one ticket in dollars. [1]

(b) During the peak season, he could buy 7 less tickets with the same amount of money. Write down an expression, in terms of x , for the cost of one ticket during the peak season. [1]

(c) Given that the increase in the cost of one ticket during the peak season is \$2, form an equation in x and show that it reduces to $x^2 - 7x - 588 = 0$. [3]

(d) What is the maximum number of tickets he can purchase with \$200 during the peak season? [3]

3. [Maximum mark: 11]

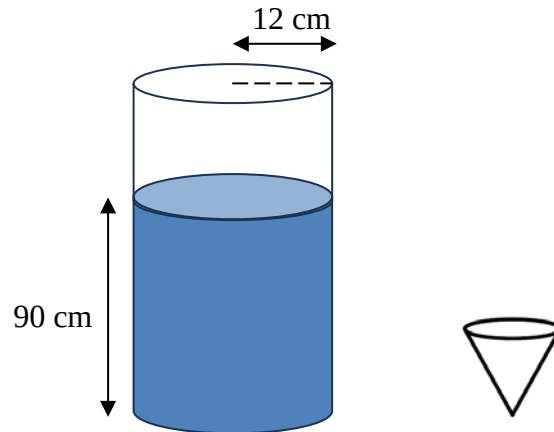
(a) Find the range of values of p for which the expression $x(x+6) - 2px + 34 - 6p$ is always positive for all real values of x . [4]

(b) Solve the equation $2x - 11\sqrt{x} + 12 = 0$. [4]

(c) Given that $x^2 + x = 7 + \sqrt{7}$, find the exact values of x , leaving your answers in the simplest form. [3]

4. [Maximum mark: 6]

During a workshop, water dispenser in the shape of a right circular cylinder of radius 12 cm, is provided for 250 participants.



[Diagrams are not drawn to scale]

The initial height of water in the container is 90 cm.

If each participant drinks once from the dispenser using a conical cup of diameter 5 cm and water level at 7 cm, determine if the water in the dispenser is enough for all the participants.

Show your workings clearly.

5. [Maximum mark: 9]

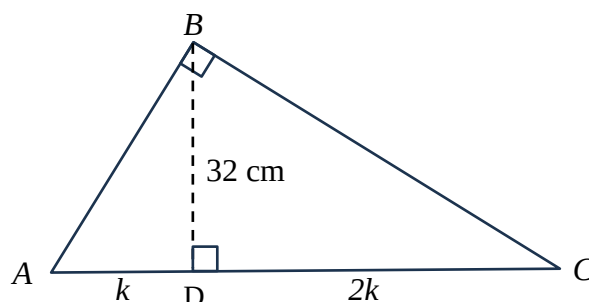
(a) The coordinates of points A and B are $(2, 5)$ and $(0, 1)$ respectively.

(i) Find the equation of the straight line that passes through the points $A(2, 5)$ and $B(0, 1)$. [2]

(ii) A quadratic curve has a maximum point at $A(2, 5)$ and it passes through the point $B(0, 1)$. Find the equation of the quadratic curve. [2]

(b) In the diagram, ABC is a right angled triangle and D is a point on AC such that BD is perpendicular to AC .

Given that $AD = k$ cm, $DC = 2k$ cm and $BD = 32$ cm, find the value of k . [5]



6. [Maximum mark: 10]

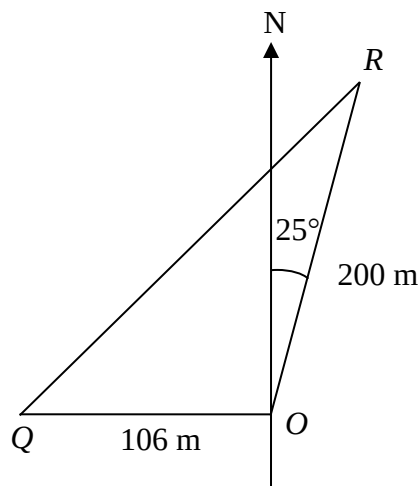
Solve the following equations

(a) $7e^x + 10e^{-x} = 37$, [5]

(b) $\log_8(x) + \log_4(x) = \frac{10}{3}$. [5]

7. [Maximum mark: 14]

A farmer held his ducks in a triangular pen QOR where Q is due west of O , $OQ = 106$ m, R is a point 200 m away from O , and its bearing from O is 025° .



(a) Find

(i) the bearing of O from R , [2]

(ii) the perimeter of the pen QOR , [4]

(iii) $\angle RQO$. [3]

(b) A signboard carrying the farm name stands vertically at point O .

The top of the signboard, T , is 7.5 m from the ground.

Is it possible to find a point, K , on RQ such that the angle of elevation of T from K is 15° ? Justify your answer with detailed working. [5]

8. [Maximum mark: 14]

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{1}{4}(20x - x^3)$.

Some corresponding values of x and y are given in the table below:

x	0	1	1.5	2	2.5	3	3.5	4	5
y	0	4.8	a	8	8.6	b	6.8	4	-6.3

(a) Find the value of a and of b . [2]

(b) Taking 2 cm to represent 1 unit on the x -axis and 1 cm to represent 1 unit on the

y -axis, draw the graph of $y = \frac{1}{4}(20x - x^3)$ for $0 \leq x \leq 5$. [4]

Use your graph to find,

(c) the maximum value of y , [1]

(d) the range of values of x for which $20x - x^3 \geq 24$, [3]

(e) the solution(s) of the equation $16x - x^3 - 4 = 0$ by drawing a suitable straight line.

[4]

----- END OF PAPER 2 -----

Answers:

1a) $y = \frac{3x^2}{5x+10}$

1b) $\frac{1}{a}$

1c) -0.041

2a) $\frac{168}{x}$

2b) $\frac{168}{x-7}$

2d) 25

3a) $-5 < p < 5$

3b) $x = 16$ or $x = 2.25$

3c) $x = \sqrt{7}$ or $x = -\sqrt{7} - 1$

4) Enough to fill all conical cups

5a) $y = 2x + 1$

5b) $y = -(x - 2)^2 + 5$

5c) 22.6

6a) $x = 1.61$ or $x = -1.25$

6b) 16

7a) 205

7b) 569

7c) 43.6

7d) No

8a) $a = 6.7$ $b = 8.3$

8c) 8.61

8d) $1.31 \leq x \leq 3.67$

8e) $x = 0.25$ or $x = 3.87$

