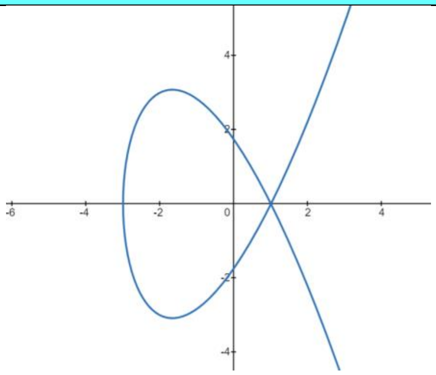
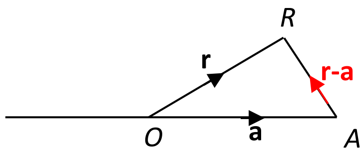
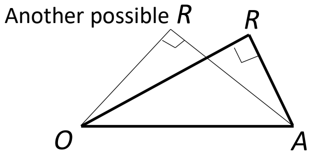
2024 NYJC J2 H2 Mathematics Preliminary Exam 9758/2

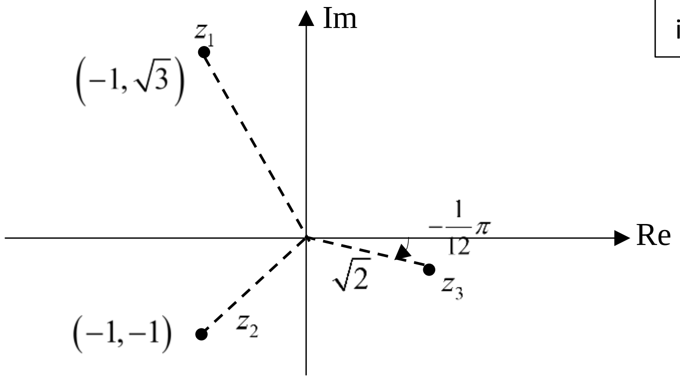
Q1	**Suggested Answers**
(a)	 Area of loop = $2\int_{\alpha}^{\beta}(1-x)\sqrt{x+3}\,\mathrm{d}x$ $\alpha=-3,\ \beta=1$
(b)	Area of loop $=2\int_{-3}^{1}(1-x)\sqrt{x+3}\,\mathrm{d}x$ $=2\int_{0}^{2}\left(4-u^2\right)u\times 2u\,\mathrm{d}u$ $=4\int_{0}^{2}\left(4u^2-u^4\right)\mathrm{d}u$ $=4\left[\frac{4}{3}u^3-\frac{1}{5}u^5\right]_0^2$ $=\frac{256}{15}$ units^2 $u=\sqrt{x+3}$ $u^2=x+3$ $2u\frac{\mathrm{d}u}{\mathrm{d}x}=1$ or $u=\sqrt{x+3}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x+3}}$ $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2u}$ When $x=-3$, $u=0$ When $x=1$, $u=2$

Q2	Suggested Answers
(a)	Vector equation $\mathbf{r}=\lambda\mathbf{a}$ where $0\le\lambda\le1$ The equation gives the position vector of **points on the line segment *OA***. (not line *OA*)
(b)	Vector equation $\mathbf{r}\times(\mathbf{a}-\mathbf{b})=\mathbf{0}$ $\Rightarrow$ $\mathbf{r}$ is parallel to $\overrightarrow{AB}$ $\mathbf{r}=k(\mathbf{a}-\mathbf{b}),\ k\in\mathbb{R}$ The equation gives the position vector of **points on the line passing through *O* parallel to *AB***.
(c)	$\mathbf{r}\cdot(\mathbf{r}-\mathbf{a})=0$ $\overrightarrow{OR}\cdot\overrightarrow{AR}=0$ $\overrightarrow{OR}\perp\overrightarrow{AR}$   Points on a sphere (or circle) with *OA* as a diameter.
(d)	$(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\mathbf{0}$ $(\mathbf{r}-\mathbf{a})$ is parallel to $(\mathbf{r}-\mathbf{b})$ $(\mathbf{r}-\mathbf{a})=\mu(\mathbf{r}-\mathbf{b})$ where $\mu\neq1$ $\mathbf{r}(1-\mu)=\mathbf{a}-\mu\mathbf{b}$ $\mathbf{r}=\dfrac{\mathbf{a}-\mu\mathbf{b}}{(1-\mu)}=\left(\dfrac{1}{1-\mu}\right)\mathbf{a}+\left(\dfrac{-\mu}{1-\mu}\right)\mathbf{b}$, where $\mu\neq1$ since $\mathbf{a}\neq\mathbf{b}$ **Alternatively**, $(\mathbf{r}-\mathbf{a})\times(\mathbf{r}-\mathbf{b})=\mathbf{0}$ $\overrightarrow{AR}$ is parallel to $\overrightarrow{BR}$ Since *R* is a common point, *A*, *B* and *R* are colinear i.e. *R* lies on the line *AB* $\mathbf{r}=\mathbf{a}+\alpha(\mathbf{b}-\mathbf{a})$, where $\alpha\in\mathbb{R}$

<table>
<tr><th>Q3</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>

$z_1=-1+\sqrt{3}\mathrm{i}$, $z_2=-1-\mathrm{i}$, $z_3=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}$

Show modulus (length) and argument (angle), if possible.



</td></tr>
<tr><td>(b)</td><td>

$$z_1=2\mathrm{e}^{\mathrm{i}\left(\frac{2}{3}\pi\right)}$$

$$z_2=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{3}{4}\pi\right)}$$

$$z_3=\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}$$

Method 1: Properties of modulus & argument

$$\left|\frac{z_1}{z_2\left(z_3\right)^2}\right|=\frac{\left|z_1\right|}{\left|z_2\right|\left|z_3\right|^2}=\frac{2}{\left(\sqrt{2}\right)\left(\sqrt{2}\right)^2}=\frac{1}{\sqrt{2}}$$

$$\arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)=\arg z_1-\arg z_2-2\arg z_3$$

$$=\frac{2}{3}\pi-\left(-\frac{3}{4}\pi\right)-2\left(-\frac{1}{12}\pi\right)$$

$$=\frac{19}{12}\pi$$

$$\therefore \arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)=\frac{19}{12}\pi-2\pi=-\frac{5}{12}\pi$$

$$\therefore \frac{z_1}{z_2\left(z_3\right)^2}=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(-\frac{5}{12}\pi\right)}$$

</td></tr>
<tr><td>(b)</td><td>

Method 2: Using exponential form

$$\frac{z_1}{z_2\left(z_3\right)^2}=\frac{2\mathrm{e}^{\mathrm{i}\left(\frac{2}{3}\pi\right)}}{\left(\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{3}{4}\pi\right)}\right)\left(\sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{1}{12}\pi\right)}\right)^2}$$

$$=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(\frac{19}{12}\pi\right)}$$

</td></tr>
</table>

	$=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(\frac{19}{12}\pi-2\pi\right)}$ $=\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(-\frac{5}{12}\pi\right)}$
(c)	Let $z_4=r\mathrm{e}^{\mathrm{i}\theta}$ **Method 1: Properties of modulus & argument** $\left\lvert\frac{z_1z_4}{z_2\left(z_3\right)^2}\right\rvert=1$ $\left\lvert\frac{z_1}{z_2\left(z_3\right)^2}\right\rvert\left\lvert z_4\right\rvert=1$ $\Rightarrow r=\sqrt{2}$ $\arg\left(\frac{z_1z_4}{z_2\left(z_3\right)^2}\right)=\arg\left(\frac{z_1}{z_2\left(z_3\right)^2}\right)+\arg z_4$ $=-\frac{5}{12}\pi+\theta$ Since $\frac{z_1z_4}{z_2\left(z_3^*\right)^2}$ is purely real, $-\frac{5}{12}\pi+\theta=k\pi,k\in\mathbb{Z}$ $\theta=k\pi+\frac{5}{12}\pi$ $=\frac{5}{12}\pi$ or $-\frac{7}{12}\pi \quad \left(\because -\pi<\theta\le\pi\right)$ $\therefore z_4=\sqrt{2}\left(\cos\frac{5}{12}\pi+\mathrm{i}\sin\frac{5}{12}\pi\right)$ or $\sqrt{2}\left(\cos\left(-\frac{7}{12}\pi\right)+\mathrm{i}\sin\left(-\frac{7}{12}\pi\right)\right)$
(c)	**Method 2: Using exponential form** $\frac{z_1z_4}{z_2\left(z_3\right)^2}=\left[\frac{1}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(-\frac{5}{12}\pi\right)}\right]\left(r\mathrm{e}^{\mathrm{i}\theta}\right)$ $=\frac{r}{\sqrt{2}}\mathrm{e}^{\mathrm{i}\left(-\frac{5}{12}\pi+\theta\right)}$ $\frac{r}{\sqrt{2}}=1\Rightarrow r=\sqrt{2}$ Since $\frac{z_1z_4}{z_2\left(z_3^*\right)^2}$ is purely real, $-\frac{5}{12}\pi+\theta=k\pi,k\in\mathbb{Z}$ $\theta=k\pi+\frac{5}{12}\pi=\frac{5}{12}\pi$ or $-\frac{7}{12}\pi \quad \left(\because -\pi<\theta\le\pi\right)$ $\therefore z_4=\sqrt{2}\left(\cos\frac{5}{12}\pi+\mathrm{i}\sin\frac{5}{12}\pi\right)$ or $\sqrt{2}\left(\cos\left(-\frac{7}{12}\pi\right)+\mathrm{i}\sin\left(-\frac{7}{12}\pi\right)\right)$

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Q4	Suggested Answers
(a)	$\frac{1}{4r^2-8r+3}=\frac{A}{2r-3}+\frac{B}{2r-1}$ $A(2r-1)+B(2r-3)=1$ When $r=\frac{1}{2}$, $B=-\frac{1}{2}$ When $r=\frac{3}{2}$, $A=\frac{1}{2}$ $\frac{1}{4r^2-8r+3}=\frac{1}{2}\left(\frac{1}{2r-3}-\frac{1}{2r-1}\right)$
(b)	$\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)=\frac{1}{2}\sum_{r=2}^{3n}\left(\frac{1}{2r-3}-\frac{1}{2r-1}\right)$ $=\frac{1}{2}\left(\begin{array}{l} 1-\cancel{\frac{1}{3}} \\ +\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}} \\ +\cancel{\frac{1}{5}}-\frac{1}{7} \\ \vdots \\ +\cancel{\frac{1}{6n-5}}-\cancel{\frac{1}{6n-3}} \\ +\cancel{\frac{1}{6n-3}}-\frac{1}{6n-1} \end{array}\right)$ $=\frac{1}{2}\left(1-\frac{1}{6n-1}\right)\quad\left(=\frac{3n-1}{6n-1}\right)$
(c)	As $n\to\infty$, $\frac{1}{6n-1}\to 0$ and so $\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)\to\frac{1}{2}$ $\therefore\ \sum_{r=2}^{\infty}\left(\frac{1}{4r^2-8r+3}\right)=\frac{1}{2}$
(d)	$\sum_{r=n+1}^{3n}\left(\frac{1}{4r^2-8r+3}\right)=\sum_{r=2}^{3n}\left(\frac{1}{4r^2-8r+3}\right)-\sum_{r=2}^{n}\left(\frac{1}{4r^2-8r+3}\right)$ $=\frac{1}{2}\left(1-\frac{1}{6n-1}\right)-\frac{1}{2}\left(1-\frac{1}{2n-1}\right)$ $=\frac{1}{2}\left(\frac{1}{2n-1}-\frac{1}{6n-1}\right)$ $=\frac{4n}{2(2n-1)(6n-1)}$ $=\frac{2n}{(2n-1)(6n-1)}$

Q5	Suggested Answers
(a)	Volume of wok $=\pi\int_{-16}^{q}\left(256-y^2\right)\mathrm{d}y$ $=\pi\left[256y-\frac{y^3}{3}\right]_{-16}^{q}$ $=\pi\left[\left(256q-\frac{q^3}{3}\right)-\left(-4096+\frac{4096}{3}\right)\right]$ $=\pi\left(256q-\frac{q^3}{3}+\frac{8192}{3}\right)$ $\pi\left(256q-\frac{q^3}{3}+\frac{8192}{3}\right)=3300$ Using GC, $q=-7.01245634$ Depth of wok = −7.01245634 – (−16) = 9 cm (correct to nearest integer)
(b)	**Method 1 (direct integration):** Volume of flat frying pan $=\pi\int_{r}^{0}\left(256-y^2\right)\mathrm{d}y$ $=\pi\left[256y-\frac{y^3}{3}\right]_{r}^{0}$ $=\pi\left[0-\left(256r-\frac{r^3}{3}\right)\right]$ $=\pi\left(\frac{r^3}{3}-256r\right)$ $\pi\left(\frac{r^3}{3}-256r\right)=1464\pi$ $\frac{r^3}{3}-256r=1464$ Using GC, $r=-6$
(b)	**Method 2 (using result from part (a)):** Volume of flat frying pan $=$ Volume of hemisphere $-\ \pi\left(256r-\frac{r^3}{3}+\frac{8192}{3}\right)$ $=\frac{2}{3}\pi(16)^3-\pi\left(256r-\frac{r^3}{3}+\frac{8192}{3}\right)$ $=\pi\left(\frac{r^3}{3}-256r\right)$ $\pi\left(\frac{r^3}{3}-256r\right)=1464\pi$ $\frac{r^3}{3}-256r=1464$ Using GC, $r=-6$

(c)	Let time taken to fill the wok to full capacity be *T* seconds **Method 1:** $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3}{55}t$ $\int \mathrm{d}V = \int \frac{3}{55}t \, \mathrm{d}t$ $V = \frac{3t^2}{110} + C$ When $t = 0$, $V = 0$, $C = 0$ When $t = T$, $V = 3.3$, $3.3 = \frac{3T^2}{110}$ $T = 11$
	Method 2: $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{3}{55}t$ $\int_0^{3.3} \mathrm{d}V = \int_0^T \frac{3}{55}t \, \mathrm{d}t$ $\left[V\right]_0^{3.3} = \frac{3}{55}\left[\frac{t^2}{2}\right]_0^T$ $3.3 = \frac{3}{55}\left(\frac{T^2}{2}\right)$ $T = 11$

Q6	**Suggested Answers**
(a)	(Treat as there is an 'invisible' car occupying a lot) Number of arrangements without restriction = $10!=3628800$
(b)	Number of arrangements with BB and RR together $= 8! \times 2! \times 2! = 161280$
(c)	**Method 1** Number of arrangements with B1B2 together $=9! \times 2! = 725760$ Similarly, number of arrangements with R1R2 together $9! \times 2! = 725760$ Number of arrangements with at least 2 adjacent cars $= 725760+725760-161280=1290240$ By complement method, required number of arrangements is $=3628800-1290240=2338560$
(c)	**Method 2** Number of arrangements with B1B2 together and R1R2 not together = $7! \times 2! \times {}^{8}C_{2} \times 2! = 564480$ Similarly, number of arrangements with R1R2 together and B1B2 not together $=564480$ By complement, required number of arrangements is $3628800-161280-564480-564480=2338560$
(c)	**Method 3** Number of arrangements with red cars separated (no restrictions on blue cars) = $8! \times {}^{9}C_{2} \times 2! = 2903040$ Number of arrangements with red cars separated and blue cars together = $7! \times 2! \times {}^{8}C_{2} \times 2! = 564480$ By complement, required number of arrangements is $2903040-564480=2338560$

Q7	**Suggested Answers**

(a)

$$\mathrm{P}(S=s)>\mathrm{P}(S=s+1)$$

$$\frac{{}^{18}C_s\ {}^{12}C_{10-s}}{{}^{30}C_{10}}>\frac{{}^{18}C_{s+1}\ {}^{12}C_{9-s}}{{}^{30}C_{10}}$$

$$\frac{18!}{s!(18-s)!}.\frac{12!}{(s+2)!(10-s)!}>\frac{18!}{(s+1)!(17-s)!}.\frac{12!}{(s+3)!(9-s)!}$$

$$(s+1)!(17-s)!(9-s)!(s+3)!>s!(18-s)!(10-s)!(s+2)!$$

$$(s+1)(s+3)>(18-s)(10-s)$$

$$s^2+4s+3>s^2-28s+180$$

$$32s>177$$

$$s>5.53$$

Thus s = 6.

(b)

Outcome Table

Absolute Difference		No. on Square					
		1	2	3	4	5	6
No. on Triangle	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2

Probability Distribution

x	0	1	2	3	4	5
$\mathrm{P}(X=x)$	$\frac{4}{24}$	$\frac{7}{24}$	$\frac{6}{24}$	$\frac{4}{24}$	$\frac{2}{24}$	$\frac{1}{24}$

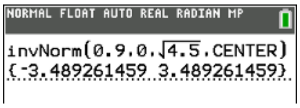
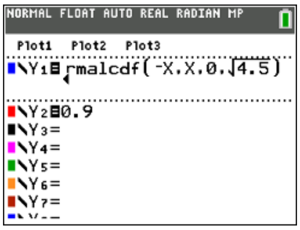
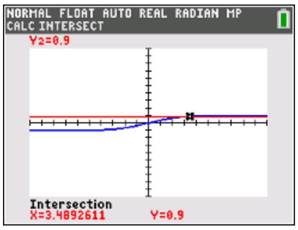
(c)

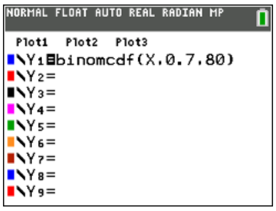
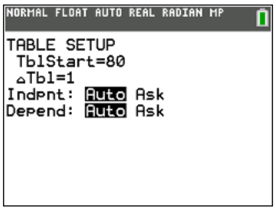
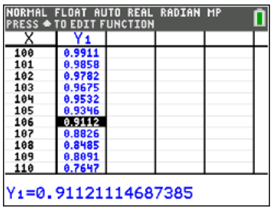
$$\mathrm{P}(X_1=2\mid X_1+X_2+X_3=3)$$

$$=\frac{\mathrm{P}(X_1=2,X_2=0,X_3=1)\times 2!}{\mathrm{P}(X_1=1,X_2=1,X_3=1)+\mathrm{P}(X_1=1,X_2=2,X_3=0)\times 3!\ +\mathrm{P}(X_1=3,X_2=0,X_3=0)\times\frac{3!}{2!}}$$

$$=\frac{\frac{6}{24}\left(\frac{4}{24}\right)\left(\frac{7}{24}\right)2!}{\left(\frac{7}{24}\right)^3+\frac{7}{24}\left(\frac{6}{24}\right)\left(\frac{4}{24}\right)3!++\frac{4}{24}\left(\frac{4}{24}\right)^2\frac{3!}{2!}}$$

$$=\frac{336}{1543}\quad\text{or 0.218 (to 3 s.f.)}$$

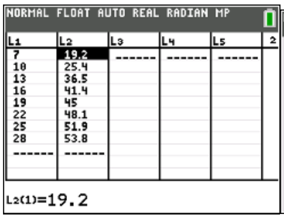
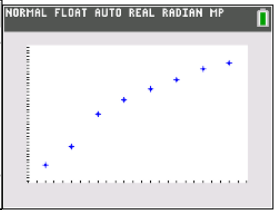
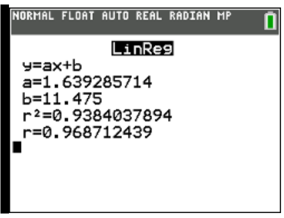
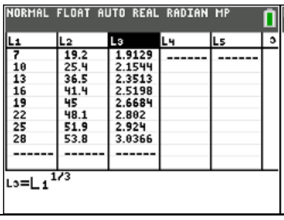
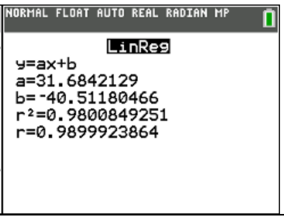
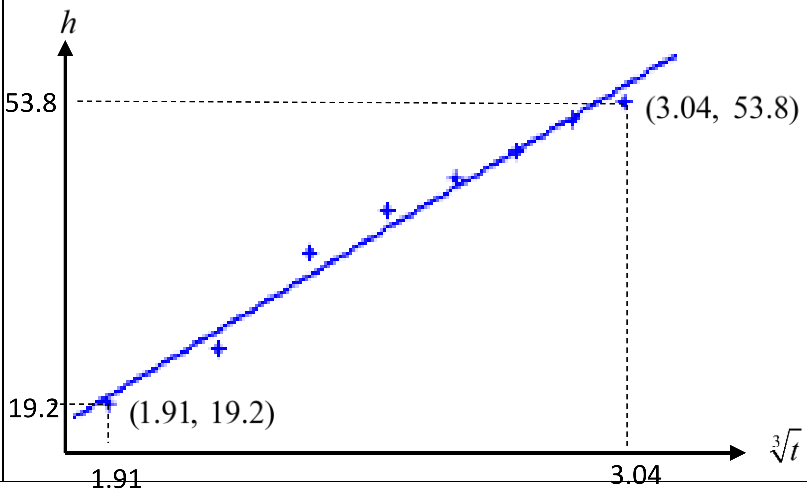
Q8	**Suggested Answers**
(a)	Let L be the length of a randomly chosen rectangular cotton fabric. $L \sim \mathrm{N}\left(24,\ 1.5^2\right)$ $\mathrm{P}\left(L < 23.5\right) = 0.36944$ (5 s.f.) $= 0.369$ (3 s.f.)
(b)	Let B be the breadth of a randomly chosen rectangular cotton fabric. $B \sim \mathrm{N}\left(20,\ 1.2^2\right)$ Perimeter $= 2L + 2B \sim \mathrm{N}\left(88,\ 14.76\right)$ $\mathrm{P}\left(2L + 2B > 90\right) = 0.30133$ (5 s.f.) $= 0.301$ (3 s.f.)
(c)	The **length and breadth** of each /a randomly chosen rectangular cotton fabric **are independent** of each other. Note that this assumption is necessary for Var(2L + 2B) = Var(2L) + Var(2B) Recall: Var(X + Y) = Var(X) + Var(Y) if and only if X and Y are independent.
(d)	$L \sim \mathrm{N}\left(24,\ 1.5^2\right)$ $\mathrm{P}\left(23 < L < 25\right) = 0.49502$ (5 s.f.) Let X be the number of rectangular cotton fabric (out of 48) with length between 23 and 25 cm. $X \sim \mathrm{B}\left(48, 0.49502\right)$ $\mathrm{E}\left(X\right) = 48\left(0.49502\right) = 23.761$ (5 s.f.) $= 23.8$ (3 s.f.)
(e)	$L_1 - L_2 \sim \mathrm{N}\left(0,\ 4.5\right)$ $\mathrm{P}\left(\left\lvert L_1 - L_2 \right\rvert \le k\right) \ge 0.9$ $\mathrm{P}\left(-k \le L_1 - L_2 \le k\right) \ge 0.9$ $k \ge 3.4893$ $k \ge 3.49$ (3 s.f.) Least $k = 3.49$ (3 s.f.) **Method 1:**  NORMAL FLOAT AUTO REAL RADIAN MP invNorm(0.9,0,√4.5,CENTER) {-3.489261459 3.489261459} **Method 2:**  NORMAL FLOAT AUTO REAL RADIAN MP Plot1 Plot2 Plot3 Y1=rmalcdf(-X,X,0,√4.5) Y2=0.9 Y3= Y4= Y5= Y6= Y7=  NORMAL FLOAT AUTO REAL RADIAN MP CALC INTERSECT Y2=0.9 Intersection X=3.4892611 Y=0.9

<table>
<tr><th>Q9</th><th>Suggested Answers</th></tr>
<tr><td>(a)</td><td>Let X be the number of diners (out of 10) who order the signature dish
$X \sim \mathrm{B}(10,\ 0.7)$
$\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X < 3)$
$= 1 - \mathrm{P}(X \leq 2)$
$= 0.99841$ (5 s.f.)
$= 0.998$ (3 s.f.)</td></tr>
<tr><td>(b)</td><td>Method 1:
$\mathrm{P}(3 < X < 8) = \mathrm{P}(X < 8) - \mathrm{P}(X \leq 3)$
$= \mathrm{P}(X \leq 7) - \mathrm{P}(X \leq 3)$
$= 0.60663$ (5 s.f.)
$= 0.607$ (3 s.f.)</td></tr>
<tr><td>(b)</td><td>Method 2 (not recommended):
$\mathrm{P}(3 < X < 8) = \mathrm{P}(X = 4) + \mathrm{P}(X = 5) + \mathrm{P}(X = 6) + \mathrm{P}(X = 7)$
$= 0.60663$ (5 s.f.)
$= 0.607$ (3 s.f.)</td></tr>
<tr><td>(c)</td><td>Let Y be the number of diners (out of n) who order the signature dish
$Y \sim \mathrm{B}(n,\ 0.7)$
$\mathrm{P}(Y \leq 80) \geq 0.9$
<table>
<tr><th>n</th><th>$\mathrm{P}(Y \leq 80)$</th></tr>
<tr><td>106</td><td>$0.9112 > 0.9$</td></tr>
<tr><td>107</td><td>$0.8826 < 0.9$</td></tr>
</table>
largest $n = 106$ (maximum number of diners)

NORMAL FLOAT AUTO REAL RADIAN MP
Plot1 Plot2 Plot3
Y1=binomcdf(X,0.7,80)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
Y8=
Y9=

NORMAL FLOAT AUTO REAL RADIAN MP
TABLE SETUP
TblStart=80
△Tbl=1
Indpnt: Auto Ask
Depend: Auto Ask

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS ▲ TO EDIT FUNCTION
X Y1
100 0.9911
101 0.9858
102 0.9782
103 0.9675
104 0.9532
105 0.9346
106 0.9112
107 0.8826
108 0.8485
109 0.8091
110 0.7647
Y1=0.91121114687385</td></tr>
<tr><td>(d)</td><td>$\text{Probability} = \left[\mathrm{P}(X \geq 3)\right]^{40} = (0.99841)^{40} = 0.938$ (3 s.f.)
OR $T \sim \mathrm{B}(40,\ 0.99841)$
$\mathrm{P}(T = 40) = 0.93833 \approx 0.938$</td></tr>
<tr><td>(e)</td><td>Let X be the number of diners (out of 10) who order the signature dish, i.e. number of portions of the signature dish served at a randomly chosen table of 10 diners
$X \sim \mathrm{B}(10,\ 0.7)$
$\mathrm{E}(X) = 7,\ \mathrm{Var}(X) = 2.1$
Average number of portions of the signature dish served per table,
$\bar{X} = \dfrac{X_1 + X_2 + \ldots + X_{40}}{40}$
Since $n = 40$ is large, by the Central Limit Theorem,</td></tr>
</table>

	$\bar{X} \sim \mathrm{N}\left(7, \frac{2.1}{40}\right)$ approximately $\mathrm{P}\left(\bar{X} \geq 6.9\right) = 0.66874$ (5 s.f.) $= 0.669$ (3 s.f.)

Q10	Suggested Answers
(a)	Unbiased estimate of population mean, $\bar{t} = \frac{543}{30} = 18.1$ Unbiased estimate of population variance, $s^2 = \frac{1}{29}\left[12722 - \frac{543^2}{30}\right] = 99.783\ (5\text{ s.f.}) = 99.8\ (3\text{ s.f.})$
(b)	Let μ be the population mean bus arrival times after the scheduled pick-up time $\mathrm{H_0}: \ \mu = 15$ $\mathrm{H_1}: \ \mu > 15$ Test at 5% level of significance Under $\mathrm{H_0}$, since $n = 30$ is large, **by the Central Limit Theorem**, $\bar{T} \sim \mathrm{N}\left(15, \frac{99.783}{30}\right)$ approximately Test statistic: $Z = \frac{\bar{T} - 15}{\sqrt{\frac{99.783}{30}}} \sim \mathrm{N}(0, 1)$ approximately p-value = 0.044586 or $z_{\text{cal}} = \frac{18.1 - 15}{\sqrt{\frac{99.783}{30}}} = 1.6998(5\text{ s.f.})$ Since p-value ≤ 0.05 (or $z_{\text{cal}} \geq 1.6449$), we reject $\mathrm{H_0}$. There is sufficient evidence at 5% level of significance to conclude that the administration manager should agree with the feedback from teachers and students.
(c)	Test $\mathrm{H_0}: \ \mu = 15$ $\mathrm{H_1}: \ \mu \neq 15$ at 10 % level of significance Under $\mathrm{H_0}$, since $n = 40$ is large, by the Central Limit Theorem, $\bar{T} \sim \mathrm{N}\left(15, \frac{99.783}{40}\right)$ approximately Test statistic: $Z = \frac{\bar{T} - 15}{\sqrt{\frac{99.783}{40}}} \sim \mathrm{N}(0, 1)$ approximately Since $\mathrm{H_0}$ is not rejected, z-value does not lie in critical region $-1.6449 < \frac{\bar{t} - 15}{\sqrt{\frac{99.783}{40}}} < 1.6449$ $12.402 < \bar{t} < 17.598$ (5 s.f.) $12.4 < \bar{t} < 17.6$ (3 s.f.)

(d)	The sample of 40 buses is a **random sample**. There is **no change in the unbiased estimate of the population variance** bus arrival times after the scheduled pick-up time after the bus company claims to have made changes to its operation.

Q11	**Suggested Answers**
(a)	Since $\left(\bar{t}, \bar{h}\right)$ lies on the regression line, $\bar{h} = 1.6393\bar{t} + 11.475$ $\dfrac{k + 273.2}{8} = 1.6393\left(17.5\right) + 11.475$ $k = 48.1$ (to 1 d.p.)  NORMAL FLOAT AUTO REAL RADIAN MP L1 L2 L3 L4 L5 7 19.2 10 25.4 13 36.5 16 41.4 19 45 22 48.1 25 51.9 28 53.8 L2(1)=19.2  NORMAL FLOAT AUTO REAL RADIAN MP  NORMAL FLOAT AUTO REAL RADIAN MP LinReg y=ax+b a=1.639285714 b=11.475 r²=0.9384037894 r=0.968712439
(b)	For $h = a\sqrt[3]{t} + b$, $r = 0.98999$ For $h = c\sqrt{t} + d$, $r = 0.98615$ Since r-value for $h = a\sqrt[3]{t} + b$ is closer to 1 than the r-value for $h = c\sqrt{t} + d$, the linear correlation between h and $\sqrt[3]{t}$ is stronger. Using GC, the equation is $h = 31.684\sqrt[3]{t} - 40.512$ (5 s.f.) $h = 31.7\sqrt[3]{t} - 40.5$ (3 s.f.)  NORMAL FLOAT AUTO REAL RADIAN MP L1 L2 L3 L4 L5 7 19.2 1.9129 10 25.4 2.1544 13 36.5 2.3513 16 41.4 2.5198 19 45 2.6684 22 48.1 2.802 25 51.9 2.924 28 53.8 3.0366 L3=L1^1/3  NORMAL FLOAT AUTO REAL RADIAN MP LinReg y=ax+b a=31.6842129 b=-40.51180466 r²=0.9800849251 r=0.9899923864
(c)	Since the age of 2 months ($t \approx 60$) is out of the data range of t, the estimate is not reliable.
(d)	

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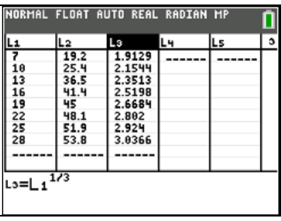
NORMAL FLOAT AUTO REAL RADIAN MP
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<tr><td>7</td><td>19.2</td><td>1.9129</td><td>------</td><td>------</td></tr>
<tr><td>10</td><td>25.4</td><td>2.1544</td><td></td><td></td></tr>
<tr><td>13</td><td>36.5</td><td>2.3513</td><td></td><td></td></tr>
<tr><td>16</td><td>41.4</td><td>2.5198</td><td></td><td></td></tr>
<tr><td>19</td><td>45</td><td>2.6684</td><td></td><td></td></tr>
<tr><td>22</td><td>48.1</td><td>2.802</td><td></td><td></td></tr>
<tr><td>25</td><td>51.9</td><td>2.924</td><td></td><td></td></tr>
<tr><td>28</td><td>53.8</td><td>3.0366</td><td></td><td></td></tr>
<tr><td>------</td><td>------</td><td>------</td><td></td><td></td></tr>
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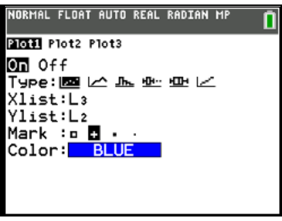
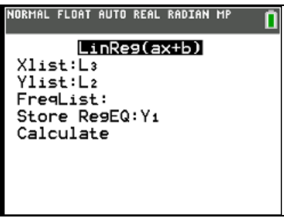
$L_3=L_1^{1/3}$

NORMAL FLOAT AUTO REAL RADIAN MP
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L3
Ylist:L2
Mark :□ ▣ • ·
Color: BLUE

NORMAL FLOAT AUTO REAL RADIAN MP
LinReg(ax+b)
Xlist:L3
Ylist:L2
FreqList:
Store RegEQ:Y1
Calculate
</td></tr>
<tr><td>(e)</td><td>Value of residual = $53.8-\left(31.684\sqrt[3]{28}-40.512\right)=-1.90$ (3 s.f.)
(observed-value – predicted value)</td></tr>
<tr><td>(f)</td><td>The values of the residuals could be positive or negative, and adding them up might cause the values to cancel out. By squaring the residuals, all the values will be positive.
The sum of squares of residuals has to be minimised in order to find the least squares regression line.</td></tr>
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