



XINMIN SECONDARY SCHOOL

新民中学

SEKOLAH MENENGAH XINMIN
Preliminary Examinations 2022

CANDIDATE NAME

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CLASS

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INDEX NUMBER

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ADDITIONAL MATHEMATICS

4049/02

Paper 2

30 August 2022

Secondary 4 Express/ 5 Normal Academic

2 hour 15 minutes

Setter : Mr Johnson Chua

Vetter : Mrs Loh Si Lan

Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the paper, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is **90**.

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		Marks Awarded	
Presentation		Marks Penalised	

For Examiner's Use
90

Parent's/Guardian's Signature:

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 (a) Solve the equation $\lg(x+3) = 1 - \lg(x-2)$. [3]

- (b) Given that $\log_2 p = a$, $\log_q 8 = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b . [3]

2 Given that $y = \frac{4x}{\sqrt{1-4x}}$, show that $\frac{dy}{dx} = \frac{4-8x}{\sqrt{(1-4x)^3}}$. [4]

- 3 The graph $y = a \cos 2x + b$, where a and b are positive integers, has a maximum value of 8 and a minimum value of -2 .

(a) Show that $a = 5$ and $b = 3$. [2]

(b) Hence, sketch the graph $y = a \cos 2x + b$ for $0 \leq x \leq \frac{3\pi}{2}$. [3]

- 4 (a) By considering the general term of $\left(x + \frac{k}{x}\right)^9$, where k is a positive constant, explain why every term is dependent of x . [2]

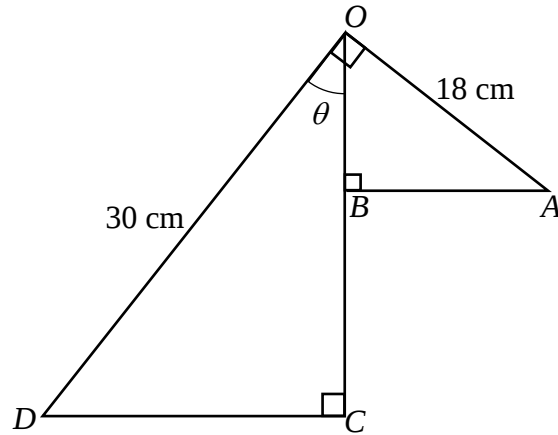
- (b) Given that the coefficients of x^3 and x^5 are the same, find the value of k . [3]

- 4 (c) Using the value of k found in (b), find the coefficient of x^7 in the expansion of

$$(1 - 5x^6) \left(x + \frac{k}{x} \right)^9.$$

[3]

5



The diagram above shows a structure.

The structure has three fixed points O , A and D such that $OA = 18$ cm, $OD = 30$ cm, and angle $AOD = 90^\circ$. The lines AB and DC are perpendicular to OC which makes an angle θ with the line OD . The angle θ can vary in such a way that B lies between the points O and C .

(a) Show that $AB + BC + CD = (12 \sin \theta + 48 \cos \theta)$ cm.

[3]

(b) Express $AB + BC + CD$ in the form $R \sin(\theta + \alpha)$ and $0^\circ \leq \alpha \leq 90^\circ$.

[3]

- 5 (c) Find the maximum value of $AB + BC + CD$ and the corresponding area of the structure.

[3]

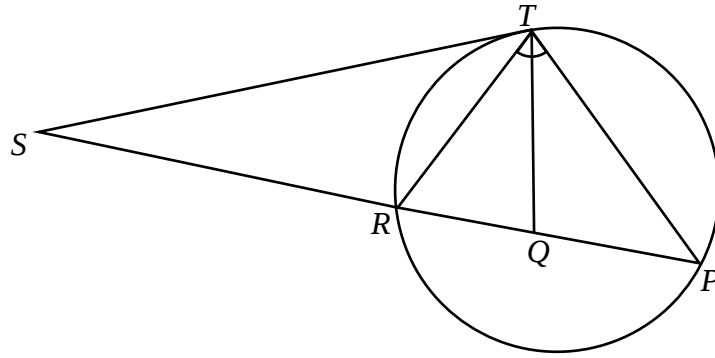
- 6 (a) The straight line $y - x = -5$ intersects the curve $2x^2 - y^2 = 2xy + 11$ at the points A and B . Find the coordinates of A and B . [3]

- (b) Find the exact values of the constant k for which the line $y = k(x + 1)$ is a tangent to the curve $y = x^2 + 6x + 3k$. [3]

- 6 (c) Find the range of values of a for which $y = (a - 6)x^2 - 8x + a$ lies completely above the x axis.

[4]

7



In the diagram, PS is a straight line intersecting the circle at P and R . T lies on the circle and TS is a tangent to the circle. Q lies on PS such that the line TQ bisects angle RTP .

(a) Prove that triangle STR and triangle SPT are similar.

[2]

(b) Prove that triangle QST is isosceles.

[3]

7 (c) Prove that $SQ^2 - SR^2 = SR \times RP$.

[3]

8

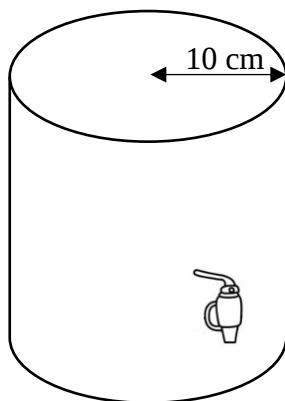


Diagram I

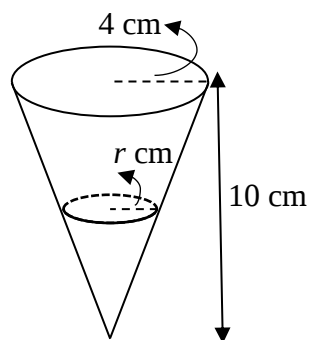


Diagram II

Diagram I shows a water dispenser, in the shape of a cylinder of radius 10 cm, which dispenses water at a constant rate into an empty conical cup, as shown in diagram II, of radius 4 cm and height 10 cm. The depth of the water in the dispenser decreases at a rate of 0.0015 cm/s.

After t seconds, radius of the horizontal surface of the water in the conical cup is r cm.

- (a) Show that the volume of water in the conical cup increases at a rate of $\frac{3\pi}{20} \text{ cm}^3/\text{s}$. [2]

8 (b) Express the volume of water in the conical cup in terms of r . [2]

(c) At the instant where the volume of the water in the conical cup is $\frac{20\pi}{3} \text{ cm}^3$, find the rate of change in the radius of the horizontal surface of the water in the conical cup. [4]

- 9 The mass, m mg, of a radioactive substance decreases with time, t hours. Measured values of m and t are recorded in the following table.

t (hours)	2	4	6	8	10
m (mg)	48.2	41.5	35.5	33.7	26.5

m and t are related by the equation $m = m_0 e^{-kt}$, where m_0 and k are constants.

It is known that one of the readings in the table is recorded wrongly.

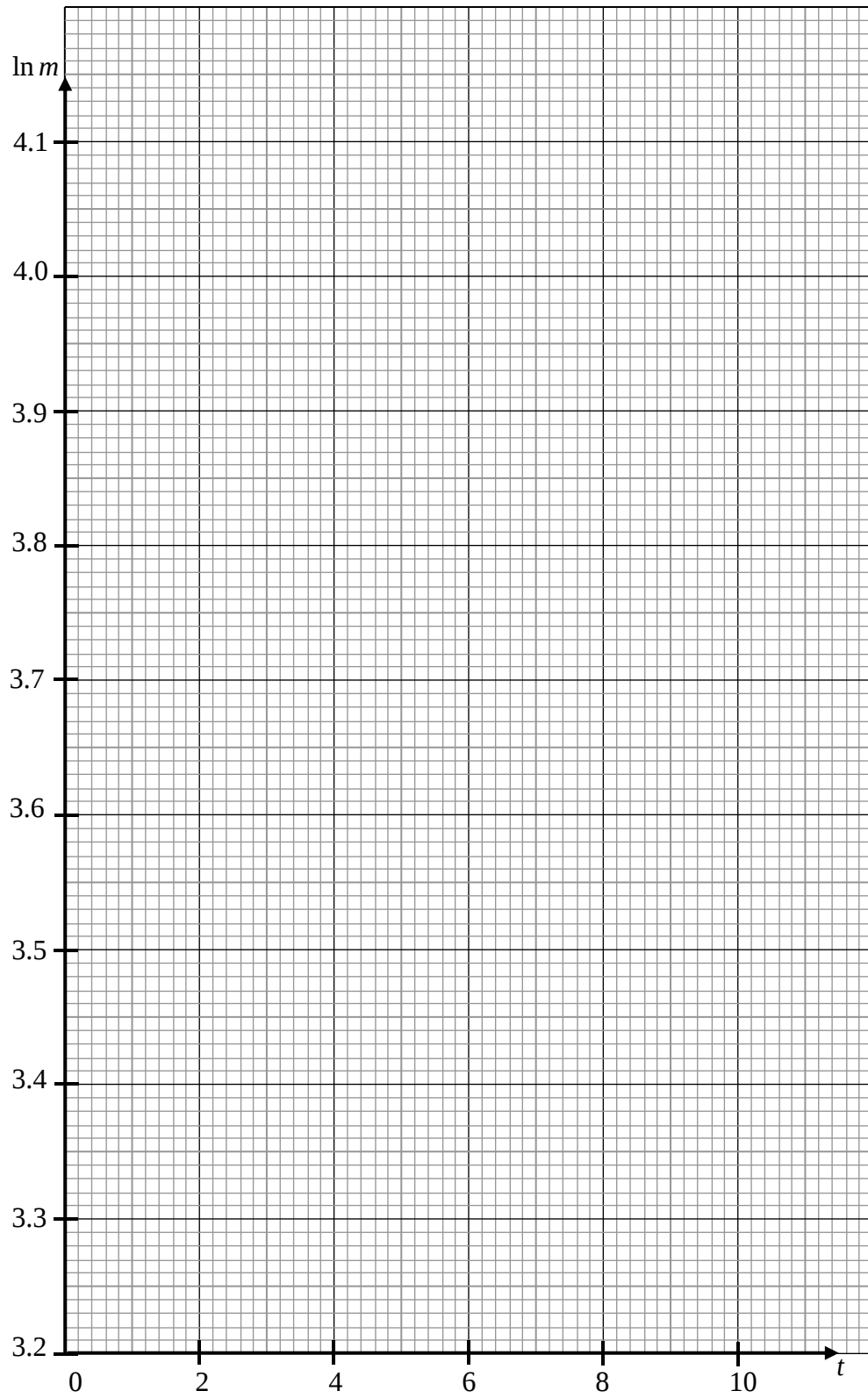
- (a) On the grid found on page 17, plot $\ln m$ against t and draw a straight line graph. Use your graph to identify the reading which has been recorded wrongly. [3]

- (b) Use your graph to estimate
(i) a value of m to replace the incorrect recording of m found in part (a), [1]

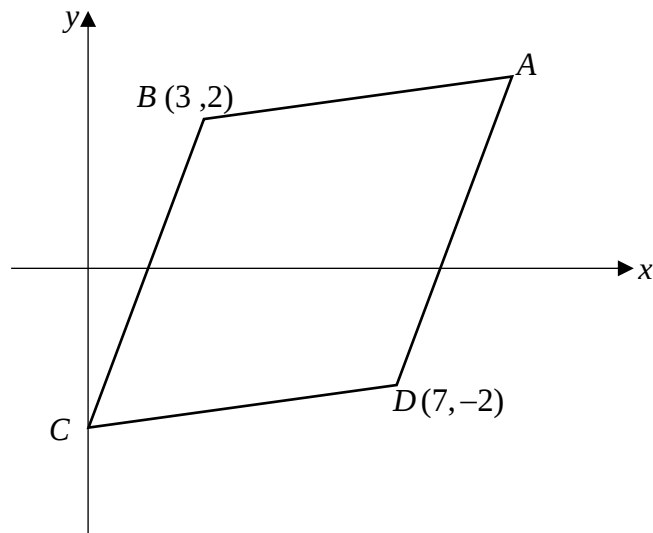
- (ii) the value of k and m_0 . [4]

- (c) Explain what m_0 represents. [1]

- (d) Ben claims that the mass of the radioactive substance reaches 0 mg after 10 to 12 hours. Do you agree with Ben? Explain your answer. [1]



10



The diagram shows a quadrilateral with vertices A , $B(3, 2)$, C and $D(7, -2)$.

C lies on the y -axis.

AC is a perpendicular bisector of BD .

(a) Show that the equation of AC is $y = x - 5$.

[3]

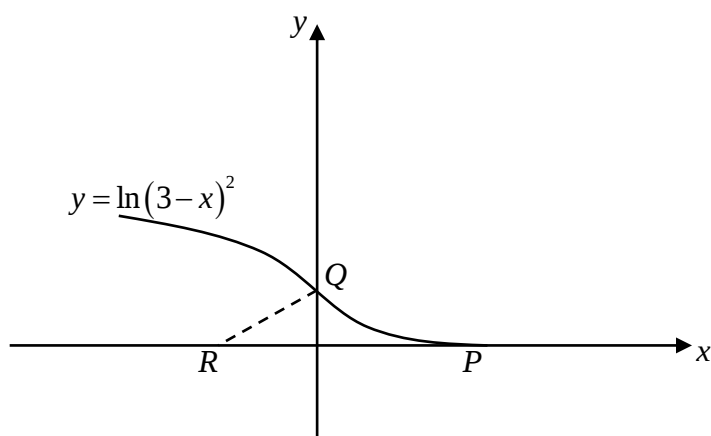
(b) State the coordinates of C .

[1]

10 (c) Given that triangle ABD has an area of 16 units^2 , find the coordinates of A . [4]

(d) Explain whether $ABCD$ is a rhombus. [2]

- 11 The diagram shows part of the curve $y = \ln(3-x)^2$ which crosses the axes at P and Q .



The normal to the curve at Q meets x -axis at R . Find the area of the region bounded by the curve, RQ and the x -axis.

[12]

Continuation of working space for question 11.