RAFFLES INSTITUTION
H2 Mathematics (9758)
2024 Year 6

Term 3 Revision
Session 5: Vectors

Questions

1 With respect to the origin O, the position vectors of the points A, B and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. Point C lies on AB such that $AC:CB=1:2$. It is given that $\mathbf{a}$ is a unit vector and the length of OB is 2 units.

(i) Give a geometrical interpretation of $|\mathbf{a}\bullet\mathbf{c}|$. [1]

(ii) It is given that the angle AOB is $60°$. By considering $(2\mathbf{a}-\mathbf{b})\bullet(2\mathbf{a}-\mathbf{b})$, find $|2\mathbf{a}-\mathbf{b}|$. [3]

(iii) Find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

(iv) Hence by considering cosine of angle AOC and cosine of angle COB, determine if the line segment OC bisects the angle AOB. [3]

2 The points A, B and R have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{r}$ respectively.

(a) The point C has position vector $\frac{2}{7}\mathbf{a}-\frac{3}{7}\mathbf{b}$ and the point D is such that the origin O is the midpoint of the line segment CD. The point R lies on BD extended such that the ratio of BD to BR is $4:7$. Show that the points A, O and R are collinear and state the ratio of OA to OR. [4]

(b) It is given that the point R has position vector $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}$, and that $\mathbf{a}=\begin{pmatrix}1\\3\\2\end{pmatrix}$, and $\mathbf{b}=\begin{pmatrix}-1\\5\\3\end{pmatrix}$.

(i) Determine the exact area of the triangle AOB. [2]

(ii) Give the geometrical interpretation of the point R, given that $\mathbf{r}\cdot(\mathbf{a}\times\mathbf{b})=0$. [2]

(iii) Find the shortest distance between the point $(-8,-2,9)$ and the collection of all points R satisfying $\mathbf{r}\cdot(\mathbf{a}\times\mathbf{b})=0$. [2]

3 The plane p passes through the points with coordinates $(-k,\ 2,\ 5)$, $(0,\ 2,\ -1)$ and $\left(-\frac{1}{2},\ 3,\ -1\right)$, and the line l has equation $\frac{x+2}{-3}=y-2=\frac{z-4}{k}$, where k is a constant.

(i) Show that the cartesian equation of the plane is $6x+3y+kz=6-k$. [2]

(ii) Show that line l cannot be perpendicular to p. [2]

For the rest of this question, let $k=-2$.

(iii) Given that l meets p at point N, find the coordinates of N. [3]

(iv) Another plane π is parallel to the plane p. Given that the distance between p and π is 11 units, find the possible points of intersection between l and π. [3]

4 One day, Eddie came home from a birthday party and brought back a helium filled balloon. After playing with it, he accidentally released the balloon at the point (1, 2, 3) and it floated vertically upwards at a speed of 1 unit per second. t seconds later, a sudden gust of wind caused the balloon to move in the direction of $\mathbf{i}+4\mathbf{j}+6\mathbf{k}$.

You may assume that $z = 0$ refers to the horizontal ground.

(i) Find the angle in which the balloon has changed in direction after the gust of wind blew it away. [3]

(ii) Find the Cartesian equation of the plane that the balloon is moving along. [3]

(iii) Given that the balloon eventually stayed at the point (2, 6, 12) on the ceiling, find the time t when the gust of wind blew the balloon away. [3]

Eddie decides to shoot the balloon down with his catapult.

(iv) Assume he was holding his catapult at (3, 2, 1) initially and he walked along the path parallel to $2\mathbf{i}+\mathbf{j}$. Find the position vector of the point where he should place his catapult so that the distance between his catapult and the balloon is at its minimum. Hence find this distance. [4]