ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749

Name: ________________________________ (   ) **Class:** 25 / ____

Topic 5: Work, Energy and Power

Content:

A. Work
B. Kinetic Energy and Potential Energy
C. Conservation of Energy
D. Power and Efficiency

Learning Outcomes:

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force.

(b) calculate the work done in a number of situations, including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$.
(To be discussed in First Law of Thermodynamics)

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation to simple examples.

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.

(e) derive, from equations for uniformly accelerated motion in a straight line, the equation $E_k = \frac{1}{2}mv^2$

(f) recall and use the equation $E_k = \frac{1}{2}mv^2$.

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy.

(h) deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph

(i) show an understanding of, and use the relationship between force and potential energy in a uniform field to solve problems.
(To be discussed in Gravitational Field)

(j) derive, from the definition of work done by a force, the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface.

(k) recall and use the formula $E_p = mgh$ for gravitational potential energy changes near the Earth's surface.

(l) define power as work done per unit time and derive power as the product of force and velocity in the direction of the force

__Relating Science and Society__

There is heavy reliance on fossil fuels (e.g. crude oil, natural gas and petroleum products) for electricity generation in Singapore. It is important to capitalise on innovative ideas and technologies to promote the efficient use of energy, especially in the major sectors of energy use, namely power generation, industry, transport, buildings and households. It is also important to grow clean energy capabilities such as solar energy to reduce the dependence on fossil fuels.

A Work

- Energy is the ability to do work; whereas work is the <u>transfer of energy</u>, or the <u>process of converting energy</u>.

- When work is done, energy is transferred.

A.1 Work Done by a Force

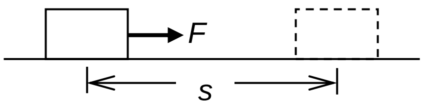
- If a body moves as a result of a force being applied to it, the force is said to be doing work on the body.

Memorise

Definition: The work done by a force is the product of the force and the displacement in the direction of the force.

- When you exert a force F on an object making it move a displacement s in the direction of the force, your work done on the object, W is expressed as:

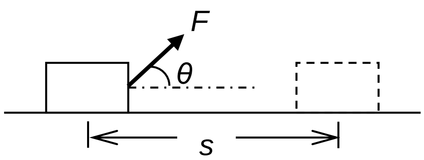
$$\boxed{W = Fs}$$



Note:
1. To use these expressions for calculation, the force has to be constant. Otherwise, we will use area under Force−displacement graph, to be discussed in A.2.
2. We can also resolve displacement in the direction of the force and the expression for work done is still the same.

- <u>When the force and displacement are not in the same direction</u>, we must use the component of the force which is parallel to the displacement.

$$\begin{aligned} W &= (\text{component of } F \text{ in direction of } s) \times s \\ &= (F\cos\theta) \times s \\ &= Fs\cos\theta \end{aligned}$$



where θ is the angle between the force and the displacement.

- The unit for work is the <u>Joule (J)</u>.

- Work is a <u>scalar</u> quantity; it has only magnitude and no direction.

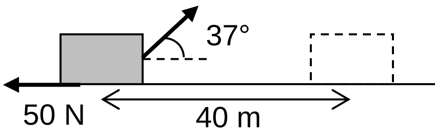
- Work done can be positive (object gains energy) or negative (object loses energy).

- The work done by a force is negative if the applied force has a component in a direction opposite to the displacement. e.g. work done by friction.

- When the displacement of the object is perpendicular to the line of action of the force, the work done on an object is zero.

Example 1

A boy pulls a 70 kg crate 40 m along a horizontal floor with a constant force of 100 N, which acts at an angle of 37° as shown below. The floor is rough and exerts a frictional force of 50 N.



Determine
(a) the work done by each force acting on the crate, and
(b) the net work done on the crate

Solution

Additional Notes

The sign of work done on the object determines whether the object gains or loses energy. It does not refer to direction as work done is a scalar quantity. When the work done by friction on a moving car is −700 J, this means that the car loses 700 J of energy, and 700 J of work is done against friction.

Thinking Process:
- What are the all the forces acting on the crate? Are there additional forces present that are not stated by the question?
- Is the question asking for **net work done** or **work done by each force**?
- Is net force / each force parallel to displacement?
- If net force / each force is not parallel to displacement, what do we need to do to find work done?

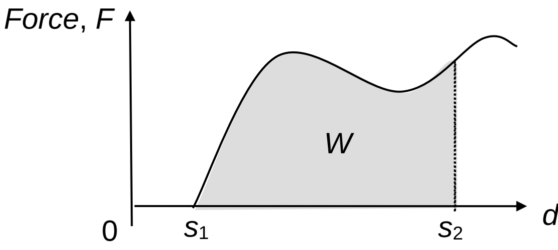
Additional Notes

Do not confuse area under *Force−displacement* graph with area under *Force−time* graph. The former represents work done, while the latter represents impulse, i.e. change in momentum.

A.2 Graphical Method for Determining Work Done by a Varying Force

Work done by a Force

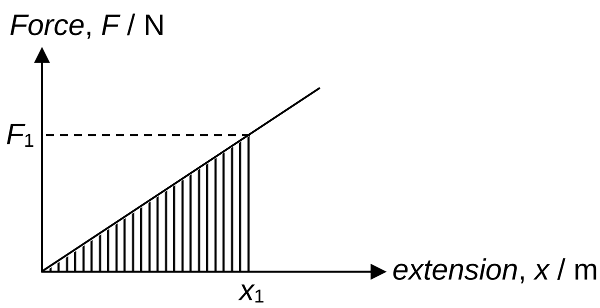
- When the force varies with respect to the displacement, the work done is represented by the area enclosed under the *Force−displacement* graph.



$$W = \int_{s_1}^{s_2} F \cdot ds$$

i.e. area under the *Force−displacement* graph.

- Recall, from the topic Forces, the *Force−extension* diagram for an elastic object that obeys Hooke's Law.

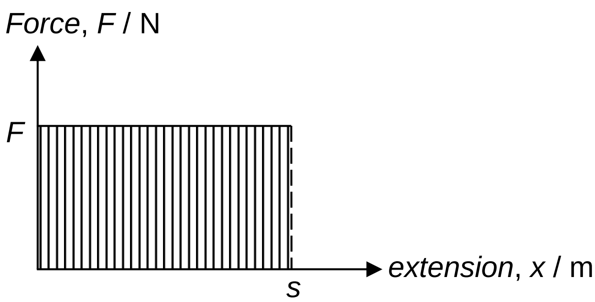


- The elastic potential energy stored in the elastic object is a result of work done by the force and is given by area under the graph.
- In this case, the work done, or elastic potential energy stored, is

$$\text{Work done} = \tfrac{1}{2} F_1 x_1 = \tfrac{1}{2} (k x_1) x_1$$
$$= \tfrac{1}{2} k x_1^2$$

where k is the spring constant.

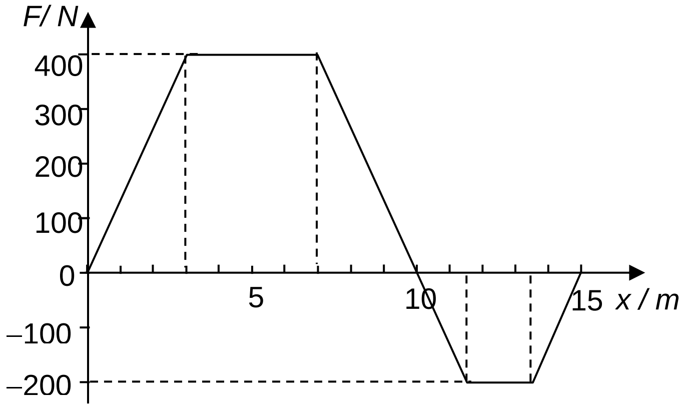
- In the case where the force applied is constant, and in the direction of the displacement, the work done is simply Fs.



Additional Notes

Check Your Understanding 1

The force on a particle varies as shown in the figure below. Determine the work done by this force to move the particle along the axis from x = 0.0 m to x = 15.0 m.



A 700 J **B** 2100 J **C** 2800 J **D** 3500 J

Solution

B Kinetic Energy and Potential Energy

B.1 Kinetic Energy

Kinetic energy is defined as the stored ability of an object to do work as a result of its motion.

- A body of mass m moving with velocity v possesses kinetic energy E_k given by

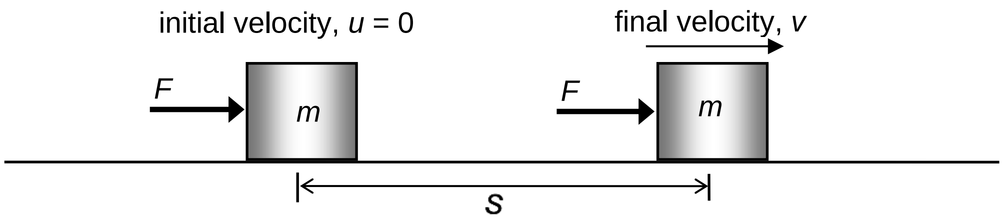
$$\boxed{\boldsymbol{E_k = \tfrac{1}{2} mv^2}}$$

- The SI unit for kinetic energy is the Joule (J).
- Kinetic energy is a positive, scalar quantity.

Memorise

Derivation of $E_k = \frac{1}{2} mv^2$ from equations of motion

- Consider a body of mass m which moves from rest a displacement s under the action of a constant force F.



- The total work done W is given by $W = Fs$
- By Newton's 2nd Law, $F = ma$
- Hence, the work done can be expressed as $W = mas$
- The body has been uniformly accelerated from rest to some velocity v, thus

$$v^2 = 0^2 + 2as$$

- Rearranging, $as = v^2/2$

- Substituting above into $W = mas$: $W = \frac{1}{2} mv^2$
- Hence, $E_k = \frac{1}{2} mv^2$

- The kinetic energy E_k of a body is equivalent to the amount of work required to accelerate it from rest to its final velocity.

- Note that kinetic energy is independent of the way in which the body acquired this velocity (i.e. regardless of how it obtained its velocity).

- The kinetic energy of an object can also be expressed in terms of its momentum.

$$\boxed{E_k = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}}$$

- This expression is useful when we need to calculate kinetic energy in a context where conservation of momentum can be applied.

Example 2

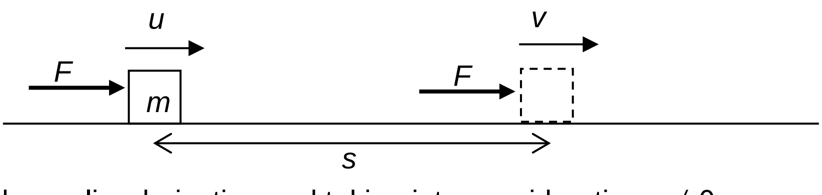
A bullet of mass 50 g is fired from a gun of mass 2.0 kg. If the total kinetic energy produced by the explosion is 3000 J, what are the kinetic energies of the bullet and the gun?

Solution

B.2 Work−Energy Theorem

- Consider the case when the velocity of a body of mass m <u>increases from u to v</u> when work is done on it by a force F acting over a displacement s.

- From kinematics, $v^2 = u^2 + 2as \rightarrow a = (v^2 - u^2) / 2s$



- Following the earlier derivation and taking into consideration $u \neq 0$

$$
\begin{aligned}
W &= Fs \\
&= mas \\
&= m\,(\tfrac{1}{2})\,(v^2 - u^2) \\
&= \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2 \\
&= \text{Final } E_k - \text{Initial } E_k \\
W &= \Delta E_k
\end{aligned}
$$

<u>Work−Energy Theorem</u>
Work done by Net Force on a Body = <u>Change</u> in Kinetic Energy of Body

Additional Notes

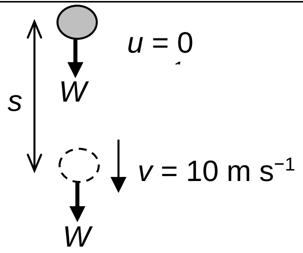
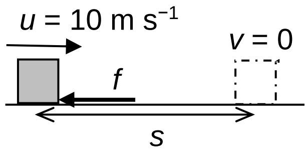
- Work done by net force on the body is the same as the net work done on the body. i.e. sum of individual components of work done on a body.

- Work done by *net force* is different from the work done by *applied force*.

Relation between net work done and kinetic energy

- If the **net** work done **on** an object is ***positive***, the object will ***gain kinetic energy*** ($W > 0 \rightarrow \Delta E_k > 0$)

- If the **net** work done **on** an object is ***negative***, the object will ***lose kinetic energy*** ($W < 0 \rightarrow \Delta E_k < 0$)

Examples	Positive or Negative work done on object by net force?	Energy conversion	Explanation
 A metal ball of weight *W* is dropped from rest in a vacuum. The ball accelerates downwards.	**Positive work done on ball** by weight	Gravitational potential energy of ball → kinetic energy of ball	Ball **gains kinetic energy** through **positive work** done on it **by weight**
 A block is sliding on a rough surface. Frictional force *f* acts on the moving block. The block decelerates to a stop.	**Negative work done on block** by friction	Kinetic energy of block → internal energy of block and floor	Block **loses kinetic energy** through **negative work** done on it **by friction**

Internal energy is the sum of a random distribution of the kinetic and potential energies associated with the molecules of a system. This will be discussed further in the topic of First Law of Thermodynamics.

Additional Notes

Check Your Understanding 2

Figure 2 shows a block of ice of mass 2.0 kg moving with a constant velocity of 2.0 m s^{-1} along a frictionless surface AB. It slows down at a uniform rate along surface BC and continues with a smaller uniform velocity of 1.0 m s^{-1} along another frictionless surface CD.

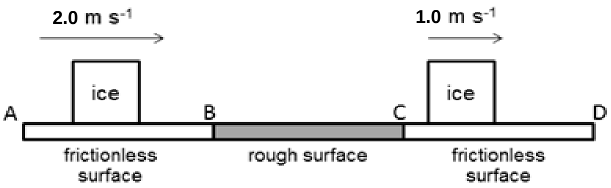


Figure 2

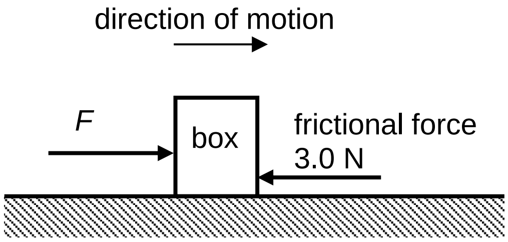
What is the work done against friction by the ice?

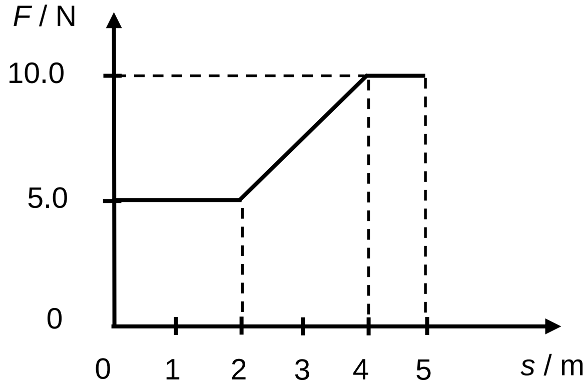
A 1.0 J **B** 3.0 J
C 4.0 J **D** 5.0 J

Solution

Example 3

A box is pushed by a variable force *F* along a rough surface. As it moves along the surface, it experiences a constant frictional force of 3.0 N with the surface. The figure below shows how the variable force varies with displacement.





Calculate the change in kinetic energy of the box in the first 5.0 m.

Solution

B.3 Potential Energy

Potential energy is defined as the stored energy of an object to do work as a result of its position or shape.

- Potential energy is a property of the *system*, not the *object*.
  (A *system* is a collection of objects or particles interacting via forces or processes that are internal to the system)

- We will learn about three types of potential energy (PE):
  1. Elastic potential energy or strain energy is the stored ability of object to do work as a result of its shape. It is the potential energy stored in a material as a result of deformation of an elastic object, such as the stretching or compression of a spring, stretching a bow.

  2. Electric potential energy is the stored ability of object to do work as a result of its charge and position. It is due to interaction between charges. The forces involved can be attractive or repulsive. Like charges repel; unlike charges attract. Consider a system consisting of a small positive charge located at some distance from a positively charged sphere. If you push the small positive charge closer to the sphere, you will need to do work to overcome the force of repulsion between them. Work done on the small charge by this external force results it to gain PE. We call the energy that the system gain as electric PE.

  3. Gravitational potential energy (GPE) is the stored ability of an object to do work as a result of its mass and position. It is due to the interaction between masses. The forces involved are attractive in nature.

We have come across elastic potential energy in the topic of Forces.

More on gravitational and electric potential energy in the topics of Gravitational Field and Electric Field respectively

Gravitational Potential Energy Near the Surface of Earth

- For an object above the ground near the surface of the earth, the change in gravitational potential energy ΔU stored in the object relative to the ground is given by

$$\boxed{\Delta U = mgh}$$

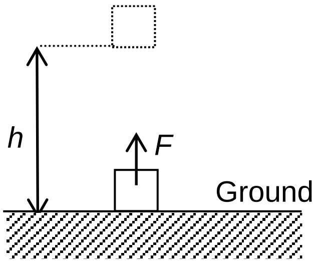
where m = mass of the object,
h = height of the object above the earth's surface,
g = gravitational acceleration = 9.81 m s^{-2} near the earth's surface.

Additional Notes

Memorise

Derivation of $\Delta U = mgh$

- Consider an object of mass m in a uniform gravitational field being lifted up by a force F to height h above the ground with constant velocity.



- From the definition of work,
  work done by the force F is the product of the force and the displacement in the direction of the force
  $\therefore$ Work done, W = Force × displacement
  $= F \times h$

- Since magnitude of force F = weight of object (as object moves at constant velocity)
- Hence, W = weight of object × h
  $= mgh$

- The gravitational potential energy **gained** is equal to the **work done by the force *F*** to **move the object to height *h***.

- Therefore, change in gravitational potential energy,
  $$\Delta U = W = mgh$$

- GPE is a relative and not an absolute measure. It tells us the amount of work needed to move an object from some arbitrarily chosen reference point (for instance, the ground) to the position it is presently in.

- The ground is usually assigned to be the "zero gravitational potential energy level".

- If the tabletop is assigned as the zero GPE level, then the potential energy of an object is based upon its height relative to the tabletop, i.e. the object would be assigned positive GPE if positioned above the tabletop and negative if below the tabletop.

- This formula assumes that g is constant throughout the height h. Near the earth's surface, g is fairly constant.

C Conservation of Energy

The **Principle of Conservation of Energy** states that energy cannot be created or destroyed, but it can be converted from one form to another.

In everyday usage, 'conservation' means saving. In physics, 'conservation' is applied to a quantity that remains strictly constant.

C.1 Principle of Conservation of Mechanical Energy

- The Principle of Conservation of Mechanical Energy is a special case of the more general Principle of Conservation of Energy, when dissipative forces such as friction is negligible.

- Mathematically,

  Kinetic Energy, KE + Potential Energy, PE = Total Energy, E

  of which E is constant.

- Conservation of mechanical energy also means that

  Total initial KE and PE = Total final KE and PE

  OR

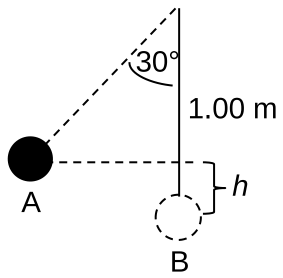
  Loss in PE = Gain in KE

Example 4

A metal ball suspended from a rope 1.00 m swings from point A to the point B (the bottom of its arc). Air resistance is negligible.

(a) Calculate the difference in height, h, from A to B.

(b) Determine the speed of the ball when it is at the position B if the ball has an initial speed of 1.20 m s^{-1} at position A.



Solution

C.2 In the Presence of Dissipative Forces

- In the presence of dissipative forces (e.g. friction) in a closed system, mechanical energy is not conserved.

- Some of the total mechanical energy of the system is lost as work done against dissipative forces:

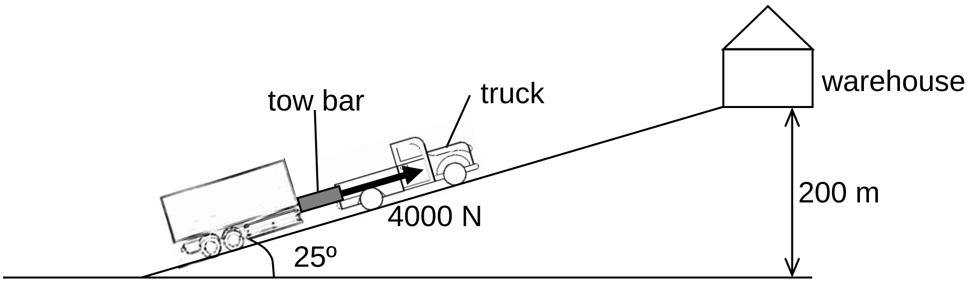
$$KE_i + PE_i = KE_f + PE_f + \text{Work done against dissipative forces}$$

- Total energy in the system is still conserved as the "lost" mechanical energy is converted into *internal energy* of the system.

Dissipative forces are also known as non-conservative forces.

Example 5

A truck tows a 500 kg storage container at constant speed from level ground to a warehouse on a slope, inclined at 25º to the horizontal as shown below. The force experienced in the tow bar is 4000 N. The warehouse is 200 m above ground level.



Determine the work done on the container due to resistive force.

Solution

Example 6

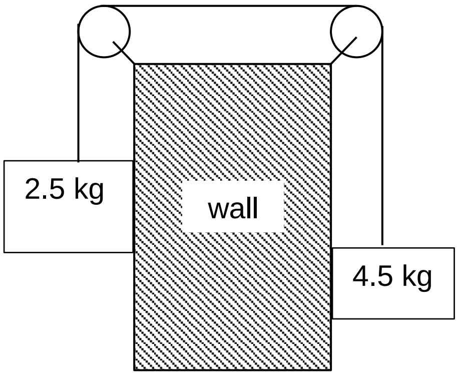
A man pulls a cart at a constant velocity along a horizontal surface. Describe the energy transformation that is taking place during the period of constant velocity.

Solution

Additional Notes

Example 7

Two boxes of mass 2.5 kg and 4.5 kg are connected by an inextensible string which is passed over smooth pulleys. Each mass experiences a constant frictional force of 1.2 N with the wall. Both masses are at rest initially.



(a) Calculate the final velocity of the masses after moving through a distance of 0.50 m each.

(b) If a 0.75 N force had acted downward on the 4.5 kg mass in the above scenario, calculate the final velocity of the masses after moving through a distance of 0.50 m each.

Solution

Applet - Conversion of energy when skating

http://phet.colorado.edu/en/simulation/energy-skate-park

Remarks: Java software required

This applet helps visualisation of the changes to the different types of energy as the skater moves.

Additional Notes

D Power and Efficiency

D.1 Definition of Power

- Power is a scalar quantity.

Definition: Power is the work done per unit time.

Memorise

- Power is also interpreted as the rate of transfer of energy, *E*. Hence,

$$\text{Power} = \frac{dE}{dt} = \frac{dW}{dt}$$

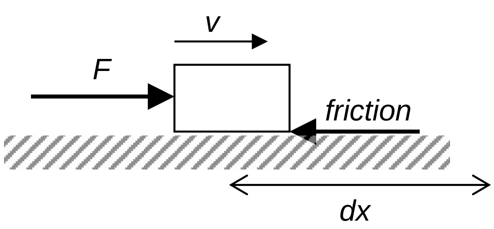
where *W* can also be the energy converted or expended.

- The SI unit for power is the Watt (W) or Joules per second (J s^{-1}).

D.2 Derive *Power* as the product of *Force* and *velocity*

Memorise

- Consider a **constant** *driving force F* exerted on a body that moves with **constant** *velocity v*.



For the body to move at **constant velocity**, the driving force and the frictional/resistive force must be of the same magnitude. Hence, there is a need to draw in friction in the diagram.

- Work done by *F* in moving the body through a distance d*x* is given by d*W* = *F* d*x*
- Hence, power *P* delivered by *F*,

$$P = \frac{dW}{dt} = \frac{F\,dx}{dt} = Fv$$

$$P = Fv$$

$$\boxed{\textit{Power} = \textit{Force} \times \textit{velocity}}$$

- If the direction of the force required by the question is opposite to *v* (e.g. friction), the expression then gives the rate at which the required force removes energy from the system.

D.3 Average Power

- When a quantity of work ΔW is done during a time interval Δt,

$$\text{average power}, \langle P \rangle = \frac{\text{total work done}}{\text{total time taken}} = \frac{\Delta W}{\Delta t}$$

- If a constant force F acts through a distance Δx for a time Δt, then average power,

$$\langle P \rangle = \frac{\text{total work done by } F}{\text{total time}} = \frac{\Delta W}{\Delta t} = \frac{F\Delta x}{\Delta t} = F\langle v \rangle$$

- Thus, average power = Force × average velocity

$$\langle P \rangle = F\langle v \rangle$$

- Note that v need not be constant to use the above expression for average power.

D.4 Instantaneous Power

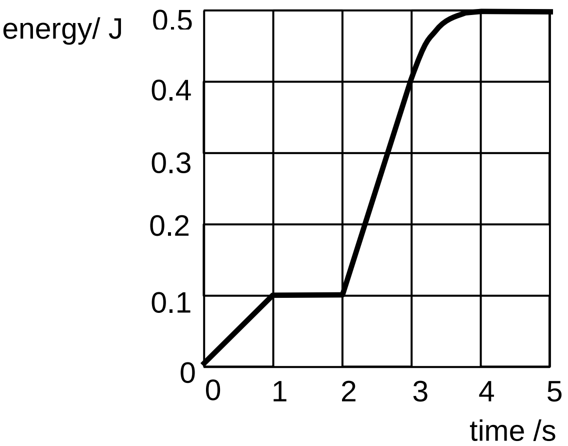
- As $\Delta t \to 0$, instantaneous power $P = \dfrac{\mathrm{d}W}{\mathrm{d}t} = \dfrac{F\,\mathrm{d}x}{\mathrm{d}t} = Fv$

- instantaneous power = instantaneous Force × instantaneous velocity.

- Instantaneous power at a specific time t can be determined from the gradient of energy−time graph at the time t.

Check Your Understanding 3

A bicycle dynamo is started at time zero. The total energy transformed by the dynamo during the first 5 seconds increases as shown in the graph.
What is the maximum power generated at any instant during these first 5 seconds?



Solution

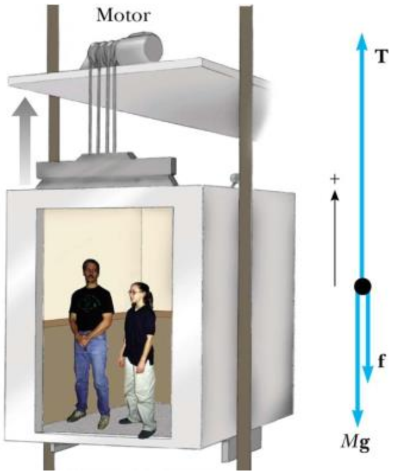
A 0.10 W
B 0.13 W
C 0.30 W
D 0.50 W

Additional Notes

Example 8

A 1000 kg lift carries a maximum load of 800 kg. A constant frictional force of 4000 N retards its motion upward. Calculate the minimum power that the motor delivers to lift the fully loaded lift at a constant speed of 3.00 m s^{-1}.

Solution



<u>Thinking Process</u>
- what is the required motion of the lift based on the question?
- to achieve this required motion, what can we find when we apply Newton's 2nd Law to the lift?
- what is the minimum power from the motor used to do?
- how can minimum power be calculated knowing the above?

D.5 Efficiency

- In machines, due to inevitable work done against frictional forces, the energy conversion can never be 100%.

- Using the Principle of Conservation of Energy,

  energy input = useful energy output + energy wasted

- The efficiency of a machine in its conversion of energy from one form to another is defined as:

$$\text{efficiency, } \eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

- Alternatively, in terms of power,

$$\text{efficiency, } \eta = \frac{\text{useful power output}}{\text{total power input}} \times 100\%$$

Example 9

Water flows to the base of a dam to generate electricity through the use of a hydro-turbine generator. This generator has an efficiency of 70% and generates 1000 MW of electrical power.

(a) Calculate the input power of the generator.

(b) The mass flow rate in this generator is 1.2×10^{6} kg s^{-1}. Determine the height that the water is falling from.

Thinking Process
- what information are we given in the question that would help with calculating the input power?
- how is the height of water falling from related to energy/power? What concept has to be applied to relate the two quantities?

Solution

D.6 Energy costs

- The kilowatt hour is another unit for energy.
- 1 kWh is the amount of energy used when 1 kW of energy is used for 1 hour.
- Hence, $\boxed{1\text{ kWh} = 1000\text{ W} \times 1\text{ hr} = 1000\text{ W} \times 3600\text{ s} = 3,600,000\text{ J}}$
- The use of electricity is usually charged by kWh.

Example 10

An electric cooker has a power rating of 5000 W

(a) How much energy is used in 30 min? Express your answer in kWh.

(b) How much will it cost to run the cooker for 30 mins if electrical energy costs 16 cents per kWh?

Solution

Additional Notes

Summary of Graphical Relationship

Graph	Equation	Significance of	
		Area under graph	Gradient of graph
Force – Displacement	$W = \int_{s_1}^{s_2} F \cdot ds$ $W = F \cdot s$	Work done	–
Energy – Time	$P = \dfrac{dW}{dt}$	–	Power
Power – Time		Work done	–

Summary

Term	Equation	Where…
Work	$W = F\,s\cos\theta$	F is the force, s is the displacement θ is the angle between the force and the displacement vectors
Work done by gas	$W = p\,\Delta V$	p is constant pressure exerted by gas ΔV is the change in volume ($V_{final} - V_{initial}$)
Elastic potential energy	$U = ½\,ke^2$	U is elastic potential energy k is spring constant e is spring extension
Gravitational potential energy	$\Delta U = mgh$	ΔU is (change in) gravitational PE m is mass of object g is gravitational acceleration 9.81 ms^{-2} h is the height from the zero PE level
PE and force relationship	$F = -\dfrac{dU}{dx}$	F can be gravity, spring force, etc. U is the corresponding PE.
Kinetic energy	$E_k = ½\,mv^2$	E_k is kinetic energy m is mass of object v is velocity of object
Work – energy Theorem	$W_{net} = \Delta E_k$	W_{net} is total work done on the object ΔE_k is the change in kinetic energy
Mechanical Energy	$E = E_k + U$	E is mechanical energy U is potential energy E_k is kinetic energy
Power	$P = W / t$	P is power W is work or energy t is time
Power	$P = Fv$	F is driving force of the object v is the velocity of the object

Energy	Stored ability within a system or body to do work. It has the SI unit of Joules (J).
Joule	The amount of work done when a force of one newton moves its point of application a distance of one metre in the direction of the applied force.
Kinetic Energy	Stored ability of an object to do work as a result of its motion.
Gravitational Potential energy	Stored ability of an object to do work as a result of its mass and position.
Electric Potential energy	Stored ability of object to do work as a result of its charge and position
Elastic Potential energy	Stored ability of object to do work as a result of its shape
Power	The work done per unit time.
Watt	SI unit of power. One Watt is equal to one Joule per second.
Work (done by a constant force on a body	The product of the force and displacement in the direction of the force.
Work − (kinetic) energy theorem	work done by a net force on a body = change in kinetic energy of the body