TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____

CIVICS GROUP: ____

H2 MATHEMATICS 9758/01

Paper 1

10 SEPTEMBER 2024

3 hours

Candidates answer on the question paper.

Additional material: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and Civics Group on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

For Examiners' Use	
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Total	

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

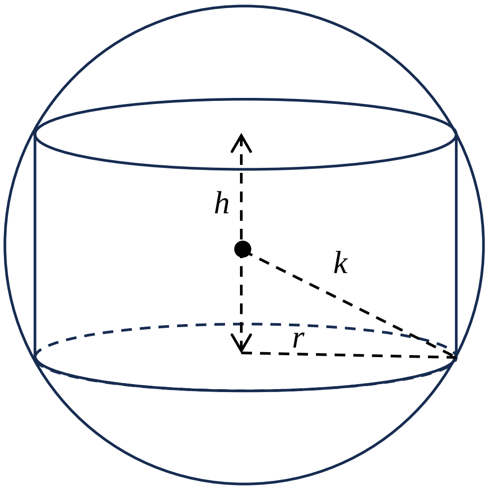
This document consists of 25 printed pages and 1 blank page.

[Turn Over

1 **(i)** Express $y = \dfrac{2x+1}{x-4}$ in the form $y = a + \dfrac{b}{x-4}$ where a and b are constants to be determined. [1]

(ii) Hence, state a sequence of transformations that will transform the curve with equation $y = \dfrac{1}{x}$ to the curve with equation $y = \dfrac{2x+1}{x-4}$. [3]

2



A cylinder with height h cm and base radius r cm is inscribed within a sphere with fixed radius k cm. The circumference of the circular top and bottom of the cylinder is in contact with the sphere (see figure above). By using differentiation, find, in terms of k, the exact value of h for which the volume of the cylinder is maximum. [6]

3 The function f is defined by

$$\mathrm{f}(x) = \begin{cases} 2\sin\left(\dfrac{\pi x}{4}\right) & \text{for } 0 \le x < 2, \\ 6 - 2x & \text{for } 2 \le x < 3, \end{cases}$$

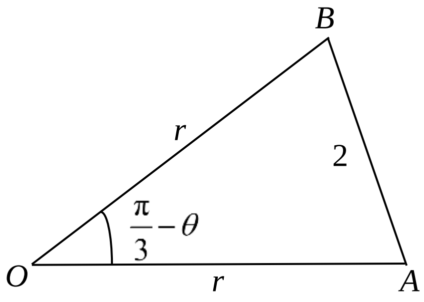
and that $\mathrm{f}(x) = \mathrm{f}(x+3)$ for all real values of x.

(i) Find the value of $\mathrm{f}(7)$. [1]

(ii) Sketch the graph of $y = \mathrm{f}\left(x\right)$ for $-4 \le x \le 7$. [3]

(iii) The region R is bounded by the curve $y = \mathrm{f}\left(x\right)$ and the x-axis from $x = 0$ to $x = 5$. Find the exact volume of solid generated when R is rotated 2π radians about x-axis. [4]

4



In the diagram above, OAB is an isosceles triangle where $OA = OB = r$ cm. It is given that the length $AB = 2$ cm and angle $AOB = \dfrac{\pi}{3} - \theta$ radians.

(i) Using cosine rule, show that $r^2 = \dfrac{4}{2 - \sqrt{3}\sin\theta - \cos\theta}$. [3]

(ii) Given that θ is a sufficiently small angle, show that

$$r \approx 2 + a\theta + b\theta^2$$

where a and b are constants to be determined. [4]

5 An arithmetic progression A has first term a and common difference d, where a and d are non-zero. A geometric progression G has first term b and common ratio r.

(i) The first, third and eleventh terms of A are equal to the fourth, third and second term of G respectively. Prove that the geometric series of G is convergent. [4]

(ii) It is given instead that $r = \frac{1}{3}$ and the terms of another sequence H is formed by squaring the terms of G. Find the range of values of b such that the sum to infinity of H exceeds the sum to infinity of G by more than $\frac{3}{2}$. [3]

6 It is given that $\mathrm{f}(r)=\dfrac{1}{2^r+1}$.

(i) Show that $\mathrm{f}(r)-\mathrm{f}(r+1)=\dfrac{2^r}{\left(2^r+1\right)\left(2^{r+1}+1\right)}$. [1]

(ii) Using the result in part **(i)**, find $\displaystyle\sum_{r=1}^{n}\frac{2^r}{\left(2^r+1\right)\left(2^{r+1}+1\right)}$. [3]

(iii) Hence, find $\sum_{r=1}^{n}\left[\frac{2^{r+3}}{\left(2^{r+2}+1\right)\left(2^{r+3}+1\right)}-\left(2r+5\right)\right]$. [4]

7 A curve C has equation $y^2 = 3x^3 - x + 1$.

(i) Find the exact coordinates of all the stationary points. [4]

(ii) A particle moves along C to the stationary point S, found in part **(i)** where both the x-coordinate and y-coordinate are positive. Given that its x-coordinate is increasing at a rate of $\frac{1}{9}$ units per second, find the exact rate of change of the gradient of C when the particle is at point S. [4]

8 Figure 1 shows a triangular prism *OABCDE*, where *O* is the origin. Point *F* lies on *AE* such that *AF*:*FE* = $1-k:k$ where $0<k<1$. Figure 2 shows a model of a crystal trophy that is made by cutting away a tetrahedral section *CDEF* from the triangular prism *OABCDE*. With respect to *O*, the coordinates of *A*, *B* and *D* are $(4,3,0)$, $(0,6,0)$ and $(0,0,20)$ respectively. It is given that $\overrightarrow{OD}=\overrightarrow{BC}=\overrightarrow{AE}$.

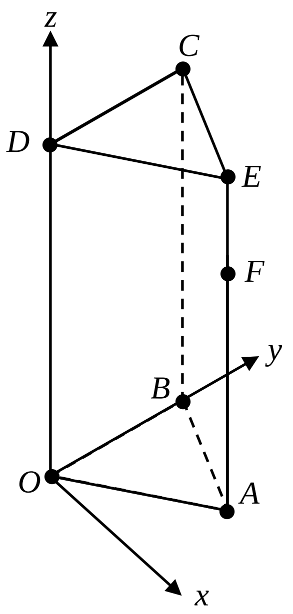


Figure 1

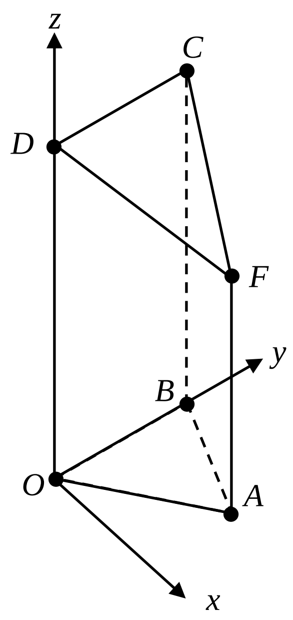


Figure 2

(i) Write down the position vector of *E* and find the position vector of *F* in terms of *k*. [2]

(ii) Show that the equation of the plane *CDF* can be written as $\mathbf{r}\cdot\begin{pmatrix}5k\\0\\1\end{pmatrix}=20.$ [3]

(iii) Find the x-coordinate of the foot of perpendicular from the point E to the plane *CDF* in terms of k. [3]

(iv) Hence, find the value of k such that the reflection of the plane CDE in the plane CDF lies on the plane $OBCD$. [4]

9 The function f is defined as follows:

$$\mathrm{f} : x \mapsto \frac{1}{(x-2)^2} \quad \text{for} \quad x \in \mathbb{R},\ x > k.$$

(i) State the least value of k for which the function f^{-1} exists. [1]

For the rest of the question, take the value of *k* as the value found in part (i).

(ii) Sketch, on the same diagram, the graph of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, showing clearly the relationship between the two graphs. [3]

(iii) Find the exact value of $\mathrm{f}^{-1}(5)$. [2]

The function g is defined as follows:

$$\mathrm{g}: x \mapsto \frac{1}{\sqrt{x}} - 4\sqrt{x} \quad \text{for} \quad x \in \mathbb{R},\ x > 0.$$

(iv) Show that gf^{-1} exists and find its range in exact form. [3]

(v) Solve the inequality $\mathrm{gf}\left(x\right) \le 0$ algebraically. [4]

10 A company proposed to build a water detention tank to address the flooding problem in a village. To test the feasibility of the proposal, the company created a model of a water detention tank, with a capacity of 300 cm^3.

Water is pumped into the model of the tank at a constant rate of 50 cm^3/min and pumped out at a rate proportional to the volume of water in the tank. At time t minutes, the volume of water in the model of the tank is V cm^3.

(i) Write down a differential equation for this situation. [1]

(ii) Solve this differential equation to obtain the general solution for V in terms of t. [4]

Initially, the volume of water in the tank is 100 cm^3. After a minute, the volume increased to 130 cm^3.

(iii) Hence, show that the particular solution for V in terms of t is $V = 289 - 189e^{-0.173t}$, correct to 3 significant figures. [4]

(iv) Find how long it will take for the tank to reach 95% of its maximum capacity. [1]

(v) Sketch the graph of V against t. [2]

(vi) Explain, with justification, what will happen to the volume of water in the tank in the long run. [1]

11 Mechanical engineers are responsible for the design of the drops and loops in roller coaster tracks. In order to design a roller coaster ride that is exciting, yet safe, mechanical engineers are required to possess a strong understanding of force, gravity, motion, momentum, and potential and kinetic energy.

A mechanical engineer first designs part of a roller coaster track using the curve C, which is defined by the parametric equations

$$x = t^2 + 2t,\ y = \ln(t+1) \text{ where } t > -1.$$

(i) Sketch the part of the roller coaster track that is defined by C. [2]

The mechanical engineer then designs a drop in the roller coaster track. The drop is modelled after the equation of the line *N*, which is the normal to *C* at the point where $t = 2$.

(ii) Find the exact equation of line *N*. [3]

(iii) For the roller coaster drop to be deemed safe, the acute angle between line *N* and the *x*-axis needs to be between $30°$ and $80°$. Comment on the suitability of using line *N* in the design of the roller coaster drop. [1]

An advertising billboard is designed to be mounted on the rollercoaster track. Its area can be modelled by the area enclosed by curve C, line N and the x-axis.

(iv) Show that the area of the billboard is $a(\ln 3)^2 + b\ln 3 + c$, where a, b and c are constants to be determined. [8]

End of Paper