BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2023

4E5N

CANDIDATE NAME	

CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS
Paper 1

4049 / 01
21 August 2023
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on ***all*** the work you hand in.
Write in ***dark blue*** or ***black*** pen.
You may use a HB ***pencil*** for any ***diagrams*** or ***graphs***.
Do ***not*** use staples, paper clips, highlighters, glue or correction fluid.

Answer ***all*** the questions.
The use of an ***approved scientific calculator*** is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give non-exact numerical answers correct to ***3 significant figures***, or ***1 decimal place*** in the case of angles in degrees.

For π, use your calculator value or 3.142.
You are reminded of the need for ***clear presentation*** in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
If working is needed for any question it must be shown in the ***space*** below the question.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is **90**.

Setter : Mr Loh Jia Perng

For Examiner's Use		
Student Check & Sign	Marks before deduction	
Parent's Signature	TOTAL	90

This paper consists of **25** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer all questions.

1. Find the coordinates of the intersection(s) for the following graphs.

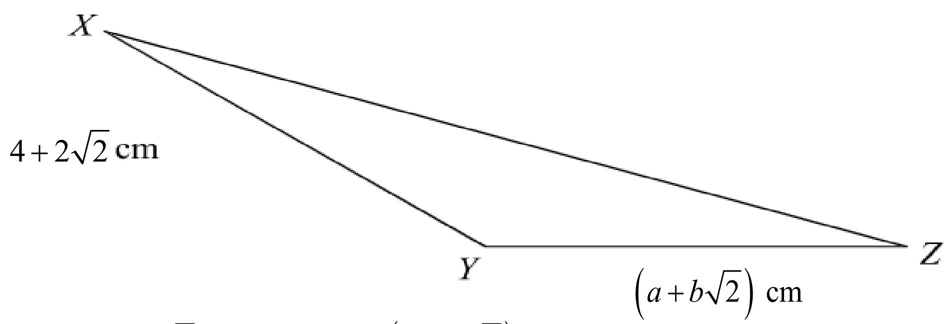
$$2x + 3y + 4 = 0,$$
$$\frac{3}{x} + \frac{2}{y} = \frac{1}{4}.$$

[5]

2.

(a) Express $\frac{30+18\sqrt{2}}{2+\sqrt{2}}$ in the form $r+s\sqrt{2}$, where r and s are integers [2]

(b) The diagram shows a triangle *XYZ*.



XY is $4+2\sqrt{2}$ cm and *YZ* is $\left(a+b\sqrt{2}\right)$ cm, where *a* and *b* are integers. The included angle *XYZ* is 135°. Given that the area of the triangle is $\left(15+9\sqrt{2}\right)\text{cm}^2$, find the value of *a* and of *b*. [3]

3. The function f is defined by $f(x) = \frac{5-x^2}{x^2+3}$, $x > 0$. A point P moves along the curve $y = f(x)$ in such a way that the y-coordinate of P is increasing at a constant rate of 0.2 units per second.

(a) Explain, with working, whether f is an increasing or decreasing function. [3]

(b) Find the rate of change of the x-coordinate of P when $x = 4$. [3]

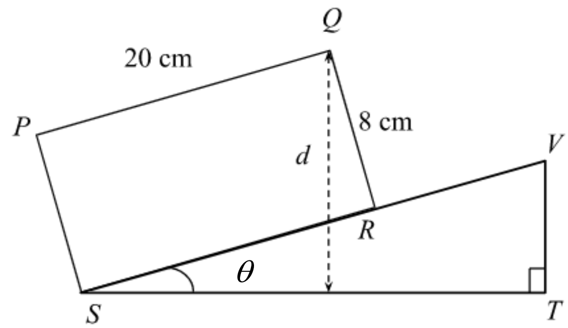
4. **(a)** Find the range of values of h for which the line $y + hx = 24$ .and the curve $5y + x^2 = 20$ do not intersect. [3]

(b) By expressing in the form of $(x-h)^2 + k$, where h and k are constants, explain why $x^2 - 4x + 7$ is always greater than or equal to 3. [2]

5. **(a)** Show that $\sin(A+B)\sin(A-B)=\cos^2 B-\cos^2 A$. [3]

(b) Hence, **without using a calculator**, show that $\sin 75^o \sin 15^o = \dfrac{1}{4}$. [2]

6. The diagram shows the frontal view of a rectangular block *PQRS*, with dimensions 20 cm by 8 cm. It is placed on a ramp, *VS*, tilted at an acute angle of θ to the horizontal surface *ST* and $\angle VTS = 90^o$.



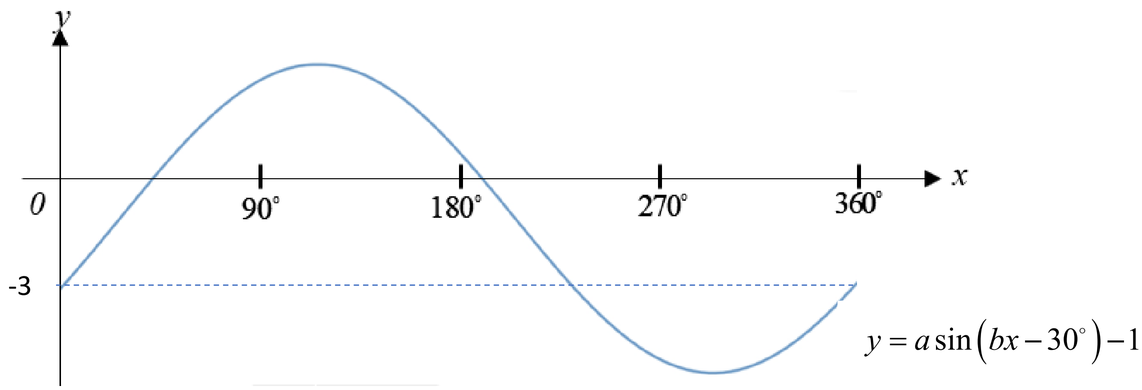
(a) Show that *d*, the perpendicular distance from *Q* to *ST*, can be expressed as $8\cos\theta + 20\sin\theta$, in centimetres. [2]

(b) Express d in the form $R\sin(\theta+\alpha)$, where $R>0$ and α is an acute angle. [2]

(c) Hence, state the maximum value of d and find the corresponding value of θ. [2]

7. **(a)** The equation of a curve is $y = \dfrac{3x+2}{\sqrt{5x-2}}$. Express $\dfrac{dy}{dx}$ in the form $\dfrac{ax+b}{2\sqrt{(5x-2)^3}}$, where a and b are constants. [2]

7 **(b)**



The diagram above shows the curve with equation $y = a\sin\left(bx - 30^\circ\right) - 1$ for $0^\circ \le x \le 360^\circ$, where a and b are constants. The curve cuts the y-axis at $\left(0,\ -3\right)$ Find the value of a and of b. [3]

8. The mass, m grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $m = 195(0.8)^t$.

(a) Find

(i) the initial mass of the substance, [1]

(ii) the mass of the substance when $t = 6$, [1]

(iii) the value of t when the mass of the substance is $\frac{1}{4}$ of its initial mass. Give

your answers correct to three significant figures. [3]

(b) Explain why the mass of the substance can never be more than 195. [1]

(c) Sketch the graph of m against t. [2]

9. **(a)** *Evaluate* $\int_0^4 \frac{3}{2x+1}\,dx$. [2]

(b) Given *that* $\int_0^3 f(x)\,dx = \int_3^5 f(x)\,dx = 3$, find the value of

(i) $\int_0^5 2f(x)\,dx - \int_5^3 f(x)\,dx$, [3]

(ii) the constant k for which $\int_3^5 \left[f(x) - \frac{k}{x^2} \right] dx = 0.$ [3]

10. **(a)** A circle, C_1, has equation $x^2 + y^2 + 8x - 12y + 27 = 0$.Find the radius and the coordinates of the centre of C_1. [3]

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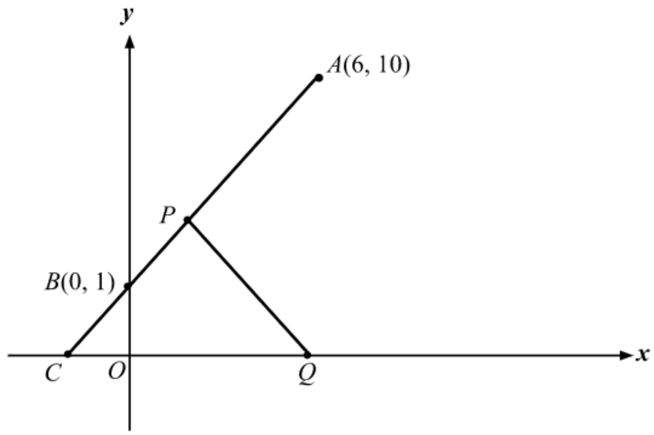
(b) A second circle, C_2 cuts C_1 on the y axis at the points F and G, where FG is the diameter of C_2. Find the equation of C_2 in the form of $(x-a)^2+(y-b)^2=r^2$. [4]

(c) Explain why the point $(-3,3)$ lies within only one of the circles C_1 and C_2. [2]

11. The line joining $A(6, 10)$ and $B(0,1)$ meets the x-axis at C. The point P lies on AC such that AP: PB = 2:1.



(a) A line through P meets the x-axis at Q and angle PCQ = angle PQC.

(i) Show that the coordinates of P is (2, 4). [1]

(ii) Find the equation of PQ, [4]

(iii) Find the coordinates of Q. [1]

(b) D is a point such that $CD = DQ$ and $CPQD$ is a kite. Given that the area of the kite is 104 units square. Find the coordinates of D. [3]

12. **(a)** Solve $2\log_7 p = 3 + \log_p 49$. [4]

(b) Given that $\log_3 x = a$ and $\log_9 y = b$, express in terms of a and b,

(i) $\log_x 9y$, [3]

(ii) $x^3 y$. [2]

13. **(a)** Given that the coefficient of x^9 in the expansion of $\left(1+kx\right)\left(x-3\right)^{11}$ is 1386, find the value of k. [7]

(b) The first three terms in the binomial expansion of $(a+b)^n$, in ascending powers of b, is given by $p+q+r$. Express $\dfrac{q^2}{pr}$ in terms of n. [3]