NATIONAL JUNIOR COLLEGE
SENIOR HIGH 1
Higher 2

NAME	

SUBJECT CLASS	1ma2	REGISTRATION NUMBER	

MATHEMATICS — 9758

Term 4 Promotional Examination — **25 September 2024**

3 hours

Candidates answer on the Answer Booklet.

Additional Materials: List of Formulae (MF27)
Printed Answer Booklet

READ THESE INSTRUCTIONS FIRST

Write your name, class and registration number on the work you hand in.
Write in dark blue or black pen.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

You are reminded of the need for clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

1 Without using a calculator, solve the inequality

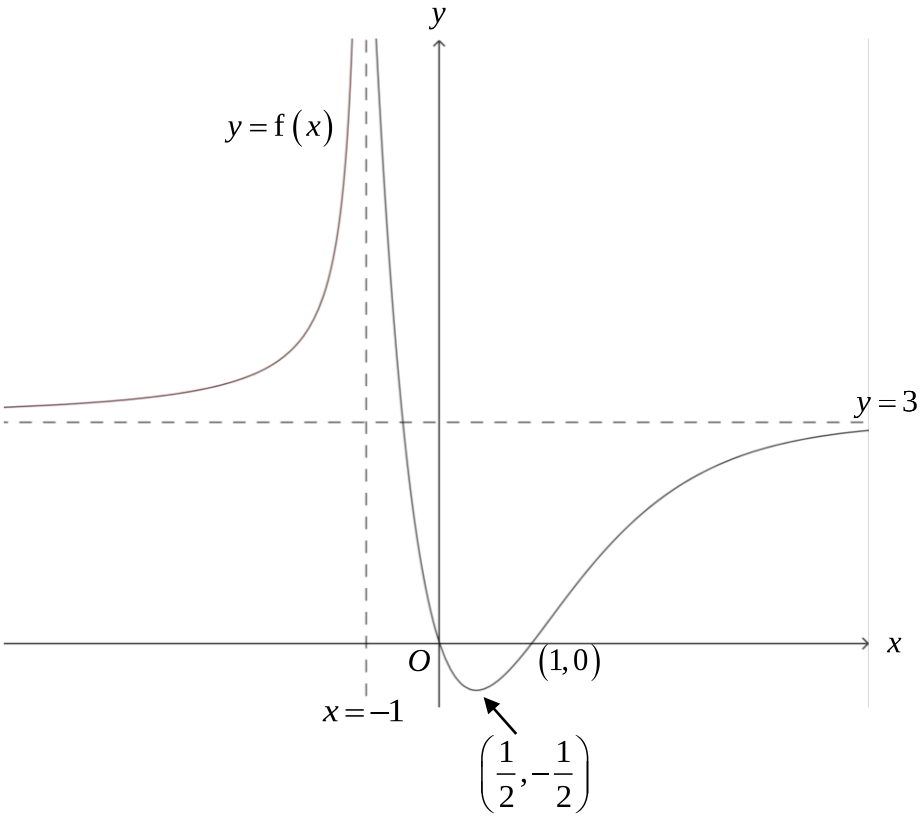
$$\frac{x+3}{x^2+x-2} \geq -1. \qquad [4]$$

2 The region bounded by $y = x\sqrt{\sin x}$, the x-axis, and the lines $x = 0$ and $x = \pi$ is rotated through 2π radians about the x-axis. Find the exact volume of the solid generated. [5]

3 **(i)** Given that k is a positive constant, sketch the curve E with equation $y = \left|x^2(x-k)\right|$, labelling the coordinates of the points where the curve cuts the axes. [2]

(ii) Hence, find, in terms of k, the area of the region(s) bounded by E and the x-axis for $0 \leq x \leq 2k$. [3]

4 **(a)** The diagram shows the curve $y = \mathrm{f}(x)$ that passes through the origin and $(1, 0)$. It has a minimum point $\left(\frac{1}{2}, -\frac{1}{2}\right)$ and asymptotes $x = -1$ and $y = 3$.



Sketch the curve $y = \dfrac{1}{\mathrm{f}(x)}$, showing clearly the coordinates of any stationary points, axial intercepts and equations of all asymptotes. [3]

(b) A curve with equation $y = \mathrm{h}(x)$ undergoes the following sequence of transformations.

Step 1: A translation of 2 units in the positive direction of the x-axis.
Step 2: A scaling parallel to the x-axis by factor of 3.
Step 3: A reflection in the y-axis.

The resultant curve has equation $y = -\dfrac{x^2}{6x+45}$. Find $\mathrm{h}(x)$. [3]

5 The line l_1 passes through the point $A(7,6,5)$ and the point $B(-5,-2,-5)$ while the line l_2 has equation $x-5 = \dfrac{y-10}{k} = \dfrac{z-8}{2}$, where k is a constant. The lines l_1 and l_2 intersect at point P.

(i) Show that $k = 2$ and find the coordinates of P. [5]

(ii) Find the position vector of the point on l_2 which is closest to A. [3]

(iii) Hence, find a vector equation of the line which is a reflection of l_1 in l_2. [2]

6 A curve C has equation $x^2y^2 + 3xy + 2y^2 - x - 2 = 0$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [3]

(ii) Find the gradients of the tangents at $x = 0$. [2]

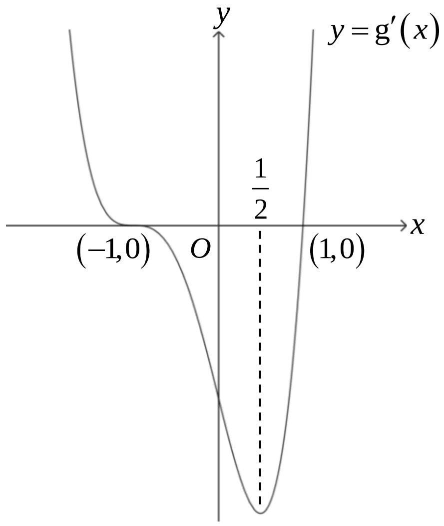
(iii) Using an algebraic method, show that there is no part of the curve for which $x < -4$. [3]

7 **(a)** **(i)** Given that $\mathbf{p} \cdot \mathbf{q} = \mathbf{0}$, what can be deduced about the vectors $\mathbf{p}$ and $\mathbf{q}$? [2]

(ii) With respect to the origin on a xy-plane, a fixed point G has position vector $\mathbf{g}$ and a variable point R has position vector $\mathbf{r}$. Describe the locus of R if $\mathbf{r} \cdot (\mathbf{r} - \mathbf{g}) = 0$. [2]

(b) With respect to the origin, plane p has an equation given by $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}), \lambda \in \mathbb{R}, \mu \in \mathbb{R}$. Points Q and R have position vectors $3\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$ and $4\mathbf{i} + 11\mathbf{j} + 7\mathbf{k}$ respectively. Find the length of the projection of $\overrightarrow{QR}$ onto p. [4]

8 The diagram below shows that the graph of $y = \mathrm{g}'(x)$ has two x-intercepts at $(-1, 0)$ and $(1, 0)$ as well as two stationary points whose x-coordinates are -1 and $\frac{1}{2}$ respectively.



(i) A stationary point on the curve of $y = \mathrm{g}(x)$ has a x-coordinate that is positive. State the value of this x-coordinate and determine the nature of this stationary point on the curve $y = \mathrm{g}(x)$. [2]

It is given that $\mathrm{g}'(x) = ax^4 + bx^3 + cx^2 + dx + e$.

(ii) Show that $a - b + c - d + e = 0$. [1]

Given further that the gradient of the tangent at the point of inflexion on $y = \mathrm{g}(x)$ is $-\frac{27}{8}$, find the values of a, b, c, d and e. [4]

9 Referred to the origin O, fixed points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where O, A and B do not lie on the same line. Point C with position vector $\mathbf{c}$ is a varied point such that $\mathbf{a} + 2\mathbf{b} + \lambda\mathbf{c} = \mathbf{0}$, where λ is a parameter.

(i) Show that $\dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ABO} = \left|\dfrac{\lambda + 3}{\lambda}\right|$. [3]

(ii) Deduce the value of λ given that A, B and C are collinear. [1]

(iii) Given instead that A, B and C are not collinear and the line OC intersects AB at the point D, find the ratio $AD : BD$. [2]

(iv) Further given that $\angle OAC = \angle OBC = 90°$, find λ in terms of $|\mathbf{a}|$ and $|\mathbf{b}|$. [4]

10 **(a)** The functions f and g are defined by

$$\mathrm{f}: x \mapsto \frac{\sqrt{2}(x-2)}{x^2+x-2}, \qquad x \in \mathbb{R},\ x>1,$$

$$\mathrm{g}: x \mapsto \mathrm{e}^{-x}-1, \qquad x \in \mathbb{R}.$$

(i) Determine if the composite function fg exists. [2]

(ii) Find the exact range of gf. [3]

(iii) The domain of f is now restricted to $0<x<1$. Find the value of $\mathrm{g}^{-1}\mathrm{f}^{-1}(2)$. [3]

(b) The function h is defined by

$$\mathrm{h}: x \mapsto \sqrt{3}(\sin x+\cos x), \qquad x \in \mathbb{R},\ -\frac{5}{2} \le x \le p,$$

where p is a real constant.

(i) Find the largest exact value of p such that h^{-1} exists. [3]

(ii) With the value of p obtained in part **(b)(i)**, find the solution of $\mathrm{hh}^{-1}(x)=\mathrm{h}^{-1}\mathrm{h}(x)$, leaving your answer to 3 decimal places. [2]

11 **(a)** **(i)** Express $\dfrac{13x-2}{(3-x)(1+4x^2)}$ in partial fractions. [3]

(ii) Hence, find the exact value of $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{13x-2}{(3-x)(1+4x^2)}\,\mathrm{d}x$, leaving your answer in the form $\ln p+q\pi$, where p and q are rational numbers to be determined. [4]

(b) You are given that $I=\displaystyle\int \frac{a^2-x^2}{(a^2+x^2)^2}\,\mathrm{d}x$, where a is a positive constant.

Use the substitution $x=a\tan t$, where $-\dfrac{\pi}{2}<t<\dfrac{\pi}{2}$, to find an expression for I in terms of a, leaving your answer in a non-trigonometric form. [5]

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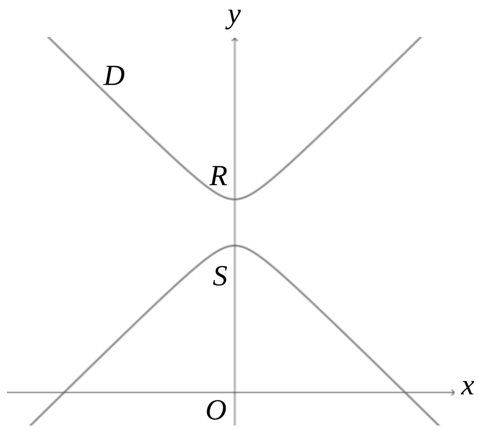


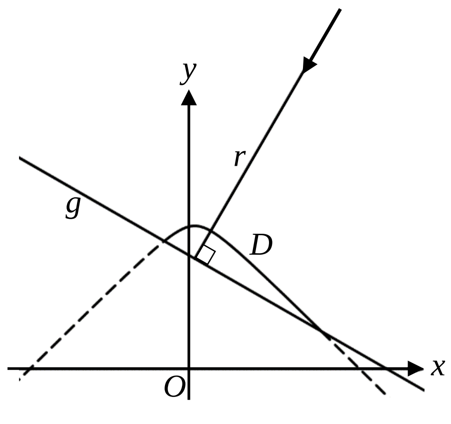
The diagram above shows Curve D which has equation $\left(y-3\sqrt{6}\right)^2-x^2=1$. Points R and S are stationary points on D. Another curve E has equation $y^2-x^2=1$.

(i) By considering a single transformation that transforms the graph of E onto D or otherwise, find the equations of the asymptotes of D and the coordinates of S. [2]

(ii) Verify that point $P\left(\tan p, \sec p+3\sqrt{6}\right)$ lies on D, where $-\pi < p \le \pi, p \ne \pm\dfrac{\pi}{2}$. [1]

(iii) Show that the equation of the normal to D at P is $(\sin p)y+x=2\tan p+3\sqrt{6}\sin p$. [4]

The story of the design of Gardens by the Bay was told in the book “Supernature”. There are many cross-section diagrams in the book and one of them consists of the region bounded by the lower piece of D and the line g with equation $y=-\dfrac{1}{\sqrt{3}}x+2\sqrt{6}$. In this cross-section diagram, the lower piece of D traces out the Flower Dome while g traces out the ground, as shown below.



This cross-section diagram is used to illustrate the condition where light rays are perpendicular to the ground. One of these light rays is normal to the lower piece of D and is represented by line r.

(iv) Find the coordinates of the point where r intersects the lower piece of D. [5]

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