Additional Notes

ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749

Topic 4: Forces

Learning Outcomes:

Candidates should be able to:

Types of forces

(a) recall and apply Hooke's law ($F = kx$, where k is the force constant) to new situations or to solve related problems.

(b) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.

(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance. (No treatment of the coefficients of friction and viscosity is required.)

Centre of gravity

(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.

Turning effects of forces

(e) define and apply the moment of a force and the torque of a couple.

(f) show an understanding that a couple is a pair of forces which tends to produce rotation only.

(g) apply the principle of moments to new situations or to solve related problems.

Equilibrium of forces

(h) show an understanding that, when there is no resultant force and no resultant torque, a system is in equilibrium.

(i) use a vector triangle to represent forces in equilibrium.

Upthrust

(j) derive, from the definitions of pressure and density, the equation $p = \rho gh$.

(k) solve problems using the equation $p = \rho gh$.

(l) show an understanding of the origin of the force of upthrust acting on a body in a fluid.

(m) state that upthrust is equal in magnitude and opposite in direction to the weight of the fluid displaced by a submerged or floating object.

(n) calculate the upthrust in terms of the weight of the displaced fluid.

(o) recall and apply the principle that, for an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object to new situations or to solve related problems.

1. How Submarines Work. Retrieved from (http://science.howstuffworks.com/transport/engines-equipment/submarine1.htm).
2. Cartesian diver. Retrieved from (http://physics.org/interact/physics-to-go/cartesian-diver/).
3. Biomechanics & High Jump. Retrieved from (https://www.topendsports.com/sport/athletics/biomechanics-highjump.htm)

A Types of Forces

Force is a physical quantity that can change the shape of an object, the direction of its motion and its acceleration.

- A force is a push or a pull, that is, a force is an interaction between two bodies or between a body and its environment. Hence we always refer to the force that one body exerts on another.
- From the topic of Dynamics, Newton's 2nd Law states that the resultant force acting on a body is proportional to the rate of change of momentum of the body and the change in momentum takes place in the direction of the force.
- Force is a vector quantity and requires both magnitude and direction for it to be fully defined. The SI unit for force is newton (N) and one newton is equal to 1 kg m s $^{-2}$.
- In the analysis of forces and interactions, the terms *objects* and *systems* will be used in various discussions. *Objects*, can be treated as having no internal structure or an internal structure that can be ignored. A *system*, on the other hand, is a collection of objects with an internal structure which may need to be taken into account.

A.1 Types of interaction

- There are four fundamental forces within all atoms that dictate interactions between individual particles, and the large-scale behavior of all matter throughout the Universe. They are the *strong* and *weak nuclear* forces, the *electromagnetic* force and *gravitational* force.

Name	Relative Strength	Comment	Example
Strong nuclear	1	Large short-ranged forces between nuclear particles. Basically attractive, but can be effectively repulsive in some circumstances.	The holding together of nucleons in a nucleus.
Electromagnetic	~ 10^{-2}	It is long-ranged but much weaker than the strong force. It can be attractive or repulsive and acts only between pieces of matter carrying electrical charge.	Electricity, magnetism, and light are all produced by this force.
Weak nuclear	~ 10^{-14}	It is very short-ranged and is very weak.	It is responsible for β-decay (the conversion of a neutron to a proton, an electron and an antineutrino).
Gravitational	~ 10^{-40}	It is long-ranged but very weak. It is always attractive, and acts between any two pieces of matter in the Universe since mass is its source.	Pull of Earth on Moon and Moon on Earth.

Additional Notes

A.2 Field of Force

- A physical body can exert a force on another similar body not in contact with it. Such interactions can be explained using the field concept.
- Field concept can be used to explain a physical phenomenon in terms of a field and the manner in which it interacts with matter or with other fields.
- A **Field of Force** is a region of space surrounding a body within which it can exert a force on another similar body not in contact with it.

They are thus sometimes known as non-contact forces.

(i) Gravitational Force

- A body of mass M establishes round itself a region of space in which it has the ability (or potential) to exert a gravitational force on another mass.
- The strength of the gravitational field established by mass, M, at a point in the region of space is given by g, and the gravitational force F exerted on another body of mass, m, placed at that point is given by $F = mg$. The gravitational force F is always attractive and points towards the mass M.

(ii) Electric Force

- A body of charge Q establishes around itself a region of space in which it has the ability (or potential) to exert an electric force on another charge.
- The two kinds of charges responsible for electric forces are the positive and negative charges. Unlike gravitational forces, electric forces can be attractive or repulsive, depending on the signs of the charges involved.
- Like charges repel and unlike charges attract.

(iii) Magnetic Force

- A magnetic field may exist in a region of space as a result of the presence of either a permanent magnet or a moving charge or current–carrying conductor.

A.3 Some common types of Forces

(i) Contact Force

When a body is in contact with another surface, the surface exerts a contact force on the body. This contact force has two components, namely the normal contact force (F_N) and the frictional force (F_r).

Normal Contact Force, F_N

- Consider a stone resting on a road. If the contact between the road and the stone is examined closely, it can be seen that the two rough surfaces make close contact only at relatively few places. Where contact is made, the road will exert a force on the stone as shown in Fig 1(a).

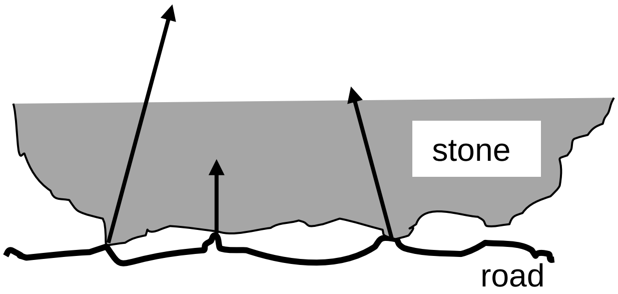


Fig 1(a)

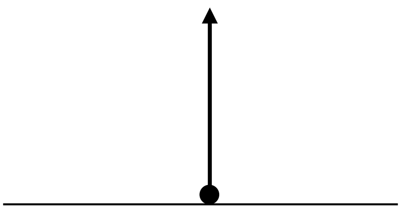


Fig 1(b)

- The sum of all these forces is shown in Fig 1(b) and this single force is the normal contact force which the road exerts on the stone. The normal contact force acts perpendicular to the surface that the body is in contact with and is always directed away from the surface towards the body.

Frictional Force, F_r

- For stationary body, the frictional force acts along the surface in a direction to prevent the body from sliding (or to oppose the tendency of motion).
- For surfaces in relative motion, the direction is to oppose the relative motion of the surfaces.
- Using the example of the stone above, if the stone is sliding across the road (e.g. to the left), Fig 1(a) then changes to Fig 2(a).

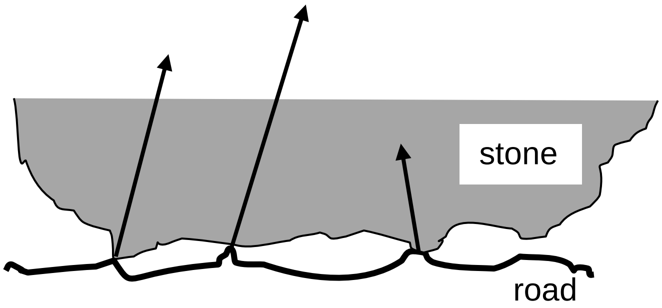


Fig 2(a)

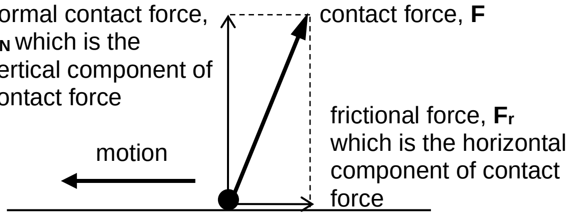


Fig 2(b)

- The contact force can be considered as being the vector sum of a horizontal component (the frictional force) and a vertical component (the normal contact force).

- For two surfaces where friction can be neglected (i.e. *smooth* surface) the only force will be the normal contact force. When a surface is *rough*, there will be frictional force as long as the body has a tendency to move.
- Friction is often regarded as a nuisance but it is often crucially necessary. For example, walking is made possible by friction. Chaos and accidents usually follow when the reduction in friction takes place on wet roads.

(ii) Viscous force

- When a solid moves in a fluid (liquid or gas) or when a fluid moves over it, it will also experience a resistance. This is called the viscous force.
- Viscous forces oppose motion, so the direction of a viscous force on a moving body is always <u>opposite to the direction of its motion</u>.
- Viscous forces are also dissipative in nature.
- The magnitude of viscous forces is <u>dependent on the speed of bodies</u>. For example, the air resistance (an example of viscous force) acting against a car is proportional to the speed of the car at low speeds ($F \propto v$). At high speeds, the air resistance is proportional to the square of the speed of the car ($F \propto v^2$).
- Viscous forces also depend on the type of fluid (viscosity) involved. For example, for a body moving at the same speed, it will encounter larger viscous force when travelling through oil compared to air or water.
- Frictional forces vs viscous forces

Frictional Forces	Viscous forces
Act along the surfaces in contact between two solid bodies.	Act along the surfaces in contact between a fluid and a solid body.
Not affected by relative motion of two surfaces.	Increase when relative velocity of fluids and moving bodies increases.
Friction exists even when the body is at rest.	Does not exist when there is no relative motion.

<u>**Check Your Understanding**</u>

Is friction an example of viscous force? **Yes** or **No**

(iii) Tension

- Tension is the force which is transmitted through a string, rope, cable or wire when it is pulled tight by forces acting from opposite ends.
- Tension is directed along the length of the string and pulls equally on the bodies on the opposite ends of the string.

<u>Tension in a spring</u>

- When a body is acted upon by a force, it can be compressed, stretched or bent. If when the force is removed, the body returns to its original shape, it is said to be elastic. Solids that do not return to their original configuration once they have been distorted are categorised as plastics.

Additional Notes

- Elastic materials will stretch until they reach their **limit of elasticity**, or yield point. After that point, they exhibit plastic deformation and will never return to their original shape. Ductile materials stretch thinner and thinner, while brittle materials break without any plastic deformation. Eventually all will rupture at their breaking point.

Hooke's Law

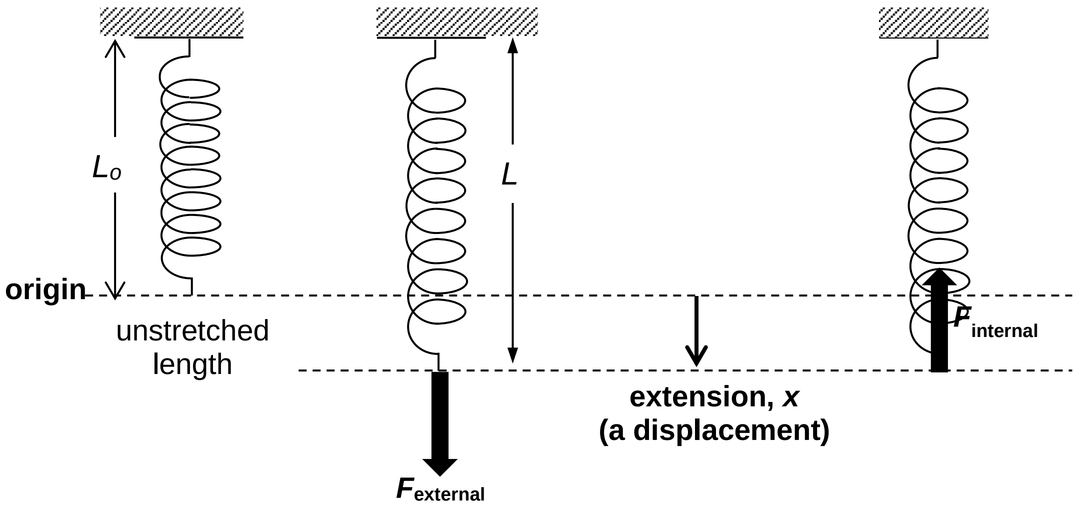
Memorize

Hooke's Law states that, provided the proportionality limit is not exceeded, the extension (or compression) of a body is proportional to the applied load.

Note that exceeding limit of proportionality just means force no longer proportional to extension, does not imply object cannot return to original length. Only when you exceed limit of elasticity will there be permanent deformation.

Mathematical form of Hooke's Law

Consider a spring as shown in the figure.



$$F_{external} = kx$$
$$F_{internal} = -kx$$

where $F_{external}$ is the force supplied by an external agent on the spring distorting it from equilibrium position in newtons (N)

$F_{internal}$ is the force exerted by the spring to restore itself to equilibrium position in newtons (N)

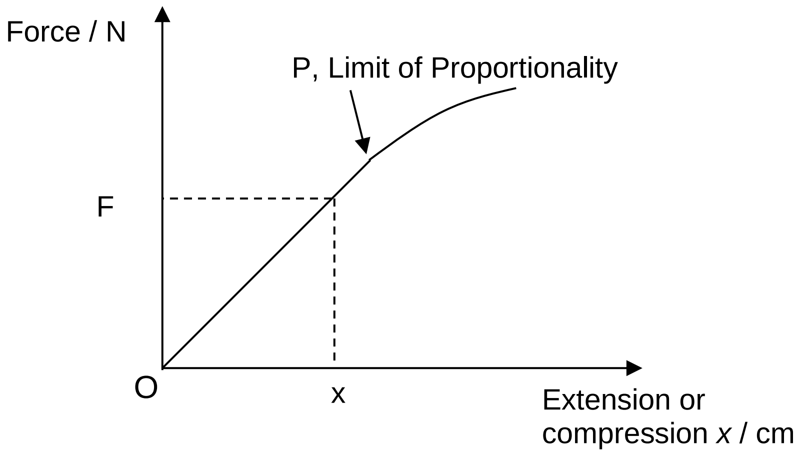
x is the extension (or compression) of the spring ($x = L - L_o$) in metres (m)

k is the stiffness / spring constant / force constant in newtons per metre (N m^{-1})

- length ≠extension
- extension = final length – original length
- compression = original length - final length

- The negative sign indicates that the spring force is a restoring force, i.e., the force $F_{internal}$ always acts in the direction opposite to the direction in which the system is displaced.
- Here we assume that the positive direction for values of x is the same as the positive values of the force supplied by the external agent.
- x is the distance that the spring has been stretched or compressed with respect to the equilibrium position. **Do not confuse *x* with the length of the spring!**

- The graph of the external force plotted against extension (or compression) for a spring under load is shown below:



- Features of the graph:

  - The magnitude of the force is directly proportional to the extension/compression of the spring (i.e. $F \propto x$) until P, which is known as the **limit of proportionality**. Beyond this point, force is no longer proportional to extension/compression.
  - The ratio of force to extension/compression (within the limit of proportionality) is the spring constant,

$$\text{spring constant } k = \frac{F}{x}$$

  - The larger the value of k, the stiffer the spring. That is, a larger force is required to extend it by a given length.

Example 1

Some weight-lifters use a 'chest expander', consisting of a strong spring with a handle at each end, to exercise chest and arm muscles. The spring obeying Hooke's law has an unstretched length of 50.0 cm and a spring constant of 400 N m^{-1}.

Calculate the tension in the spring when the overall length is 70.0 cm.

Additional Notes

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Elastic Potential Energy (Strain Energy)

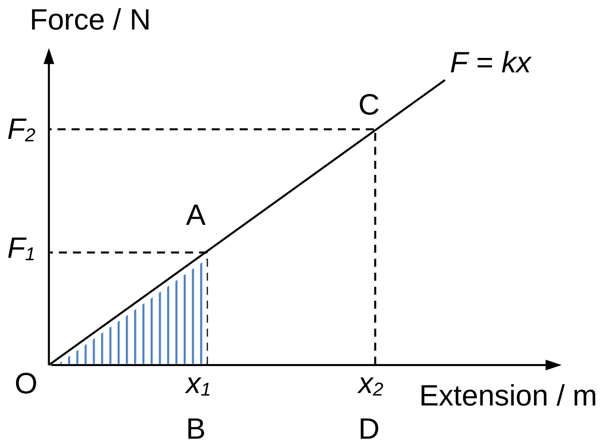
Strain energy is the energy stored in a material when it is stretched or compressed.

- An external force F acting on a spring or wire (causing it to extend to x_1) performs work W given by

W = area under F-x graph from x = 0 to x_1

- This work is stored as elastic potential energy (strain energy) in the spring or wire.
- Elastic potential energy stored is represented by the area under the force-extension graph.

Derivation of Elastic Potential Energy:



Energy stored when spring is extended from 0 to x_1 $=$ area under $F-x$ graph from 0 to x_1

$=$ area of triangle OAB

$= \frac{1}{2}F_1x_1$

$= \frac{1}{2}kx_1^2$ (since $F_1 = kx_1$)

Additional energy stored when spring is extended from x_1 to x_2

$=$ area under the $F-x$ graph from x_1 to x_2

$=$ area of triangle OCD $-$ area of triangle OAB

$= \frac{1}{2}F_2x_2 - \frac{1}{2}F_1x_1$

$= \frac{1}{2}k\left(x_2^2 - x_1^2\right)$

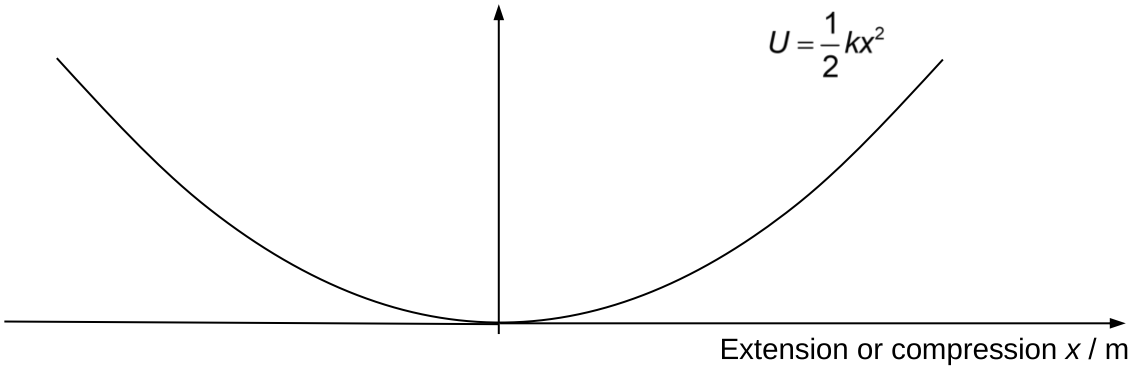
This area is the area enclosed by the trapezium BACD.

Note : Common mistake

$\frac{1}{2}k\left(x_2^2 - x_1^2\right)$ is not equal to $\frac{1}{2}k\left(x_2 - x_1\right)^2$

Additional Notes

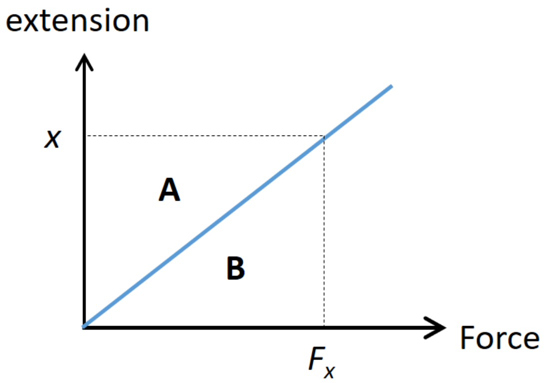
- The graph of total elastic potential energy stored (U) against extension/compression (x) is shown below.



The equation $U = \frac{1}{2}kx^2$ has the same format as $y = mx^2$.

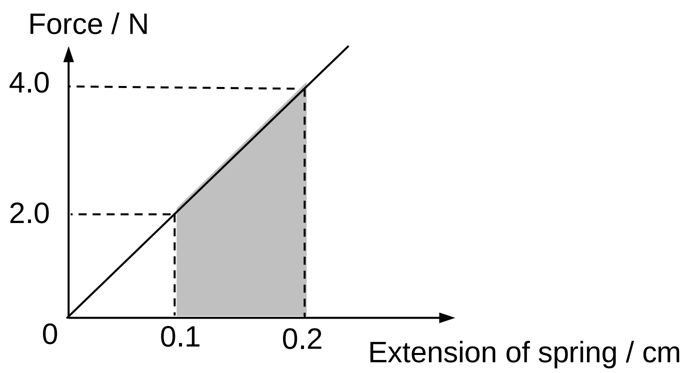
Check Your Understanding

Shade the area which gives the work done on the spring by the force F_x in the figure below.



Example 2

The extension of Spring A when a variable force is applied is given by the graph below.



(a) Calculate the increase in potential energy stored when the spring extends from 0.10 cm to 0.20 cm.

Note that the effective spring constants of springs arranged in series is given by:

$\frac{1}{k_{eff}} = \frac{1}{k_1} + \ldots + \frac{1}{k_N}$

& for springs arranged in parallel, it is given by

$k_{eff} = k_1 + \ldots. + k_N$

Do not memorise them. You can learn to derive them in Tutorial Question A1.

(b) Spring B is added in series to Spring A such that when a force of 4.0 N is applied, the total extension is 0.30 cm. Sketch the force-extension graph of Spring B in the figure above.

(iv) Forces due to fluid

<u>Pressure at a point in a Liquid</u>
Derivation:

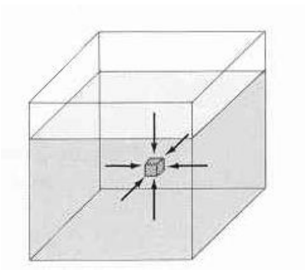


Fig 1

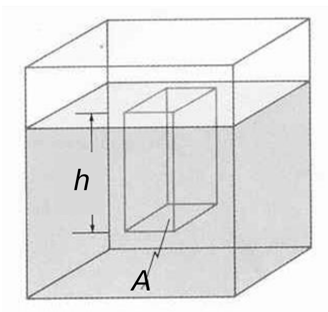


Fig 2

- Pressure is defined as the force per unit area, where the force is acting at right angles to a unit area.
- At a particular point in a liquid at rest, the pressure is the same in all directions, as illustrated in Fig 1.
- A fluid at rest exerts a force due to fluid pressure and the force always acts perpendicularly to any surface it is in contact with.

Additional Notes

- Consider a point which is at a depth *h* below the surface of the liquid in Fig 2. The pressure due to the liquid at this depth *h* is due to the weight of the column of liquid *F* above it over an area of *A*.

- Pressure $P = \dfrac{F}{A}$

- Density ρ is defined as the mass *m* per unit volume *V* of a substance.

- Hence, Pressure $P = \dfrac{F}{A} = \dfrac{m_{fluid}g}{A} = \dfrac{(V_{fluid}\rho_{fluid})g}{A} = \dfrac{(hA\rho_{fluid})g}{A} = h\rho_{fluid}g$

Remember this derivation

h is measured from the surface of the liquid downwards.

h is always the vertical height.

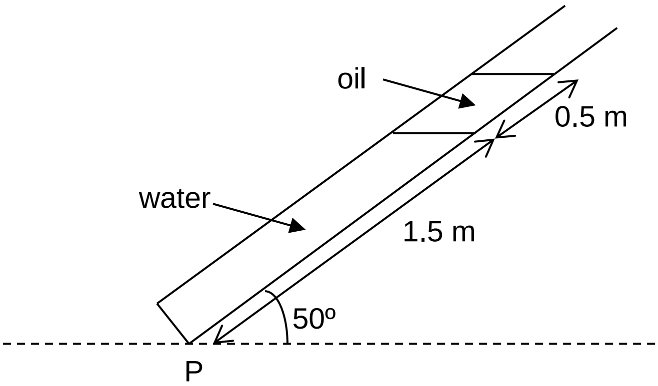
- Pressure in a fluid increases with depth of the fluid.

- The pressure at equal depths within a uniform liquid is the same.

- The equation is valid for fluids whose density is constant and does not change with depth, that is, if the fluid is incompressible.

Example 3

A long narrow tube is filled with water of density 1000 kg m^{-3} to a depth of 1.5 m, and oil of density 800 kg m^{-3} to a depth of 0.5 m. The tube is then inclined at 50° to the horizontal. Given that the atmospheric pressure is 1.01 x 10^{5} Pa, estimate the pressure at the bottom of the tube, point P.



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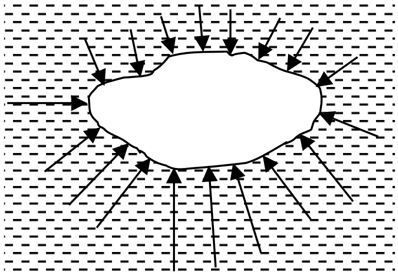
Upthrust



- The figure above shows a body submerged in a fluid. Drawn on the body are forces exerted on the body due to fluid pressure, and they are acting at right angles to the surface of the body.
- The horizontal components of these forces cancel out each other. As for the vertical components, the upward components on the bottom are larger than the downward components at the top. Hence, the resultant of all of these forces gives an upward force. This resultant upward force is known as the **upthrust** or **buoyancy force**.

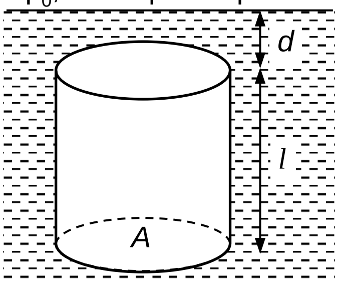
Remember this

- Upthrust **orignates from the difference in pressure between the top and bottom surfaces of the immersed body**, a difference that always exists due to the fact that pressure varies with depth. This implies that there is upthrust as long as a body is in a fluid.

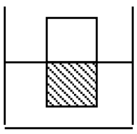
Derivation of magnitude of Upthrust :

- Consider a body of uniform cross-sectional area A of length l placed so that its top is a distance d beneath the surface of a liquid of density ρ_f.

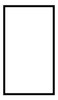
p_0, atmospheric pressure



Pressure at top surface	$= d\,\rho_f\,g + p_0$
Force (downward) on top surface	$= (d\,\rho_f\,g + p_0)\,A$
Pressure at bottom surface	$= (d + l)\,\rho_f\,g + p_0$
Force (upward) on bottom surface	$= [(d + l)\,\rho_f\,g + p_0]\,A$
Upthrust	$= (d + l)\,\rho_f\,g\,A - d\,\rho_f\,g\,A = l\,A\,\rho_f\,g$



Hence, $\boxed{\text{Upthrust} = V_f\,\rho_f\,g}$


= volume of object

where $V_f = l\,A$ is the volume of the body in the fluid
OR the volume of the fluid displaced


= volume of object below fluid surface

Additional Notes

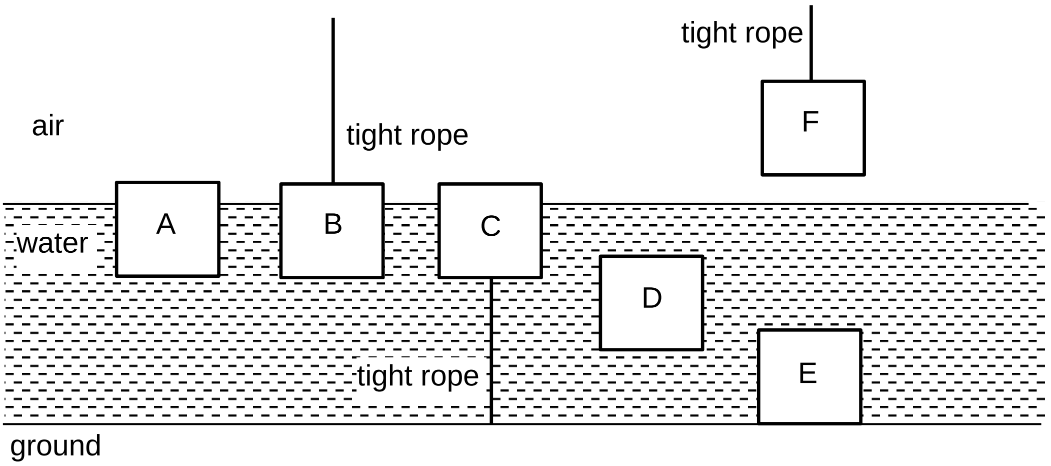
- **Archimedes' Principle** states that for a submerged or floating object in a fluid, it experiences an **upthrust** equal in magnitude and opposite in direction to the **weight of fluid displaced**.

- For an object floating in equilibrium, since the only two forces acting on the floating object are its weight and the upthrust, the **Principle of Flotation** states that for **an object floating in equilibrium**, the upthrust is equal in magnitude and opposite in direction to the weight of the object.

- Extending Archimedes' Principle to **an object floating in equilibrium**, the **weight of the fluid displaced** is therefore equal in magnitude and opposite in direction to the **weight of the object**.

<u>Check Your Understanding</u>

Which of the following object(s) can **(i)** Archimedes' Principle and **(ii)** Principle of Flotation be applied?



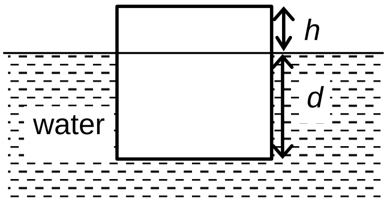
Example 4

A spherical buoy of radius 10 cm floats on the surface of the water with half of its volume submerged in the water. The density of water is 1.0×10^3 kg m^{-3}.

(a) Calculate the upthrust.
(b) Calculate the mass of the buoy.

Example 5

A wooden body is floating as shown. Density of the body and water is ρ_b and ρ_w respectively. The body has base area *A*. Determine the ratio ρ_w / ρ_b.
Note: Be careful and meticulous in handling subscripts!



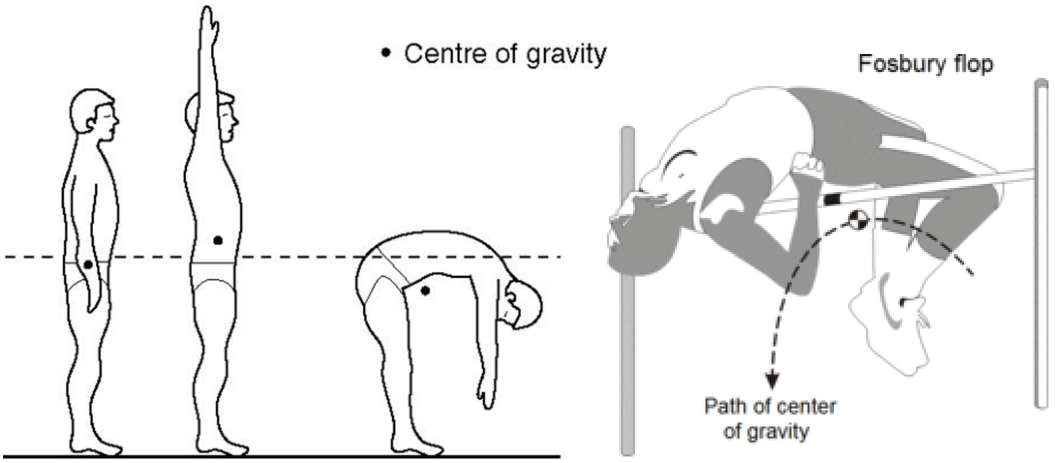
B Centre of Gravity

- Any body other than the idealised point body has size. It therefore has its mass distributed over its volume and forces acting on parts of it. If it were necessary to consider every bit of the body's mass separately, the mathematical manipulation would be very complex. To simplify this, we use the term centre of mass.

- The centre of mass is the point at which the whole mass of the body may be considered to be concentrated.

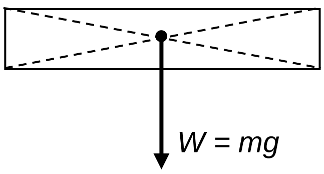
- The **centre of gravity** of an object is the point at which the whole weight of the body may be considered to act.

- The position of the centre of mass is usually in the same place as the centre of gravity of the body if the body is situated in a uniform gravitational field. The weight of a body may be taken as acting through the centre of gravity of the body.



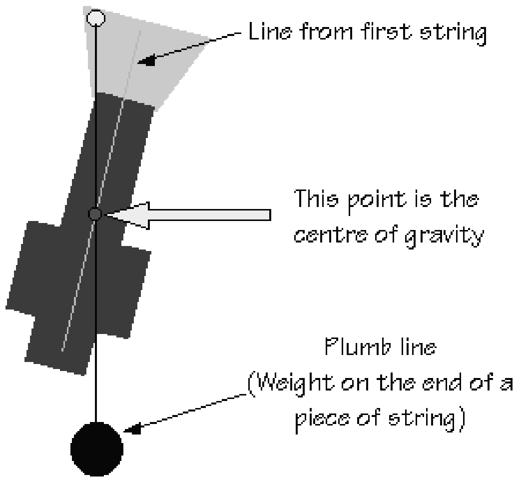
Applying centre of gravity to biomechanics in the area of sports

- The centre of gravity of a regularly shaped body of uniform thickness and density is at its geometrical centre.



Please refer to Appendix to see how to calculate cg.

- For an irregularly shaped body, or for a body of non-uniform thickness or density, the centre of gravity can be found by using calculation or a plumb line.



Plumb line method to find centre of gravity

C Turning Effects of Forces

- A force, besides being able to cause a body to change its linear momentum, can also cause the body to rotate. This is the moment of a force or the turning effect of a force.

C.1 Moment of a force

Memorize

- Moment of a force is the turning effect of a force. It is equal to the product of the force and the perpendicular distance of the line of action of the force from the pivot.

What is the 'perpendicular distance' perpendicular to?

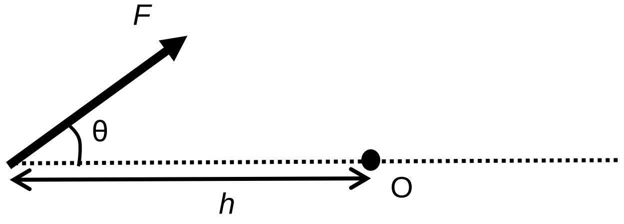
What is the 'line of action of the force'?

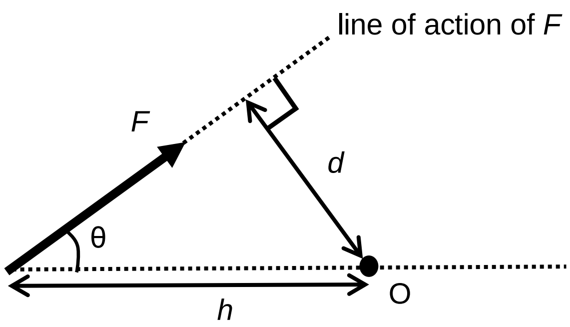
Where are the start and end points of the 'perpendicular distance'?

- Mathematically,

Moment of a Force about a point O = Force x Perpendicular Distance to point O

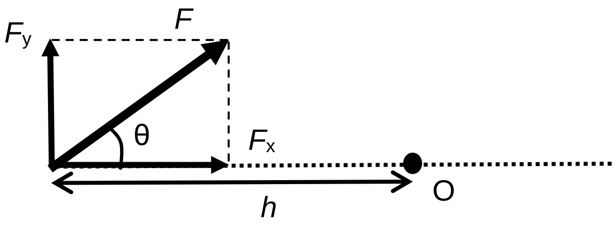
- The unit of moment is the newton-metre (N m).
- It is a vector quantity. Its direction is either clockwise or anti-clockwise.
- When finding moment of a force, we always find moment about any point. There is no need for a physical pivot in the question for us to find the moment of a force.
- Examples of moment at work include: closing a door, lifting a car hood.





<u>Method 1:</u> Find the perpendicular distance d from O to the line of action of the force F

moment of F about point O
$= F\, d$
$= F\, h \sin\theta$



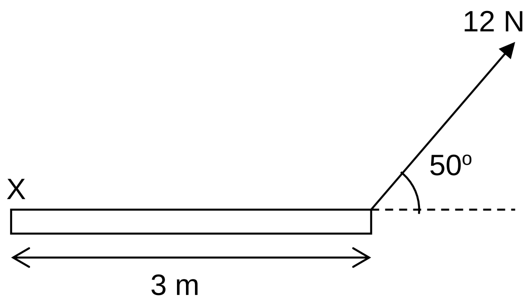
<u>Method 2:</u> Resolving F into two perpendicular components, F_x and F_y

moment of F about point O
= moment of F_y about point O
$= (F_y)\, h = F \sin\theta\, h$

Note that component F_x has no turning effect about point O since its line of action passes through O.

Worked Example

Calculate the moment of the 12 N force about the given point X.



Solution

Moment of 12 N force about X
$= 12 \sin 50^{o}\ (3)$ or $12\ (3 \sin 50^{o})$
$= 28$ N m

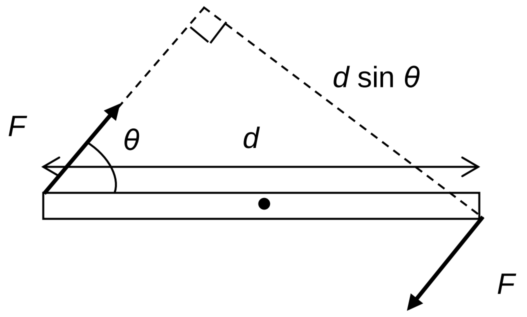
Additional Notes

C.2 Torque of a Couple

Memorize

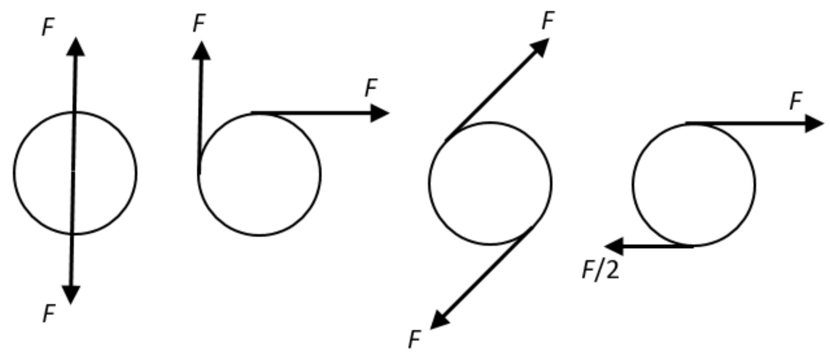
- A <u>couple</u> consists of two equal and opposite parallel forces whose lines of action do not coincide.

- It is a system of forces which produces a turning effect only (i.e. zero translational effect), resulting in rotation.



Check Your Understanding

Which of the following shows a couple?



A **B** **C** **D**

Memorize

<u>Torque of a couple</u> is the turning effect of a couple. It is equal to the product of one of the forces and the perpendicular distance between the forces.

Torque = Force F x Perpendicular Distance between the Two Parallel Forces
= F x $d \sin \theta$

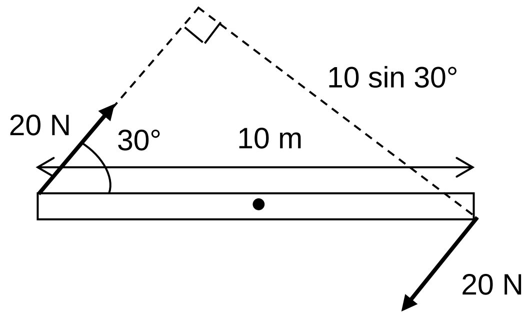
- The unit of torque is the <u>newton-metre (N m)</u>.

- An example of the torque of a couple at work: turning a tap

Additional Notes

Example 6

Calculate the torque of the couple.



D Equilibrium of Forces

- A system may consist either

  1. a point mass (body has no dimension), or
  2. an extended mass (body has dimension).

D.1 Point Mass

- For a point mass (body with no dimension), the condition for equilibrium is:

  Resultant force must be zero (Translational Equilibrium)

- There are 2 methods to solve a problem involving a point mass in equilibrium:

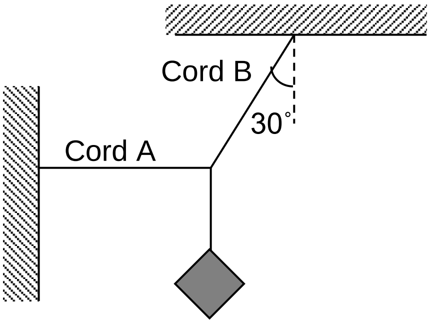
  1. Method 1 (Force Diagram)
     Force diagram must form a closed polygon.

  2. Method 2 (Resolving Forces)
     Vector sum of forces in any direction = 0
     For example, vector sum of forces in x-direction = 0
     vector sum of forces in y-direction = 0

From Newton's Law, it therefore implies that when a point mass is in translational equilibrium, it is either at rest or moving with constant velocity.

Example 7

A body of weight 200 N is suspended by two cords, A and B as shown below. Determine the tension in each cord.



Additional Notes

D.2 Extended Mass (or rigid body)

- For an extended mass (body with dimensions) or a system in general, the conditions for equilibrium are:

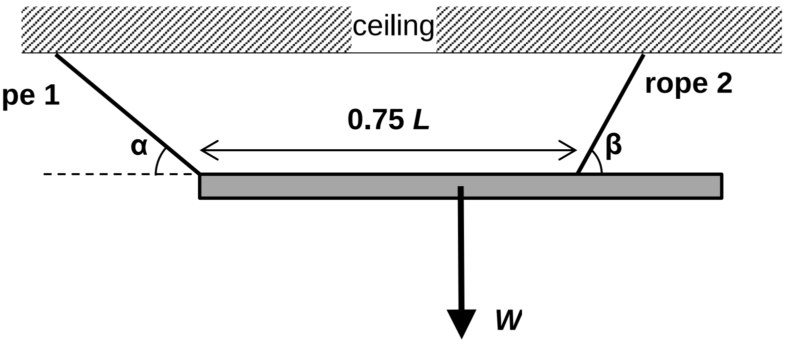
  1. No resultant force (Translational Equilibrium)
  2. No resultant torque about any point (or axis) (Rotational Equilibrium)

When an object is at rest (no translation or rotation), the object is said to be in *static equilibrium* (and the 2 conditions must be met). The same conditions apply to an object in uniform translational motion (without rotation) such as an airplane in flight with constant speed, direction and altitude. Such a body is in equilibrium but is not static.

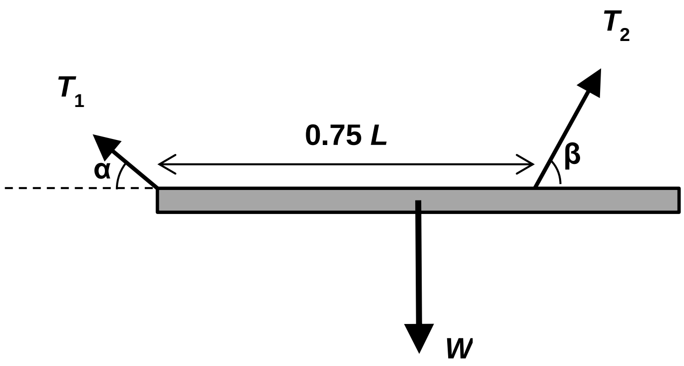
- To solve problems involving an extended mass, the principle of moments may be applied.

- The principle of moments states that for a system in equilibrium, there is no resultant moment about any point (or axis). In other words, the sum of the clockwise moments about any point (or axis) must be equal to the sum of anticlockwise moments about that same point (or axis).

Consider a uniform beam of weight W and length L hanging from the ceiling, supported by two ropes as shown in the figure below.



Draw a Free Body Diagram (FBD) of the beam and identify the forces acting on the beam.



Resolving the tension forces and applying translational equilibrium (condition 1), we have:

[horizontal direction → +ve] $\sum F_x = 0$

$\Rightarrow T_2 \cos\beta - T_1 \cos\alpha = 0$ …………… [1]

[vertical direction ↑ +ve] $\sum F_y = 0$

$\Rightarrow T_1 \sin\alpha + T_2 \sin\beta - W = 0$ …………[2]

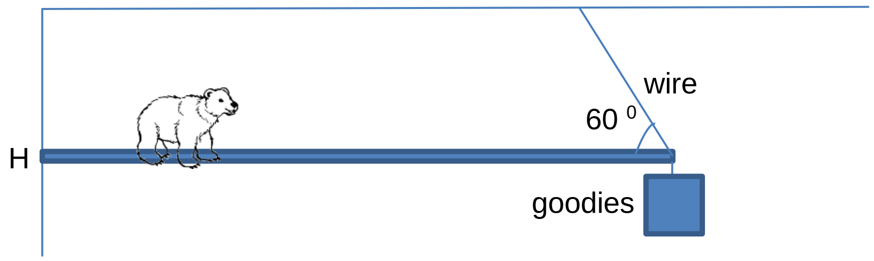
Using principle of moments and taking moments about the centre of gravity (applying rotational equilibrium, condition 2), we have:

$\sum M_{cg} = 0 \Rightarrow (T_1 \sin\alpha)(0.5\,L) - (T_2 \sin\beta)(0.25\,L) = 0$ …………………[3]

When applying Condition 2, the condition holds true for any point (or axis) that you take moments about. It need not necessarily be the cg, and for that matter, the point (axis) need not be on the object.

Example 8

A hungry bear weighing 700 N walks out on a beam in an attempt to retrieve some “goodies” hanging at the end of the beam. The beam is uniform, weighs 200 N, and is 6.00 m long. The goodies weigh 80.0 N.



(a) By drawing a labelled free body diagram of the beam, determine the tension in the wire when the bear is at a distance of 1.00 m away from hinge H.

(b) Determine the force from the hinge when the bear is at this distance.

Additional Notes

D.3 Equilibrium of Rigid Body under Three Concurrent Forces

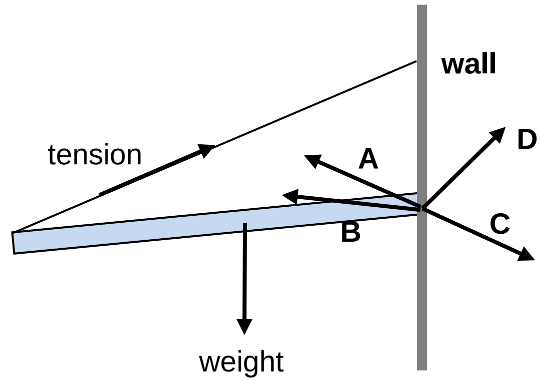
- When an extended mass (rigid body)
  1. is in equilibrium and
  2. there are only 3 coplanar non-parallel forces acting on the body

  the forces must be concurrent forces (forces whose lines of action pass through a single point).

The converse is not necessarily true, ie if there are 3 concurrent forces acting on an object, the object may not necessarily be in equilibrium. You have to check if the net force acting on the object is zero.

Example 9

A uniform beam shown below is in equilibrium. It is supported by a wire and a rough wall. Which of the following forces correctly show the direction of the force exerted by the wall on the beam?

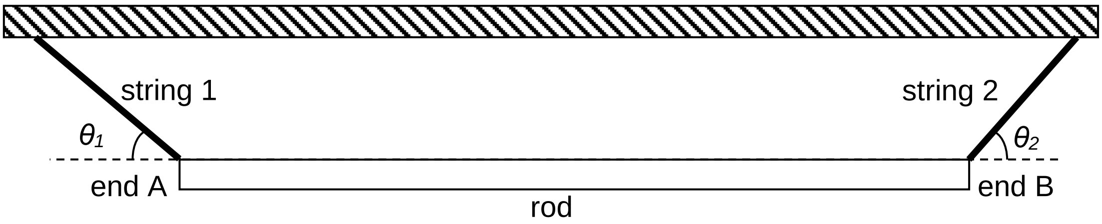


Example 10

A non-uniform rod of length 1.0 metre is held horizontally in place by two inextensible strings.

(a) Without calculation, indicate the location of the centre of gravity of the rod in the diagram below.

(b) If θ_1 = 30°, θ_2 = 45° and the weight of the rod is 30 N, calculate the distance of the centre of gravity from end A of the rod.



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SUMMARY

Term	Definition
Hooke's Law	Provided the proportionality limit is not exceeded, the extension (or compression) of a body is proportional to the applied load.
Archimedes' principle	For a submerged or floating object in a fluid, it experiences an upthrust equal in magnitude and opposite in direction to the weight of fluid displaced.
Principle of Flotation	For an object floating in equilibrium, the upthrust is equal in magnitude and opposite in direction to the weight of the object.
Centre of gravity	The point at which the whole weight of that body may be considered to act.
Moment of a force (about a point)	Moment is the turning effect of a force and is calculated as the product of the force and the perpendicular distance of the line of action of the force from the pivot.
Couple	Two equal and opposite parallel forces whose lines of action do not coincide.
Torque of a couple	Torque of a couple is the turning effect of a couple and is calculated as the product of one of the forces and the perpendicular distance between the forces.
Conditions for a system to be in equilibrium	1. No resultant force. 2. No resultant torque about any point (or axis).
Principle of Moments	The principle of moments states that for a system in equilibrium, there is no resultant moment about any point (or axis). In other words, the sum of the clockwise moments about any point (or axis) must be equal to the sum of anticlockwise moments about that same point (or axis).

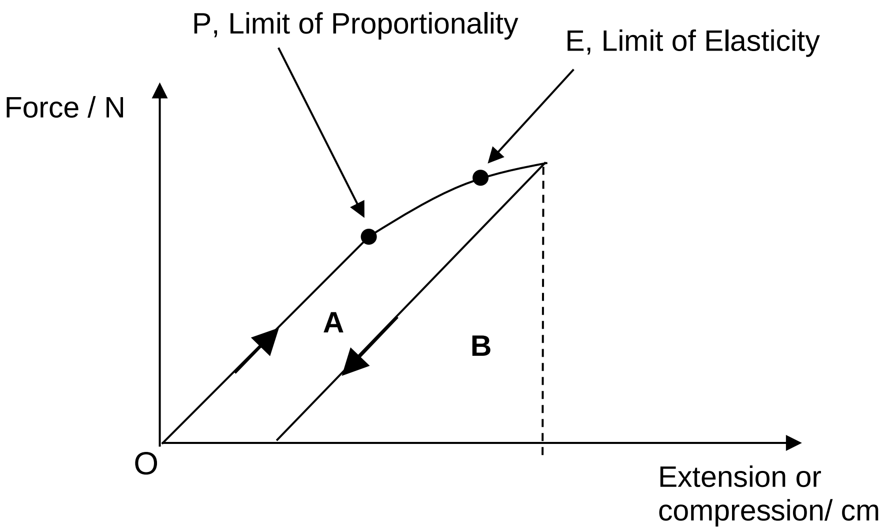
IMPORTANT EQUATIONS

Hooke's Law	$F = kx$	F = tension in the spring k = spring constant x = extension/compression of the spring
Elastic Potential Energy (E_e)	$E_e = \frac{1}{2}kx^2$	k = spring constant x = extension/compression of the spring
Pressure (P)	$P = \frac{F}{A}$ $P = h\rho g$	F = force acting perpendicularly on unit area A= area h = depth of fluid ρ = density of fluid
Upthrust	$U = V\rho g$	U = upthrust V = volume of fluid displaced by object ρ = density of fluid g = gravitational acceleration
Moment	$m = F \times d$	m = moment F = applied force d = perpendicular distance from the line of action of the force to the pivot

APPENDIX

<u>Spring that has exceeded the limit of elasticity</u>

Note that exceeding limit of proportionality just means force no longer proportional to extension, does not imply object cannot return to original length. Only when you exceed limit of elasticity will there be permanent deformation.



- Beyond the limit of elasticity, the spring will not return to its original length even when applied force to extend it is removed.

- Area **A** + **B** represents the work done necessary to extend the elastic material.

- Area **B** represents the energy released when the applied force to extend it is released. This is also the elastic potential energy stored within the elastic material.

- Not all the work done was transformed into elastic potential energy stored within the elastic material. The energy which was not transformed is represented by area **A**. This amount of energy is used to cause deformation of the material and is also converted into heat.

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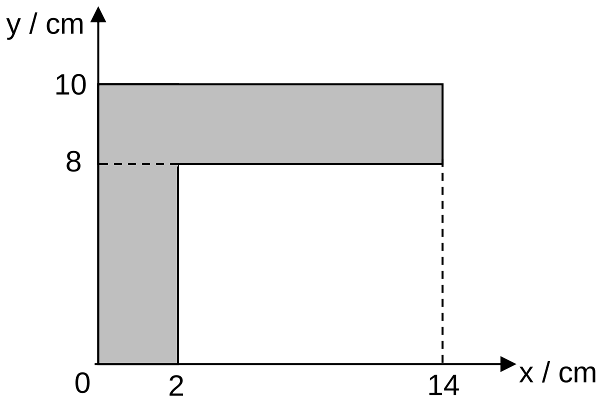
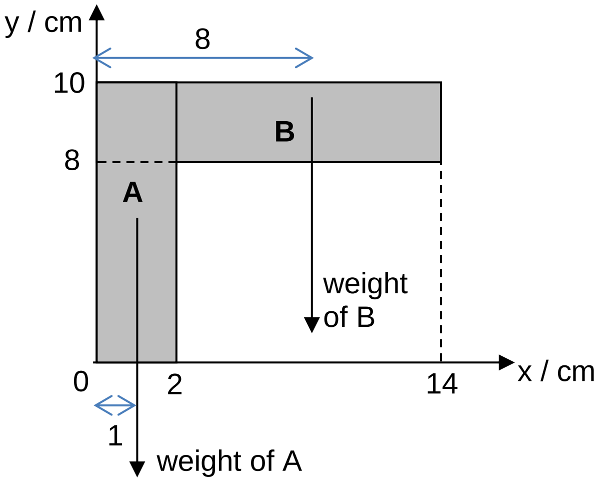
Finding Centre of Gravity

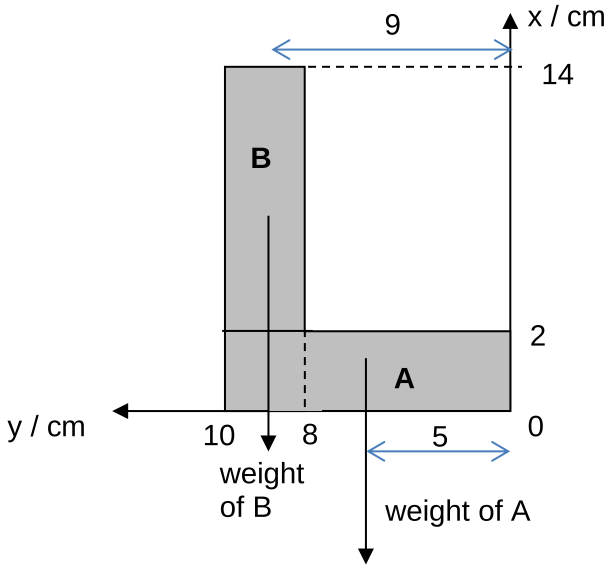
A uniform L-shape body of dimensions in centimetres is placed on Cartesian plane as shown. The following steps show how the coordinates of the location of centre of the gravity of the body is obtained.



Consider that the block is made of two parts A and B.
Let t be the thickness of the block, ρ be the density of the body



Consider the x-direction, taking moment about the origin (0, 0)
Moment due to weight of A + Moment due to weight of B
= Moment due to the weight of the block
$(10 \times 2)(t\,\rho\,g)(1) + (12 \times 2)(t\,\rho\,g)(8) = (10 \times 2 + 12 \times 2)\,t\,\rho\,g\,x$
$x = 4.8$



Consider the y-direction, taking moment about the origin (0, 0)
Moment due to weight of A + Moment due to weight of B
= Moment due to the weight of the block
$(10 \times 2)(t\,\rho\,g)(5) + (12 \times 2)(t\,\rho\,g)(9) = (10 \times 2 + 12 \times 2)\,t\,\rho\,g\,y$
$y = 7.2$

Coordinates of the location of centre of the gravity of the body is (4.8, 7.2)