Chapter 11 WAVE MOTION

H2 PHYSICS 9646

Content

* Progressive waves
* Transverse and longitudinal waves
* Polarisation
* Determination of frequency and wavelength

Learning Outcomes

Candidates should be able to:

(a) show an understanding and use the terms displacement, amplitude, phase difference, period, frequency, wavelength and speed.

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.

(c) recall and use the equation $v = f\lambda$.

(d) show an understanding that energy is transferred due to a progressive wave.

(e) recall and use the relationship, intensity $\propto$ (amplitude)^2.

(f) show an understanding of and apply the concept that wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems

(g) analyse and interpret graphical representations of transverse and longitudinal waves.

(h) show an understanding that polarisation is a phenomenon associated with transverse waves.

(i) recall and use Malus' law (intensity $\propto \cos^2\theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.

(j) determine the frequency of sound using a calibrated oscilloscope.

(k) determine the wavelength of sound using stationary waves.
(* to be taught in Chapter 12)

11 Introduction

A wave can be described as a **disturbance** that travels through a medium from one location to another location. Waves can be considered as an **energy** transport phenomenon because energy is transferred from one point to another some distance away in the direction of wave propagation. This chapter will focus on ***progressive waves***.

Definition of a *progressive wave*

A *progressive wave* transports energy from one point to another in the direction of wave propagation.

The particles in the medium do not move along with the wave - they merely *oscillate* about their equilibrium positions i.e. their undisturbed positions.

11.1 Characteristics of Progressive waves

11.1.1 Wave Terminologies

Displacement (x or y) of a particle is the distance in a specific direction of a particle of a wave from its equilibrium position.

Amplitude (x_0 or A) is the magnitude of the maximum displacement of a particle in the wave from its equilibrium position.

Period (T) is the time taken for a particle of a wave to complete one oscillation.

(For progressive waves, period is also the time required for the waveform to travel one wavelength.)

Definition of *frequency*

Frequency (f) of a wave is the number of oscillations made per unit time.

$\boxed{f = \frac{1}{T}}$

Unit: hertz (Hz)

Definition of *wavelength*

Wavelength (λ) of a wave is the distance between two consecutive points which are in phase.

(such as two successive crests or two successive troughs)

Definition of *speed* (of a wave)

Speed (v) of a wave is the speed at which the wave shape (or wave profile) moves.

Phase or phase angle (ϕ) of a wave is an angle that gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave.

(a phase angle of 2π rad represents one complete oscillation)

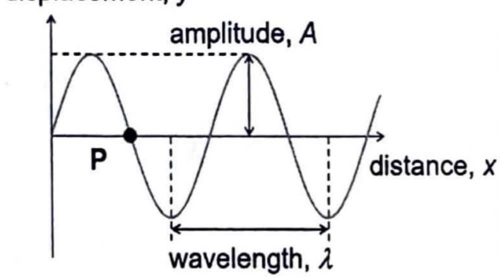
Phase difference between two particles in a wave or between two waves at a point is a measure of the fraction of a cycle which one is ahead of the other.

Wavefront is a line or surface joining points on a wave that are **in phase**. The wave travels in a direction that is perpendicular to the wavefront.

11.1.2 Graphical Representations of Waves

Waves can be represented graphically in two ways.

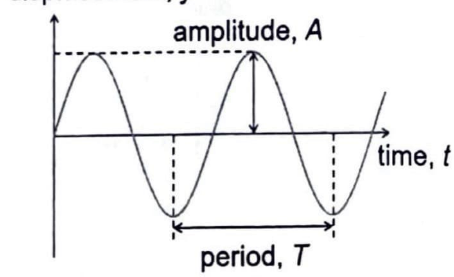
1. **Displacement *y*** (of particles) vs **Distance *x*** (from a reference position)



A graph of y vs x shows the displacement of **ALL** the particles of a wave at a particular instant of time.

You can imagine it as a snapshot of a wave.

2. **Displacement *y*** (of one particle **P**) vs **time *t***



A graph of y vs t shows the displacement of **ONE** particle **P** of a wave over time.

You can imagine it as a motion video of a particle in a wave.

11.1.3 Speed of a Progressive Wave

The particles of a wave do not move with the propagation of the wave.

They only oscillate about their equilibrium positions with different phases within one wavelength. (For simple harmonic oscillations, the waves are sinusoidal).

The result is that a waveform is created that moves in the direction of energy transfer.

Derivation of Speed of a Wave

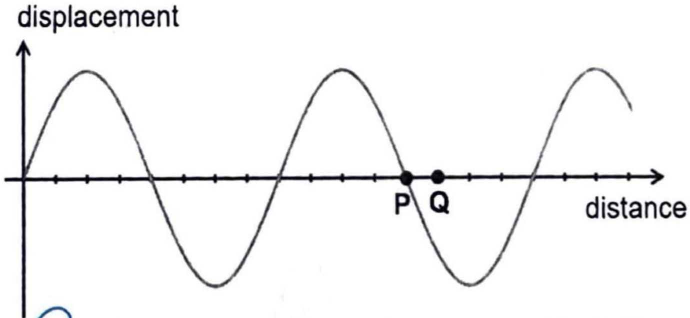
In a time of one period ($t = T$),
the waveform moves a distance of one wavelength($x = \lambda$).

$$\therefore \quad \text{speed, } v = \frac{\text{distance}}{\text{time}} = \frac{x}{t} = \frac{\lambda}{T}$$

Since $f = \frac{1}{T}$, $\boxed{v = f\lambda}$

Example 1

The diagram shows a transverse wave at a particular instant. The wave is travelling to the right. The frequency of the wave is 12.5 Hz.



At the instant shown, the displacement is zero at the point P. What is the shortest time to elapse before the displacement is zero at point Q?

A 0.01 s **B** 0.03 s **C** 0.08 s **D** 0.10 s

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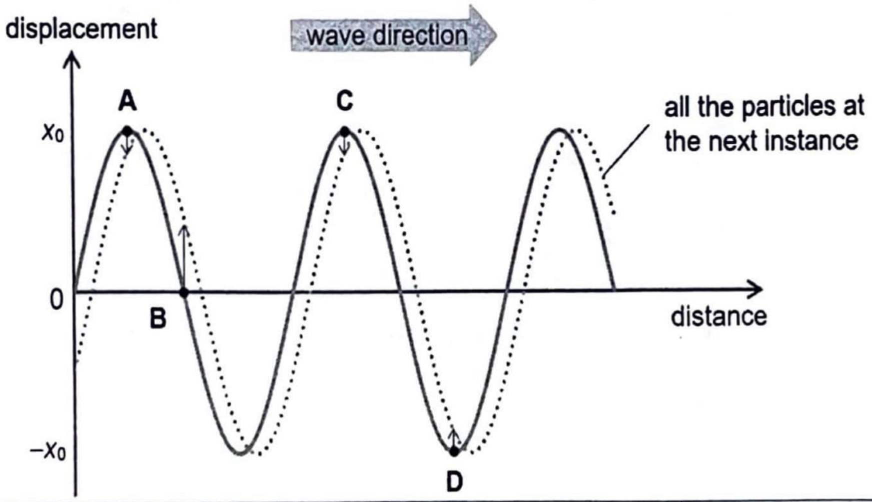
11.1.4 Phase and Phase Difference

Phase and Phase Difference

The phase of a particle in a wave can be considered as the stage of motion of the particle with respect to the other particles in the same or different wave.

Consider the various particles in a wave at different displacements at one instant of time. The wave profile at the next instance is illustrated by the dotted line.



	displacement		velocity		acceleration	
	magnitude	*direction*	*magnitude*	*direction*	*magnitude*	*direction*
A	x_0	up	0	down	$\omega^2 x_0$	down
B	0	-	ωx_0	up	0	-
C	x_0	up	0	down	$\omega^2 x_0$	down
D	x_0	down	0	up	$\omega^2 x_0$	up

In Phase

Particles of a wave are considered to be **in phase** when they execute the **same motion** at the **same time**.

E.g. Consider two different particles **A** and **C** oscillating in the same wave.

They will always pass through the equilibrium position at the same time in the same direction (or in the same stage of oscillation). Hence, they are in phase.

Out of Phase

Particles that show different stages of motion are **out of phase** with each other. They have a non-zero **phase difference** ϕ, e.g. 45°, 90°, π rad.

E.g. Consider particle **A** and **any** two different particles <u>between</u> **A** and **C** oscillating in the same wave, e.g. **B**. Their displacement, velocity and acceleration will always be different. For **A** and **B**, they are out of phase by 0.5π rad or 90°.

(Recall one cycle corresponds to an angle of 2π rad).

Anti-phase

Particles in a wave are **in anti-phase** when they execute motions that are out of phase by π rad or 180°. (In such case, the particles have equal but opposite displacements, velocities and accelerations.)

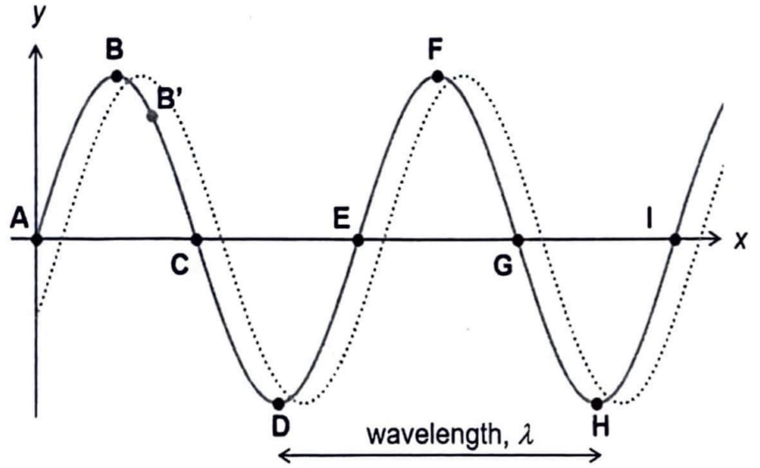
E.g. Compare particles **A** and **C** (crests) with **D** (trough).

Their displacement, velocity and acceleration will always be equal in magnitude but opposite in direction.

Example 2 Consider a wave moving to the right at a certain instant as shown below where the diagram represents the variation of y with x at time t_1.

9 particles on the wave are labelled A to I. (Ignore B' for now)

The dotted line shows the position of the wave at time t_2 (a short time interval later) and the motion of the 9 particles during this time interval are indicated by the arrows.



Particle		Particle
A	is in phase with	E / I
B	is in phase with	F
C	is in phase with	G
D	is in phase with	H
E	is in phase with	A / I

Note In a progressive wave, all particles within a wavelength are **out of phase** with one another.

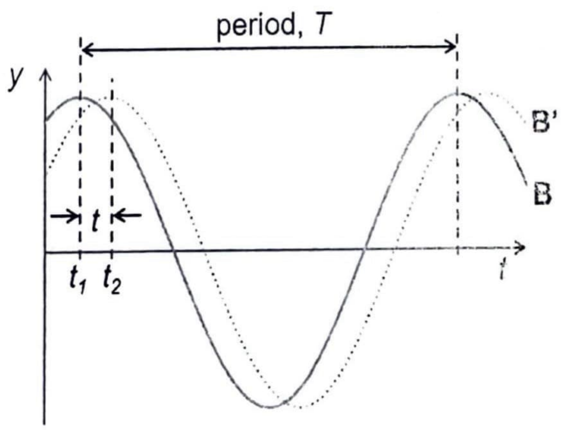
A wavelength can be defined as the shortest distance between any two particles in a wave that are in phase.

Comparing two particles B & B' **B** is on its way downwards while **B'** is on its way up.

It can therefore be said that **B'** is slower than **B** and <u>is lagging **B**</u> by a phase difference of ϕ (measured in radians) where $0 \le \phi \le 2\pi$.

Determining ϕ

<u>Displacement vs Time</u>

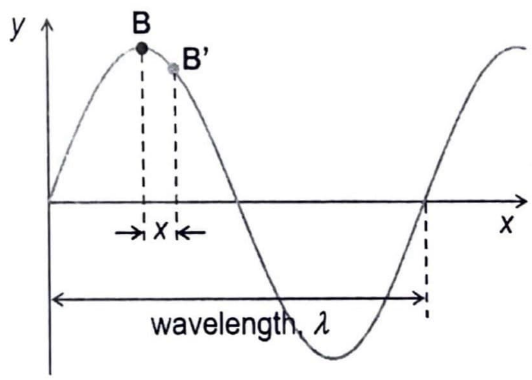


B' is lagging behind B by a time t.

By comparing t to T, $\dfrac{t}{T} = \dfrac{\phi}{2\pi}$

$$\therefore \phi = \left(\frac{t}{T}\right)2\pi$$

<u>Displacement vs Distance</u>



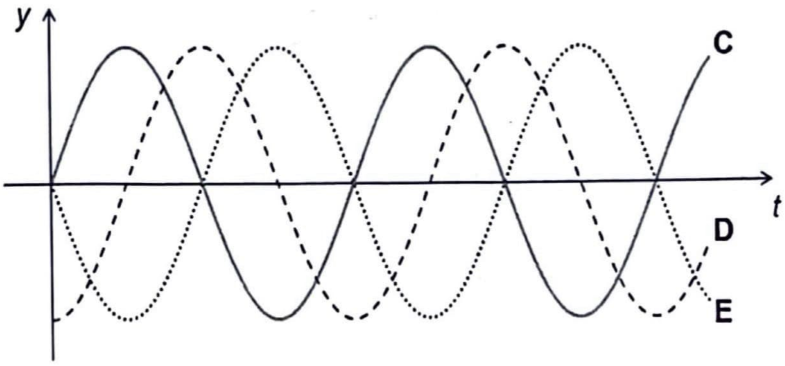
B' is lagging behind B by a distance x.

By comparing x to λ, $\dfrac{x}{\lambda} = \dfrac{\phi}{2\pi}$

$$\therefore \phi = \left(\frac{x}{\lambda}\right)2\pi$$

In the displacement vs time graph, the dotted graph belongs to particle B' which lags behind particle B. The phase difference between the two particles corresponds to the time difference between the peaks of the two graphs.

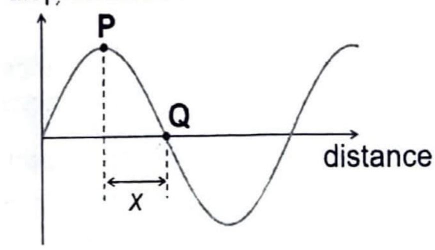
Example 3 Refer to the points C, D and E in Example 2.
The diagram below shows the displacement against time graph of point C. On the same axes, sketch a graph which represents the point (i) D and (ii)E.



Determine the phase difference between particles **C** and **D**?

Solution Solution -- The phase difference between particles C and D is 0.5π rad.

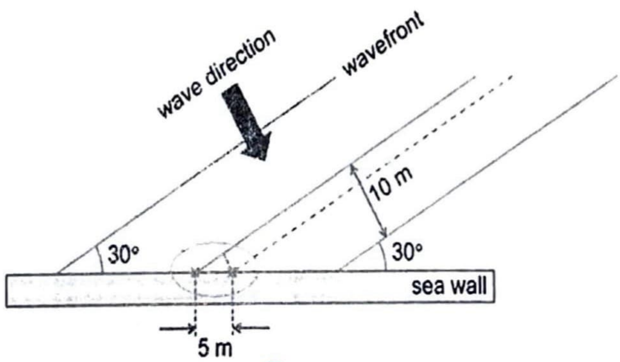
Example 4 For a wave that propagates to the right, determine the phase difference ϕ between points P and Q.



Solution

$$\frac{\phi}{2\pi}=\frac{x}{\lambda} \quad \Rightarrow \phi=\frac{1}{4}(2\pi)=\frac{\pi}{2} \text{ rad}$$

Example 5 Parallel water waves of wavelength 10 m strike a straight sea wall. The wavefronts make an angle of 30° with the wall as shown. What is the phase difference at any instant between the waves at two points 5 m apart along the wall?



A 45° **B** 55° **C** 90° **D** 180° N95/I/10

Solution

5 m along the wall would mean that the wavefront has travelled 5 sin 30° or 2.5 m.

$$\therefore \phi=\frac{2.5}{10}\times 360°=90°$$

11.1.5 Intensity of a Wave

Wave Intensity

The intensity I of a wave is defined as the **rate of transfer of energy per unit area normal** to the **direction of energy transfer** of the wave.

$$\boxed{I = \frac{P}{S}}$$

where P is the power of the source and S is the surface area.

The energy of a wave comes from the oscillations of its fields (electromagnetic radiation) or from the oscillation of the particles in the medium (mechanical waves). For the latter, the total energy of an oscillating particle is $E = \frac{1}{2}m\omega^2 x_0^2$.

Its intensity I is also proportional to the square of its amplitude A and the square of its frequency f, i.e. $I \propto A^2 f^2$.

If frequency f is constant,

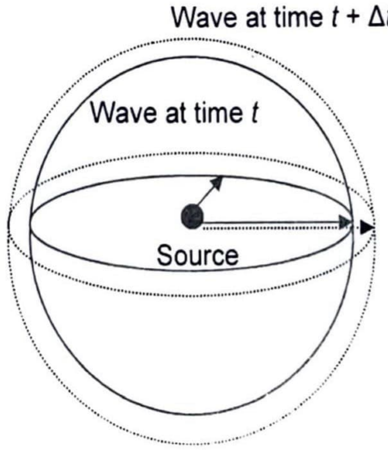
I* is proportional to square of *A

$$\boxed{I = kA^2}$$

where k is a proportionality constant

Point Source of Wave & the Inverse Square Law

Consider a point source emitting waves that spread out radially in all directions. Energy will spread over an expanding spherical wavefront. The further away from the source, the larger the area the energy is being distributed. Hence, the intensity of the wave decreases.



Intensity at a point on a spherical wavefront:

$$\boxed{I = \frac{P}{4\pi r^2}}$$

where

P is the power of the source, and

r is its distance from the source.

If the <u>power P of the source is constant</u>, then the intensity is inversely proportional to the square of the distance from the source r:

$$\boxed{I \propto \frac{1}{r^2}}$$

This is known as the **inverse square law**.

Note: For an ideal plane-wave source, the intensity of the plane waves remains unchanged as it progresses.

Example 6

A constant power source of 500 W emits light uniformly in all directions.
(a) Calculate the intensity of the light detected by a man located 3.0 m from the source.
(b) The power of the source is now halved. In order to detect the same intensity as before, determine the distance of the man from the source.

Solution

(a) $I = \dfrac{P}{4\pi r^2} = \dfrac{500}{4\pi(3.0)^2} = 4.42\ \text{W m}^{-2}$

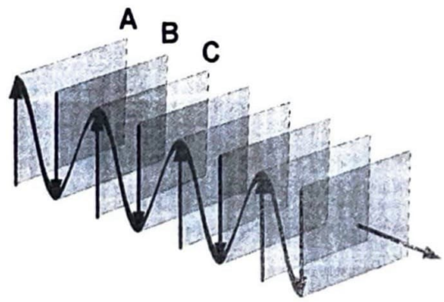
(b) $\dfrac{P}{4\pi r^2} = \dfrac{P'}{4\pi R^2}$

$$R^2 = \frac{P'}{P} r^2 = \frac{1}{2}(3.0)^2$$
$$R = 2.12\ \text{m}$$

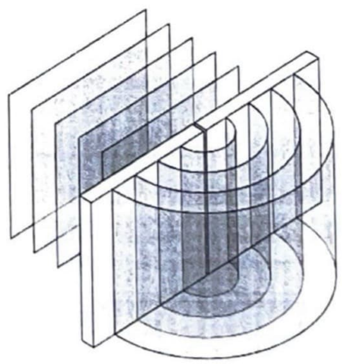
Plane Waves & Cylindrical Waves

Plane waves are waves whose wavefronts are parallel planes normal to the direction of energy transfer. The wavefronts are equally spaced, a wavelength apart. Initially EM waves emitted from a point source have spherical wavefronts, but as the distance travelled by wave increases towards infinity, the wavefronts can be considered as plane waves.

Cylindrical waves are waves whose wavefronts form a family of coaxial cylinders. Cylindrical waves can be formed when a wave is emitted from a line source such as a narrow, rectangular slit.

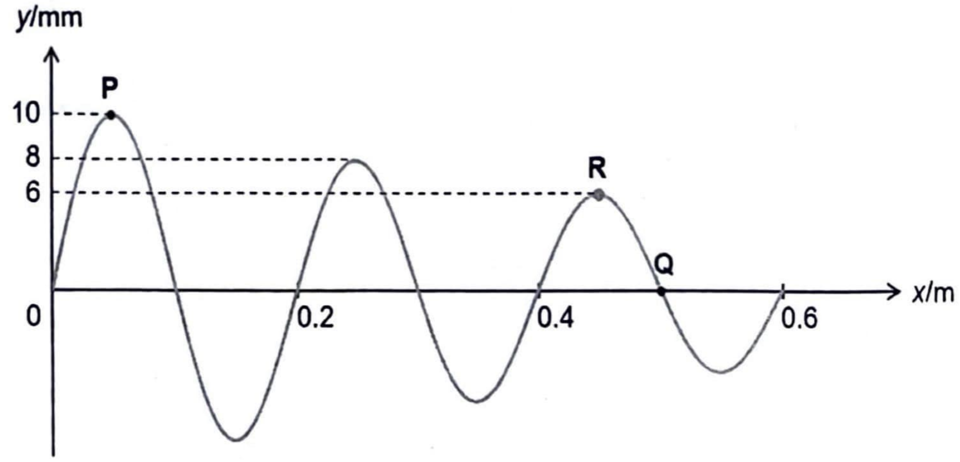


Diagrammatic representation of a **plane wave**
Wavefronts **A** and **B** are anti-phase; Wavefronts **A** and **C** are in phase.



A plane wave undergoing diffraction to become a **cylindrical wave**

Example 7 The diagram below shows a portion of the displacement against distance graph of a wave. Use the diagram to answer the following questions



(i) State the wavelength of the wave?

(ii) Given that the frequency of the wave is 10.0 Hz, calculate the speed of the wave.

(iii) Calculate the phase difference between particles P and Q.

(iv) Calculate the ratio $\dfrac{\text{intensity at Q}}{\text{intensity at P}}$.

Solution

(i) State the wavelength of the wave? $\lambda = 0.20\text{ m}$

(ii) Given that the frequency of the wave is 10.0 Hz, calculate the speed of the wave.

$$v = f\lambda = (10.0)(0.20\text{ m}) = 2.0\text{ m s}^{-1}$$

(iii) Calculate the phase difference between particles P and Q.

$$\frac{\phi}{2\pi} = \frac{x}{\lambda} = \frac{2.25\lambda}{\lambda} \quad \Rightarrow \phi = \frac{9}{4}(2\pi) = 4.5\pi\text{ rad OR }0.5\pi\text{ rad}$$

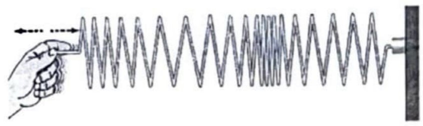
(iv) Calculate the ratio

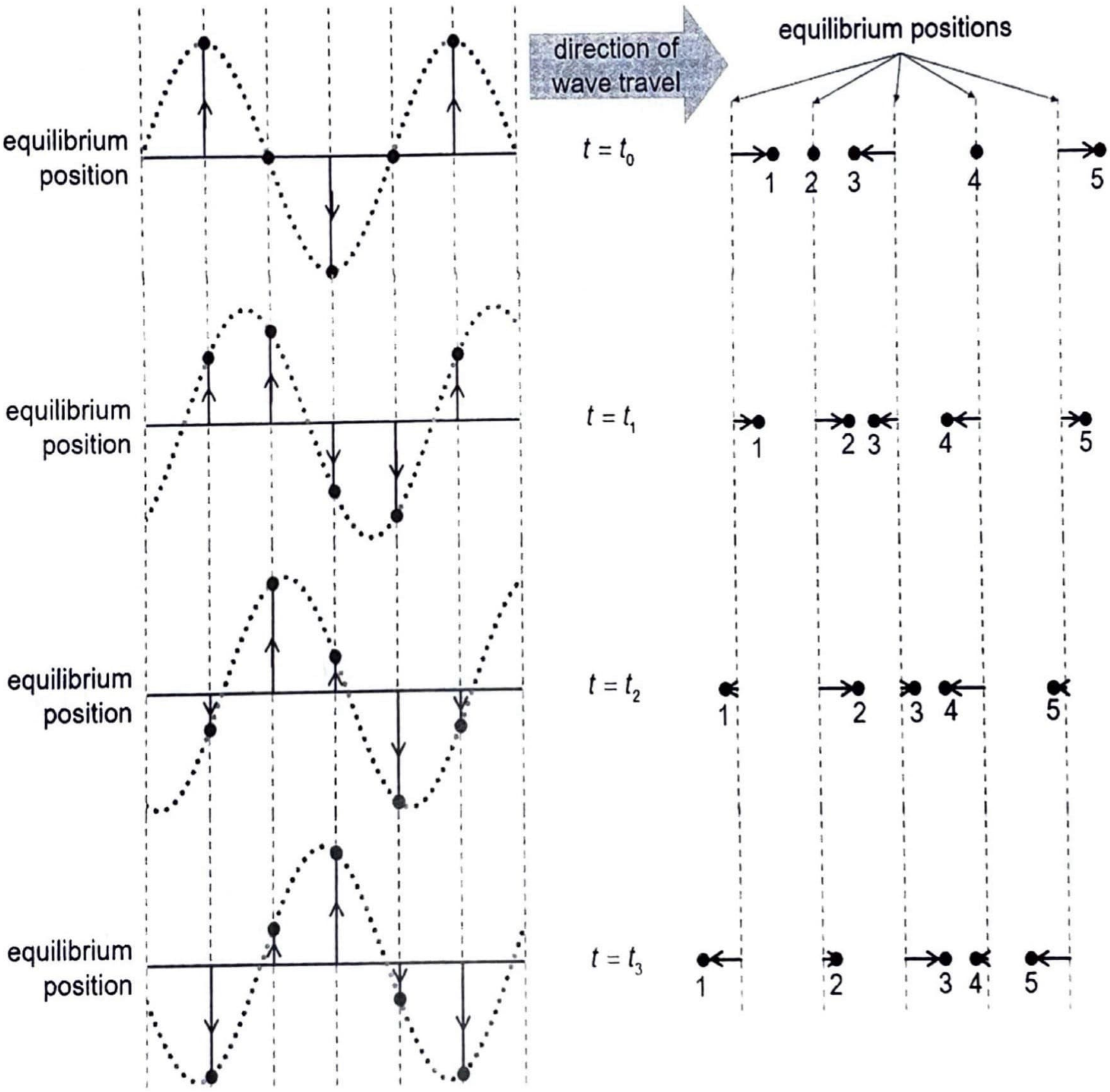
$\dfrac{\text{intensity at Q}}{\text{intensity at P}}$ $\qquad I = kA^2 \quad \Rightarrow \dfrac{I_Q}{I_P} = \dfrac{A_Q^{\,2}}{A_P^{\,2}} = \left(\dfrac{6}{10}\right)^2 = 0.36$

11.2 Transverse and Longitudinal Waves

11.2.1 Comparing Transverse and Longitudinal Waves

	Transverse	Longitudinal
Definitions	A **transverse wave** is one in which its particles oscillate in a direction <u>perpendicular</u> to the direction of energy transfer.	A **longitudinal wave** is one in which its particles oscillate in a direction <u>parallel</u> to the direction of energy transfer.
Example	• electromagnetic (EM) waves • pulses in ropes	• sound • longitudinal pulses in springs

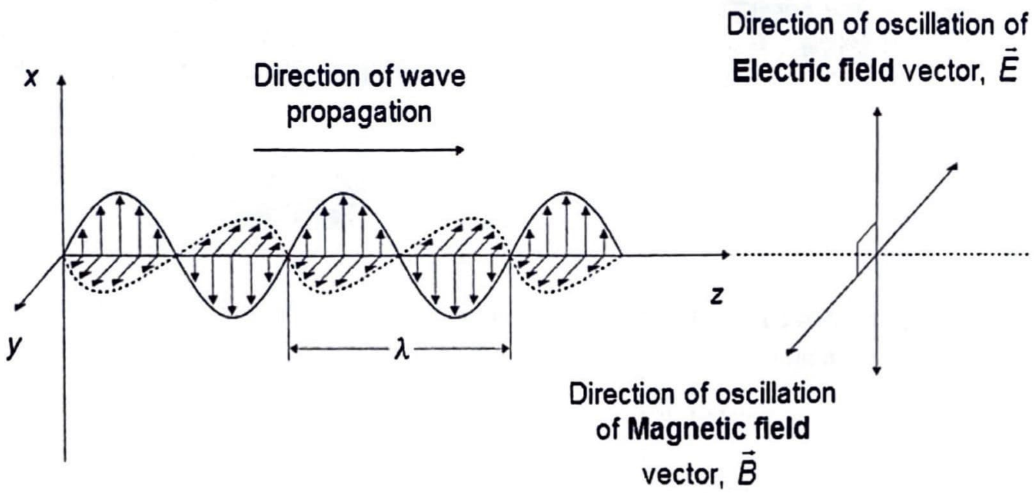




11.2.2 Transverse Waves

Electromagnetic Waves (EM Waves)

The most important and common type of transverse waves is the electromagnetic (EM) waves. They are generated by oscillating electric charges, and are made up of oscillating electric and magnetic fields which are perpendicular to each other and the direction of energy transfer.

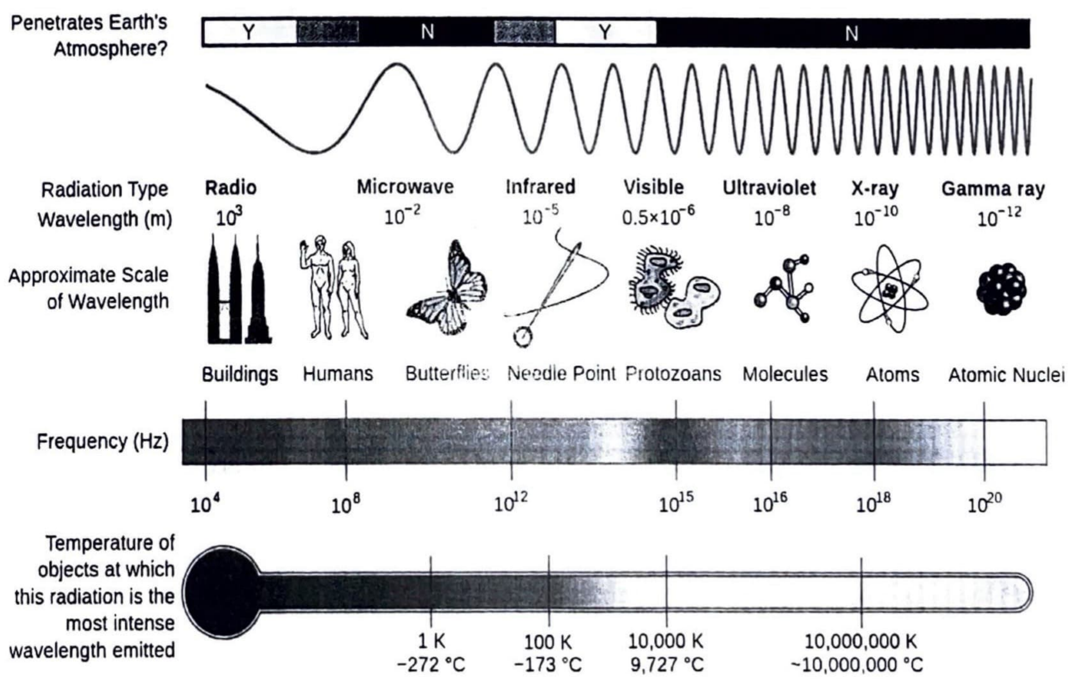


Important Properties of EM Waves

1. EM waves do not require a medium to propagate; they can travel through vacuum.
2. EM waves travel at the speed of $c = 3.00 \times 10^8$ m s^{-1} in vacuum. (No object can travel at the speed of light.)
3. EM waves are transverse waves, and hence they can be polarised.

The Electromagnetic Spectrum

Electromagnetic radiation is classified into types according to the frequency of the wave.



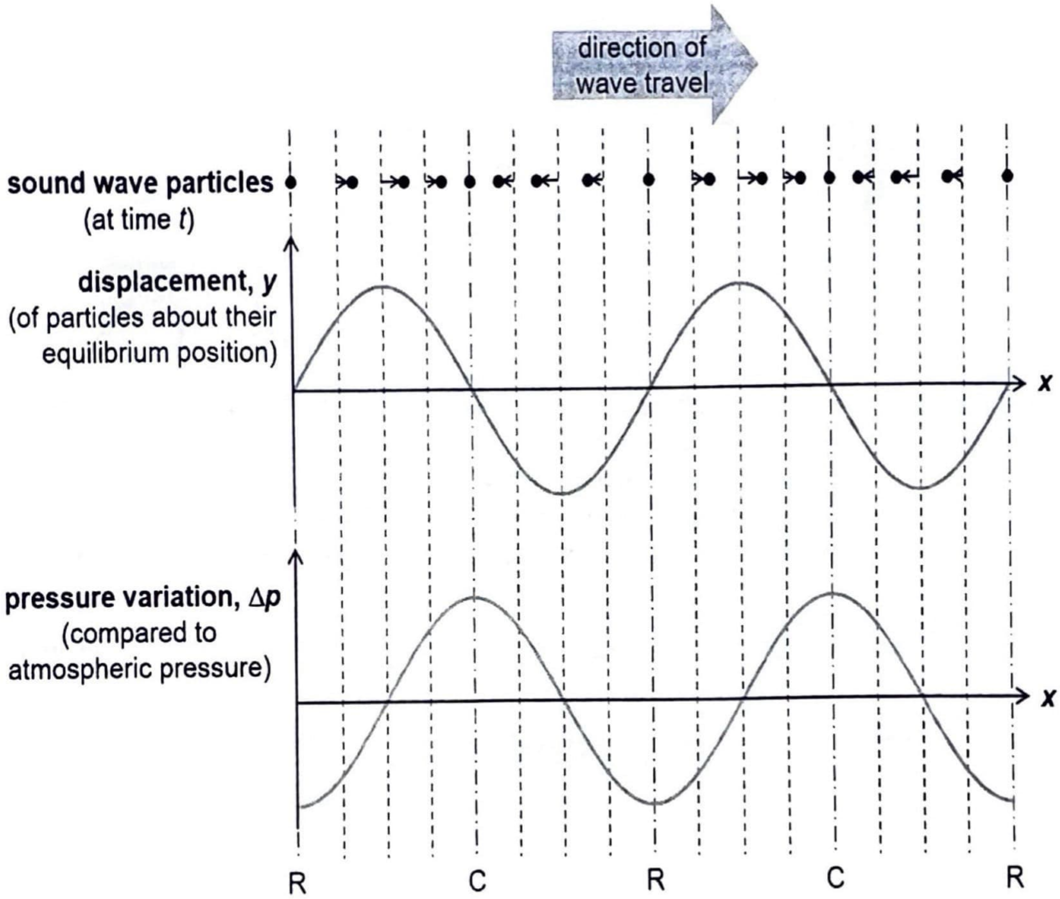
11.2.3 Longitudinal Waves

Sound Waves

Sound waves are the most important and common type of longitudinal waves.

The particles of the medium in which sound wave propagates, oscillate about their equilibrium positions in a direction **parallel** to the direction of energy transfer of the wave.

The positions of the air particles in a sound wave at a time *t* is shown in the diagram below. Their individual equilibrium positions are indicated by the dashed vertical lines, and their displacement with respect to their equilibrium positions are indicated by the arrows.



The regions where the air particles are compressed together and other regions where the air particles are spread apart are known as compressions (C) and rarefactions (R) respectively.

The compressions are regions of high air pressure while the rarefactions are regions of low air pressure. In between a compression and a rarefaction is a point where the pressure is equal to the atmospheric pressure.

Since a sound wave consists of a repeating pattern of high-pressure and low-pressure regions moving through a medium, it is sometimes referred to as a pressure wave.

11.3 Polarisation

11.3.1 Polarisation of Transverse Waves

Definition of Polarisation

Polarisation is a phenomenon whereby the oscillations of the wave particles in **transverse** waves are restricted to one direction only and this direction is perpendicular to the direction of energy transfer.

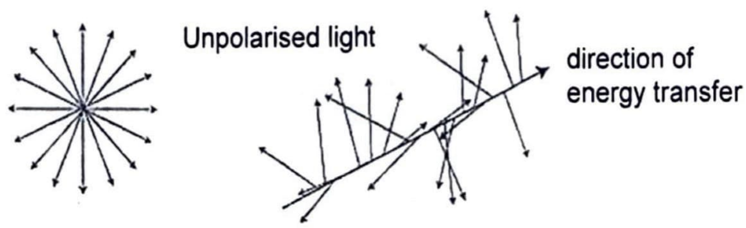
* This phenomenon does not apply to longitudinal waves.

** For electromagnetic waves which are polarised, the direction of polarisation of each wave is defined to be the direction in which the **electric field** (vector) is vibrating.

Polarisation and EM (transverse) Waves

Light is a transverse electromagnetic wave and is generally unpolarised, all planes of polarisation being present.

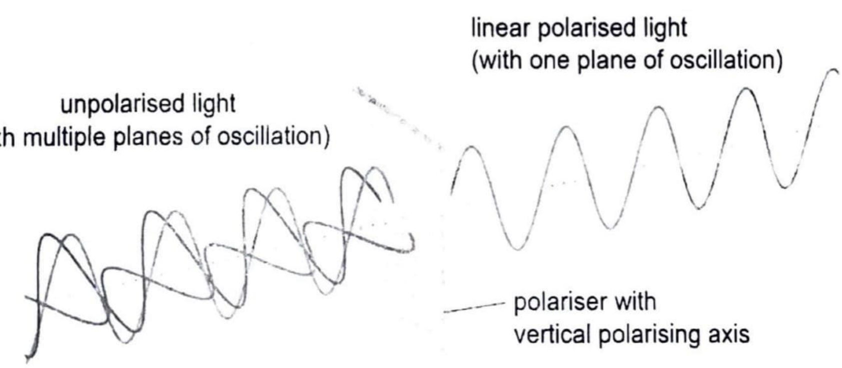
For unpolarised light, the electrical field can oscillate in any direction perpendicular to the direction of energy transfer as shown below.



Front View
Arrows show the oscillations of the electric field vector

Side View
The electric field vector can oscillate in **any direction** perpendicular to the direction of energy transfer

In general, when an unpolarised electromagnetic wave is passed through a (linear) polariser, a **plane-polarised wave** is produced. This wave is also said to be **linearly polarised**.



Not in syllabus

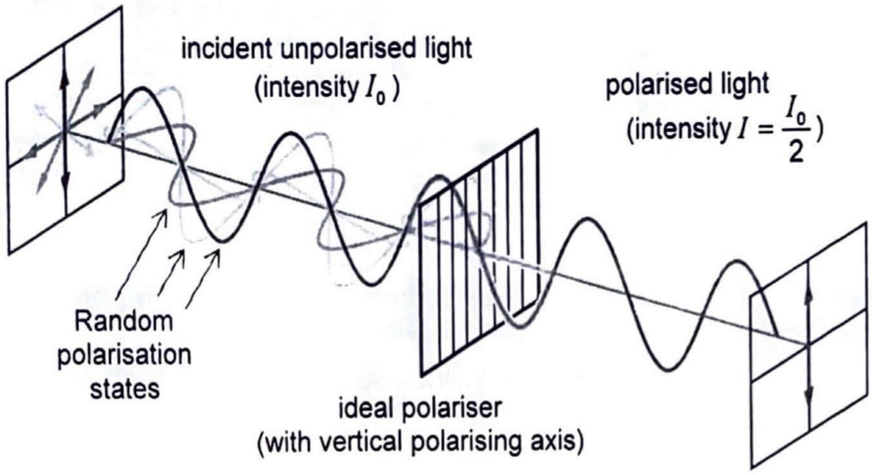
Polarisation of light may be achieved by the following methods:

1. Reflection of light (off plane surfaces)
2. Scattering of light (off air molecules)
3. Refraction of light (using birefringent materials)
4. Preferential absorption of light in certain planes (using dichroic materials)

11.3.2 Intensity of Plane Polarised Light

Intensity of plane polarised light

When unpolarised light is incident on an ideal polariser*, the intensity of the transmitted light is exactly half that of the incident unpolarised light, no matter how the polarising axis is oriented.



Since incident light is a random mixture of all polarisation states, the components of the electric field vectors that are perpendicular or parallel to the polarising axis is equal.

Hence, by only transmitting the component that is parallel to the polarising axis, only half the incident intensity is transmitted.

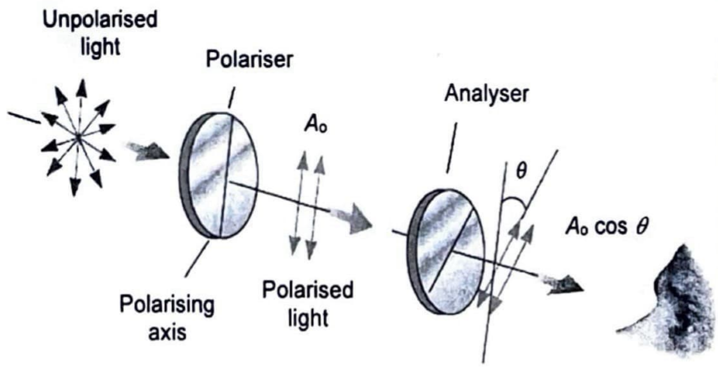
Note: Even though $I_0 = 2I$, it is not possible to compare amplitudes between unpolarised and polarised waves.

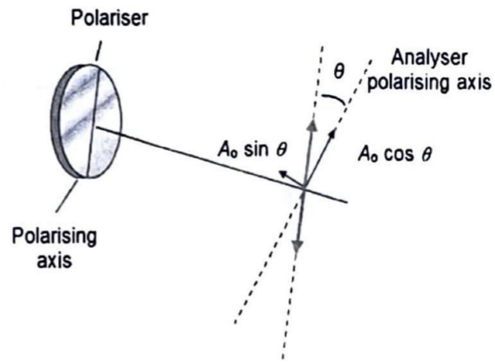
* An **ideal polariser** is a material that only allows the electric field vector of an EM wave that is parallel to its polarizing axis to pass through and not the electric field perpendicular to its polarising axis.

11.3.3 Malus' Law

Amplitude of a plane polarised wave passing through a polariser

When polarised light is incident on a second polariser (usually called an analyser) placed with its polarising axis at an angle θ with the polarising axis of the first polariser, only the electric field component parallel to the polarising axis of the analyser will be transmitted.





The resultant amplitude A of the transmitted wave is therefore

$$\boxed{A = A_0 \cos\theta}$$

The amplitude of the transmitted wave is greatest when $\theta = 0$ and zero when the polariser and analyser are crossed so that $\theta = 90°$.

Malus Law

Intensity of an electromagnetic wave is proportional to the square of the amplitude of the wave. This implies that the intensity of the transmitted wave can be expressed as

$$\boxed{I = I_0 \cos^2\theta}$$

where I_0 is the intensity of the light before passing through the analyser.

This is known as Malus' Law.

Definition

Malus' Law states that the intensity of a beam of plane-polarised light after passing through a polariser varies with the square of the cosine of the angle through which the polariser is rotated from the position that gives maximum intensity.

Example 8 A beam of plane polarised light of intensity I_0 falls normally onto a thin sheet of polaroid. If the transmitted beam has an intensity of $\frac{I_0}{4}$, what is the angle θ between the plane of incident polarisation and the polarising axis of the polaroid?

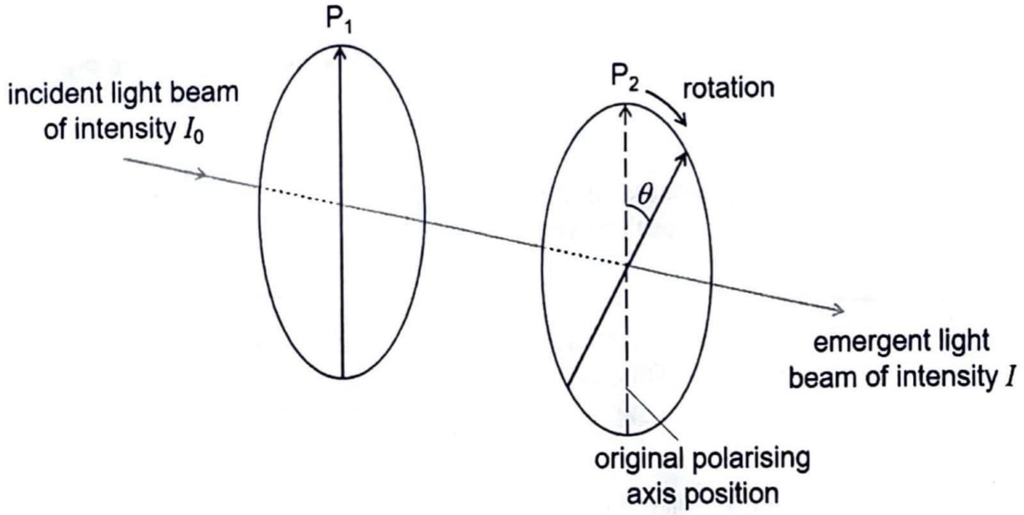
Solution

$$I = I_0 \cos^2\theta$$

$$\frac{I_0/4}{I_0} = \cos^2\theta$$

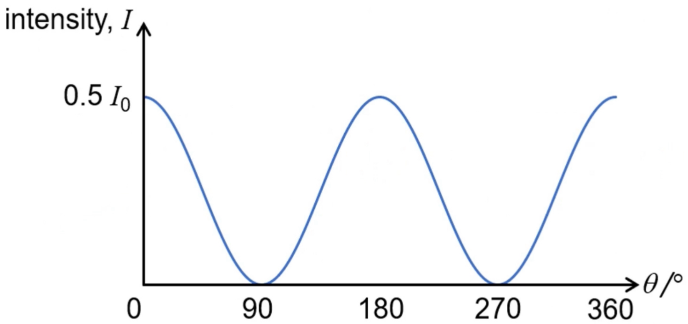
$$\theta = \cos^{-1}(0.5) = 60°$$

Example 9 A beam of unpolarized light with intensity I_0 passes in turn through polaroids P_1 and P_2. P_1 is fixed in position while P_2 is initially aligned with P_1. P_2 is then rotated through 360° as shown.



Sketch a graph to show how the intensity I of the emergent beam varies with the angle of rotation.

Solution



$$I = 0.5 I_0 \cos^2\theta$$

11.4 Frequency of a Sound Wave

11.4.1 Oscilloscopes

An oscilloscope is a type of electronic test instrument that graphically displays varying signal voltages, usually as a two-dimensional plot of one or more signals as a function of time. Other signals (such as **sound or vibration**) can be converted to voltages and displayed.

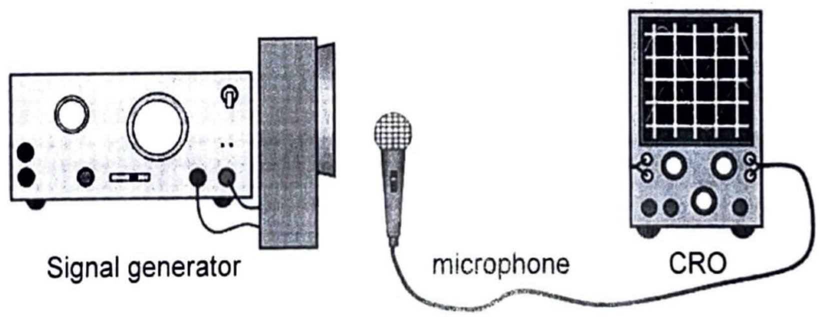
Oscilloscopes display the change of an electrical signal over time, with voltage and time as the *y* and *x* axes, respectively, on a calibrated scale. The waveform can then be analyzed for properties such as amplitude, frequency, rise time, time interval, distortion, and others.

Modern digital instruments may calculate and display these properties directly. For the H2 syllabus, the calculation of these values require measuring the waveform against the scales built into the screen of the instrument.

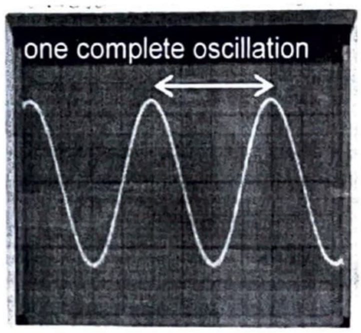
Determination of Frequency of a sound wave

As discussed earlier, sound waves produce regions of compressions and rarefactions as they travel through air. This gives rise to pressure variations as the wave travels.

Hence by placing a microphone in front of a sound source (or a signal generator), the microphone should be able to detect a continuous series of compressions and rarefactions over time. If the microphone is connected to a cathode ray oscilloscope (CRO), the CRO will be able to display a variation of the pressure experienced by the microphone with respect to time.



set-up of apparatus



display on oscilloscope

When a signal is viewed on the CRO display, the period of the signal can be measured by counting the number of horizontal divisions that the signal "covers" on the display and multiplying by the scale of each division. This scale is the "time base".

For the display show, the time base has been set to 1 ms div^{-1}.

Hence, $T = 4 \text{ divisions} \times 1 \text{ ms div}^{-1} = 4 \text{ ms}$

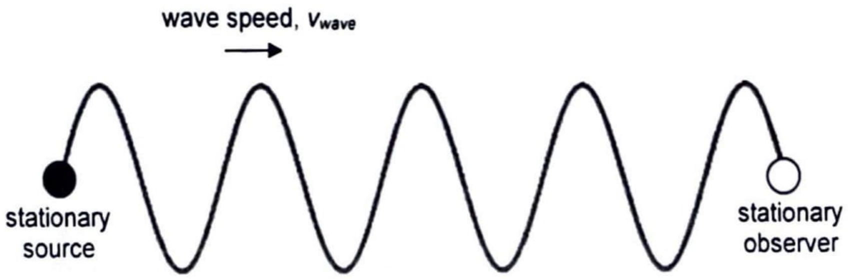
$$\therefore f = \frac{1}{T} = \frac{1}{0.004} = 250 \text{ Hz}$$

Annex

Doppler Effect for Sound Waves (Not in Syllabus)

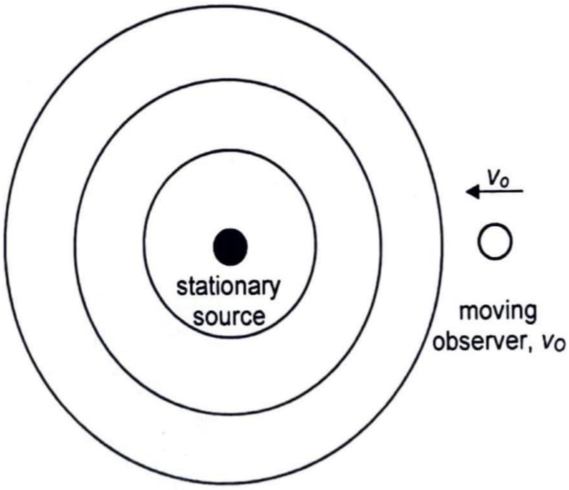
Stationary Source, Stationary Observer

When the source of a wave and an observer are both stationary, the number of waves reaching the observer per second is the same as the number of waves leaving the source per second. Hence, the frequency of the wave produced is the same as that perceived by the observer.



Moving Observer, Stationary Source

However, when the source of the waves remains stationary and the observer moves, the frequency of the waves perceived by the observer is now different from the frequency of the waves produced. This is known as the Doppler Effect.



Suppose the speed of the wave being produced is v_{wave}, and the speed of the observer is v_O, then the perceived frequency f' of the wave when the observer is moving ***towards*** the source is higher and can be found by:

$$f' = f\left(1+\frac{v_o}{v_{wave}}\right)$$

For appreciation.

The perceived frequency f' of the wave when the observer is moving ***away*** from the observer is lower and can be found by:

$$f' = f\left(1-\frac{v_o}{v_{wave}}\right)$$

For appreciation.

Analogy:

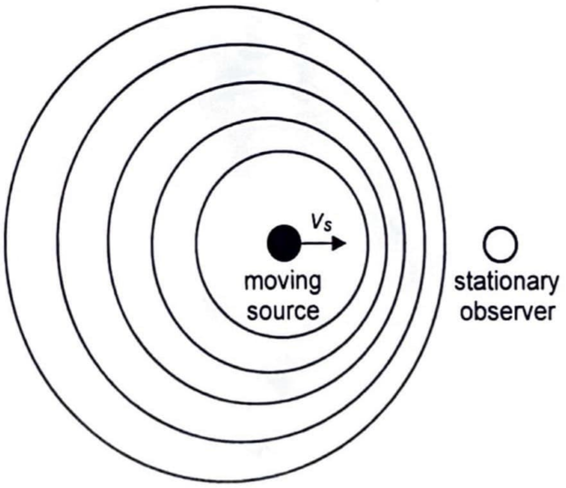
To understand this scenario, imagine a boat near the coast of an island and assume that waves are being produced in the middle of the ocean. If the boat remains at the same spot, it will bob up and down with a certain frequency f. However, once the boat moves towards the ocean, it will bob up and down more quickly because it will pass through the crests and troughs faster than before. Notice that the wave being produced at the source has not changed – it has the same frequency as before. However, the perceived frequency f' of the wave is now higher than before.

Conversely, when the boat now moves towards the shore and is travelling in the same direction as the waves, it passes through the crests and troughs at a slower rate. Hence the frequency of the waves appears to have fallen. Again, the wave being produced at the source has not changed – it has the same frequency as before. However, this time the perceived frequency f' of the wave is now lower than before.

Moving Source, Stationary Observer

The analysis when the wave source moves is slightly different. Once a wave is produced, it is no longer affected by the source of the wave.

When a wave source moves towards the observer, waves are observed to arrive at the observer more rapidly than that being produced. The wavelength of the waves appear to be shorter and the the perceived frequency higher.



Suppose the wave source is moving with speed v_S ***towards*** the observer, the perceived frequency f' of the wave is higher and can be found by:

$$\boxed{f' = f\left(\frac{v_{wave}}{v_{wave} - v_s}\right)}$$

For appreciation.

If the wave source is moving ***away*** from the observer, the perceived frequency f' of the wave is lower and can be found by:

$$\boxed{f' = f\left(\frac{v_{wave}}{v_{wave} + v_s}\right)}$$

For appreciation.

RAFFLES INSTITUTION
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Tutorial 11 WAVE MOTION

Self-Check Questions

S1 Explain the term progressive wave.

S2 Define the displacement, amplitude, period, frequency, wavelength and speed of a wave.

S3 Explain the term phase difference?

S4 What is a wavefront?

S5 Starting with the definition of speed, frequency and wavelength, deduce the formula for speed of a wave.

S6 a) Define the intensity of a wave.
b) State the relationship between the intensity and the amplitude of a wave.
c) State the relationship between intensity and distance from a point source.

S7 Distinguish between longitudinal and transverse waves.

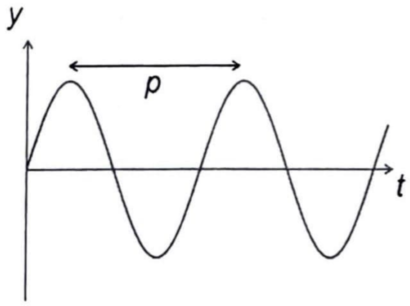
S8 Polarisation is a phenomenon associated with transverse waves and not longitudinal waves. Discuss.

S9 Describe how a cathode ray oscilloscope (c.r.o) may be used to determine the frequency of a sound wave.

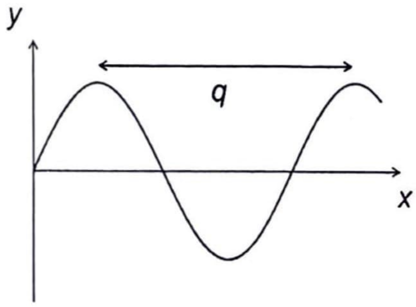
Self-Practice Questions

SP1 A progressive wave can be represented by the following graphs.

(i) Displacement y against time t



(ii) Displacement y against position x

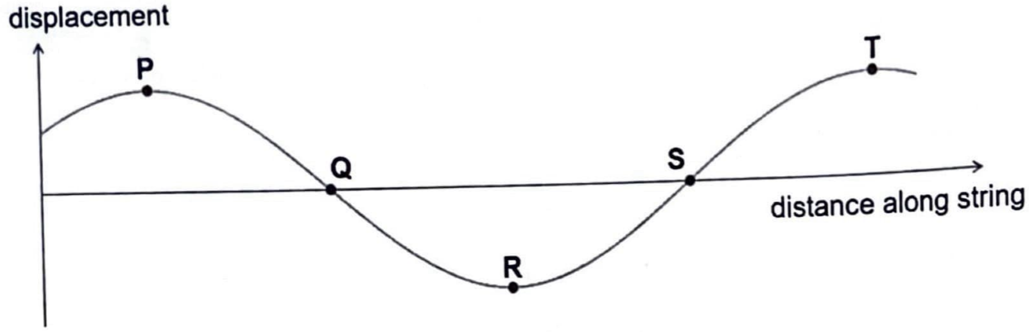


Which one of the following gives the speed of the wave?

A pq **B** $\frac{p}{q}$ **C** $\frac{q}{p}$ **D** $\frac{1}{qp}$

(N91/I/11)

SP2 The graph shows the shape at a particular instant of part of a transverse wave travelling along a string. Which one of the following statements about the motion of points along the string is correct?



A The speed of point P is a maximum.

B The displacement of point Q is always zero.

C The energy of point R is entirely kinetic.

D The acceleration of point T is a maximum.

(N84/II/11)

SP3 Referring to the same diagram in SP2, which one of the following statements is correct?

A The periodic time is represented by the length PT.

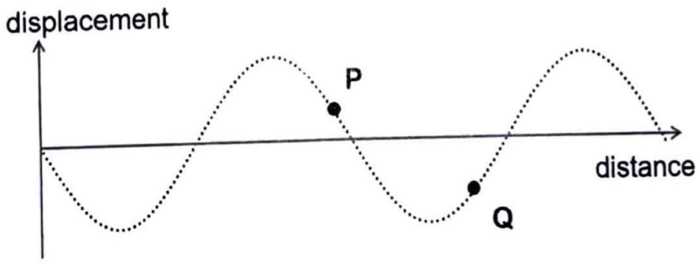
B The particles Q and S have the same velocity.

C The particles R and T are in phase.

D The amplitude of the wave is the vertical separation RT.

E The length QS is half a wavelength.

SP4 A transverse progressive wave travels along a rope. The graph shows the variation of displacement with distance along the rope at a certain time. The wave is travelling to the right. In which directions are P and Q moving?



	Movement of P	Movement of Q
A	downwards	downwards
B	downwards	upwards
C	upwards	downwards
D	upwards	upwards

(N03/I/15)

SP5 Visible light has wavelengths between 400 nm (blue) and 700 nm (red). What is the maximum frequency of visible light?

SP6 The diagram represents the concentration of air molecules in two progressive sound waves. Which statement can be deduced from this diagram?

A The amplitude of wave 1 is greater than the amplitude of wave 2.

B The frequency of wave 1 is greater than the frequency of wave 2.

C The speed of wave 1 is greater than the speed of wave 2.

D Wave 1 and wave 2 are both polarised.

(H1- N14/I/16)

SP7 A transverse wave is passed through a piece of polarizing material. The resulting amplitude of the wave is A. The polarizing material is now rotated through angle θ and the amplitude of the wave becomes $A\cos\theta$.

The new intensity of the transmitted wave is proportional to

A $A\cos\theta$ **B** $A\cos^2\theta$ **C** $A^2\cos\theta$ **D** $A^2\cos^2\theta$

(N14/I/20)

Discussion Questions

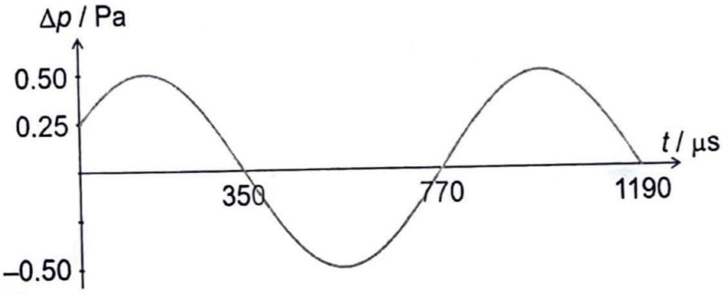
D1 Transverse progressive sinusoidal waves of wavelength λ are passing along a horizontal rope. P and Q are points on the rope $\frac{5}{4}\lambda$ apart and the waves are travelling from P to Q. Which one of the following correctly describes Q at an instant when P is displaced upwards but moving downwards?

	Displacement of Q	Movement of Q
A	upwards	downwards
B	upwards	upwards
C	downwards	upwards
D	downwards	downwards
E	downwards	stationary

(N87/I/8)

D2 **(a)** Distinguish between longitudinal and transverse progressive waves. [2]

(b) The figure below shows the variation with time t of Δp, the excess pressure at a point in a progressive sinusoidal wave in air. The speed of the wave is 340 m s^{-1}.

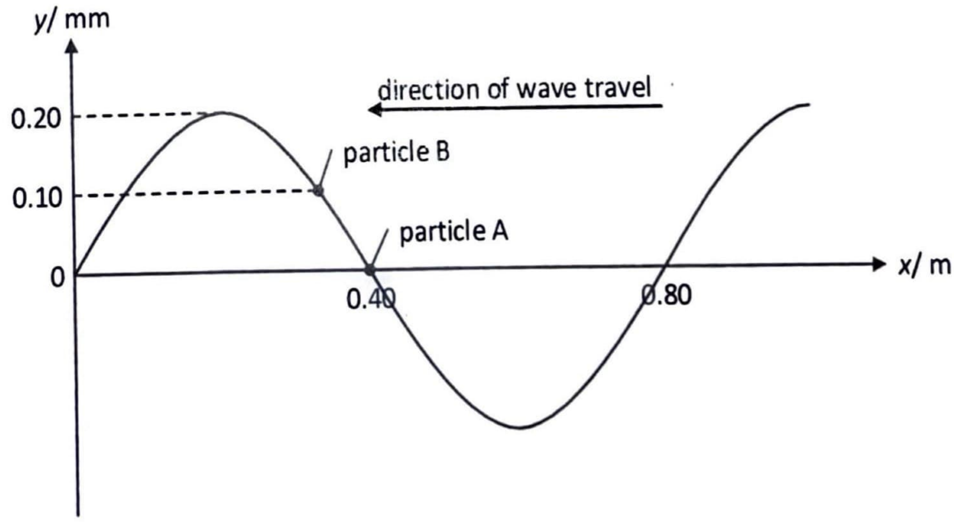


Determine the

(i) amplitude A (of the excess pressure), [1]
(ii) frequency, [2]
(iii) wavelength. [2]

(J89/III/8)

D3 The graph of a wave at time $t = 0$ is shown in the figure below. The wave is travelling to the left with a speed of 340 m s^{-1}.



(a) Calculate the frequency of the oscillation of particle A. [2]

(b) Show that the total energy E_T of the oscillation of particle A is given by

$$E_T = 2\pi^2 m f^2 a^2$$

where a is the amplitude of vibration of particle A and m is the mass of particle A. [2]

(c) The mass of particle A is 4.7×10^{-26} kg. Use the expression in **(b)** to calculate the energy of the oscillations of particle A. [2]

(d) Sketch a graph showing the variation with time of the displacement y for particle A. [2]

(e) Calculate the phase angle between the two particles A and B. [2]

(AJC/H2/Prelims 2018/P3/4)

D4 A ship siren vibrates with displacement $y = A\sin(200\pi t)$, where t is in seconds. This sound causes vibration of the diaphragm of an eardrum of an observer 500 m away. The speed of sound is 335 m s^{-1}. Calculate

(a) the frequency of sound [2]
(b) the number of waves between the siren and the eardrum (to 2 dp) [2]
(c) the phase difference between the motion of the siren and the eardrum. [2]

(J82/III/1)

D5 A plane wave of amplitude A is incident on a surface of area S placed so that it is perpendicular to the direction of travel of the wave.

The energy per unit time intercepted by the surface is P. The amplitude of the wave is increased to $2A$ and the area of the surface is reduced to $\frac{S}{2}$.

How much energy per unit time is intercepted by this smaller surface?

[3]
(N97/I/10)

D6 A point source of sound radiates energy uniformly in all directions. At a distance 3.0 m from the source, the amplitude of vibration of air molecules is 1.0×10^{-7} m.

Assuming that no sound energy is absorbed, calculate the amplitude of vibration 5.0 m from the source.

[3]
(N85/III/2)

D7 A satellite passing the planet Neptune communicates with its controller on Earth using a microwave transmitter with output power 22.0 W and wavelength of 600 μm. Neptune is 4.35×10^{12} m away from Earth at the time when the communication takes place.

(a) State whether the microwaves are progressive or stationary. [1]

(b) State whether the microwaves are longitudinal or transverse. [1]

(c) Calculate the time taken for a signal to travel from the satellite to Earth. [2]

(d) (i) Assuming that the power transmitted by the satellite is radiated uniformly in all directions, calculate the power received on Earth by a dish aerial of effective area 260 m^2. [3]

(ii) The actual power received at the dish aerial is 1.2×10^{-15} W. Suggest why the actual power received is greater than that calculated in (d)(i). [1]

(N02/III/6)

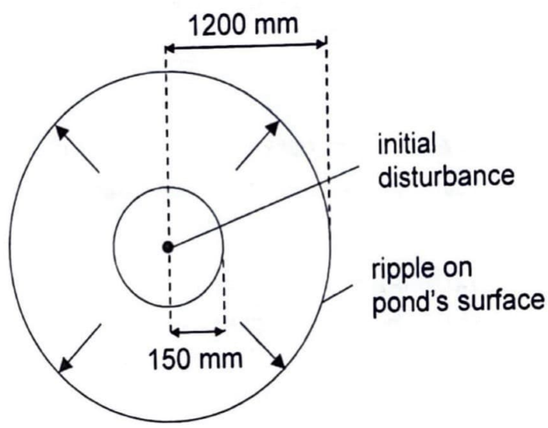
D8 Ripples on the surface of a pond spread out in circles from the point of an initial disturbance. Assume that the energy of the wave is spread over the entire circumference of the ripple.

For one such ripple, the amplitude of the ripple at a distance of 150 mm from the disturbance is 2.0 mm.

What will be the amplitude of the ripple at a distance of 1200 mm from the distrubance?

(Assume that no energy is lost in the propagation of the ripple.)

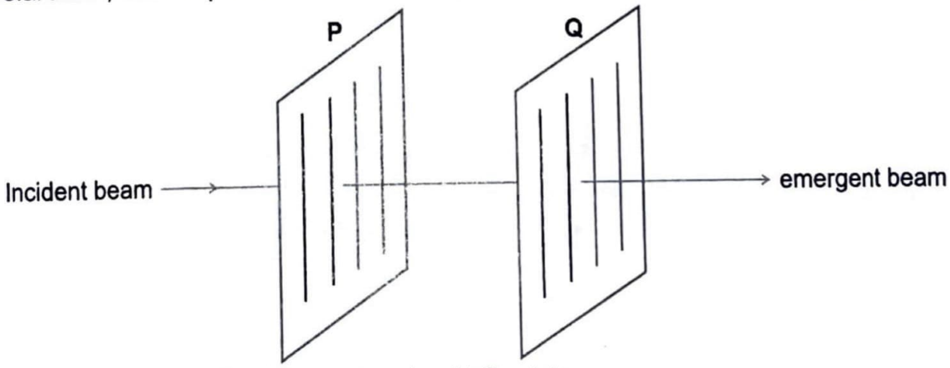
A 0.031 mm
B 0.13 mm
C 0.25 mm
D 0.71 mm



[N10/I/20]

D9 (a) Two sheets of polaroid P and Q are placed close to each other. Their directions of polarization are parallel to one another, as shown below.

A parallel beam of light passes through polaroid P. The beam, after passing through polaroid P, has amplitude *A* and intensity *I*. The beam then passes thorough polaroid Q.



For the light transmitted through polaroid Q, state

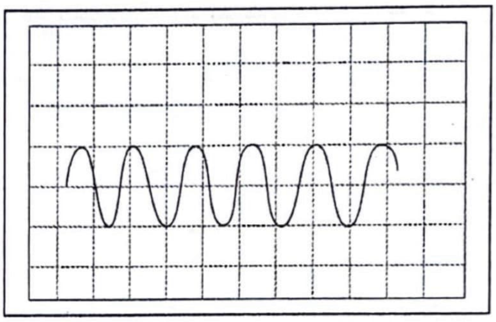
(i) the amplitude (in terms of *A*), [1]
(ii) the intensity (in terms of *I*), [1]
(iii) the relation between the answers to (i) and (ii). [1]

(b) The polaroid Q in **(a)** is now rotated about the axis of the incident beam, while keeping its plane parallel to the plane of polaroid P. The angle between the direction of polarisation of polaroid P and of polaroid Q is θ. Complete the table below to show the amplitude of the emergent beam, in terms of A, and the intensity, in terms of I, for angle θ equal to 180°, 90° and 60°.

angle θ	amplitude	intensity
180°		
90°		
60°		

[3]

D10 The trace below is for a sound wave reflected from a smooth surface. A microphone placed between the loudspeaker and the reflecting surface is fed into a C.R.O. The time base of the C.R.O. is set at 1 ms cm^{-1}.



(a) Calculate the frequency of the sound. [2]

(b) Calculate the distance between two consecutive points with maximum amplitude as the microphone is moved between the wave generator and reflector. Take the speed of sound to be 330 m s^{-1}.
(Note: Distance between 2 consecutive points with maximum amplitude is 0.5 λ.)

[2]

Challenging Question

C1 Two points, A and B, on the Earth are at the same longitude and 60.0° apart in latitude. An earthquake at point A sends two waves towards B. A transverse wave travels at 4500 m s^{-1} along the surface of the Earth while a longitudinal wave travels through the Earth at 7800 m s^{-1}. What is the time difference between the arrivals of the two waves at B?

(Radius of the Earth is 6.37×10^3 km).

Answers

D1 B
D2 (b)(i) 0.50 Pa **(ii)** 1190 Hz **(iii)** 0.29 m
D3 (a) 430 Hz **(b)** 6.7×10^{-27} J **(iii)** 30°
D4 (a) 100 Hz **(b)** 149.25 **(c)** 1.57 rad
D6 6.0×10^{-8} m
D7 (d) (i) 2.41×10^{-23} W
D8 D
D10 (a) 600 Hz **(b)** 0.28 m
C1 666 s

Tutorial 11 Waves Suggested Solutions

Self-Check Questions

S1 A progressive wave is one that transfers energy from one location to another.

S2 **Displacement** of a particle on a wave is the distance it travelled in a specific direction from its equilibrium position.
Amplitude of a wave is the magnitude of the maximum displacement of a particle in the wave from its equilibrium position.
Period of a wave is the time taken for a particle of a wave to complete one oscillation.
Frequency of a wave is the number of oscillations per unit time made by a particle of a wave.
Wavelength of a wave is the distance between two consecutive points which are in phase, such as two successive crests.
Speed of a wave is the distance travelled by the wave over the time taken.

S3 **Phase difference** between two particles in a wave is fraction of a complete oscillation by which one is ahead of the other. It is usually expressed as an angle in radians.

S4 A **wavefront** is a line or surface joining points on a wave that are in phase.

S5 In a time of one period, the waveform moves a distance of one wavelength.
Using the formula $\text{speed} = \frac{\text{distance}}{\text{time}}$, $v = \frac{\lambda}{T}$. Since $f = \frac{1}{T}$ $\Rightarrow v = f\lambda$

S6
a) The intensity I of a wave is defined as the rate of transfer of energy per unit area normal to the direction of propagation of the wave.
b) Intensity is proportional to the square of the amplitude of the wave.
c) Intensity is inversely proportional to the square of the distance from a point source.

S7 The particles of a **longitudinal** wave oscillate parallel to the direction of energy transfer, whereas the particles of a **transverse** wave oscillate perpendicularly to the direction of energy transfer.

S8 Polarisation is a phenomenon whereby the oscillations of **transverse** waves are restricted to a single plane.
Transverse waves can be polarised because the oscillations of the particles are perpendicular to the direction of propagation.
Longitudinal waves cannot be polarised because the direction of oscillations of the particles and direction of propagation are the same.

S9
1. Connect a microphone to a cathode ray oscilloscope.
2. Point the microphone at the source of the sound (e.g. a loudspeaker connected to a signal generator).
3. When the signal is viewed on the CRO display, the period T of the signal can be measured by counting the number of horizontal divisions that the signal covers for one cycle and multiplying this number by the time base,
   e.g. $T = 4 \text{ div} \times 1 \text{ ms div}^{-1} = 4 \text{ ms}$
4. Frequency is then obtained by taking the reciprocal of the period
   i.e. $f = \frac{1}{T}$

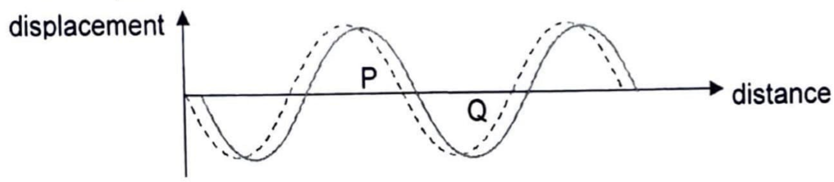
Self-Practice Questions

SP1 **C** $T = p,\ \lambda = q \ \therefore\ v = \frac{\lambda}{T} = \frac{q}{p}$

SP2 **D**
Option A & C: At maximum displacement, the particle is changing direction. Its speed is thus zero.
Option B: The particles in a progressive wave are constantly moving.
Option D: At maximum displacement, $a = -\omega^2 x_0$, so acceleration is maximum.

SP3 **E**
Option A: PT represents a wavelength as the x-axis represents distance
Option B: Q & S have the same speed but are moving in different directions (anti-phase)
Option C: R & T have opposite displacements (anti-phase)
Option D: the vertical separation between RT represents twice the amplitude.

SP4 **C** The graph of the next instant is indicated by the solid curve (shift to the right).
P moves upwards, Q moves downwards to form the solid curve.



SP5

$$c = f\lambda$$

$$3.00 \times 10^8 = f\left(400 \times 10^{-9}\right) \ \Rightarrow f = 7.50 \times 10^{14} \text{ Hz}$$

SP6 **A**
Option A: The amplitudes of both waves can be compared from the separation between the dots.
Option B: The wavelength of wave 2 is obviously shorter as compared to wave 1. Since they both travel at the same speed, the frequency of wave 2 is higher than wave 1.
Option C: Both waves are sound waves, so they travel at the same speed.
Option D: Longitudinal waves cannot be polarized.

SP7 **D** By Malus' Law
$I = I_0 \cos^2\theta = k\left(A^2 \cos^2\theta\right)$