Additional Notes

ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749 / 8867

Name: ______________________________ (   )   **Class:** 25 / ____

Topic 1: Measurement

Content

- Physical quantities and SI Units
- Scalars and vectors
- Errors and uncertainties

Learning Outcomes:

Candidates should be able to:

<u>Physical quantities and SI units:</u>

(a) recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol)

(b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.

(c) use SI base units to check the homogeneity of physical equations

(d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000*)*

(e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)

(f) make reasonable estimates of physical quantities included within the syllabus

<u>Scalars and vectors:</u>

(g) distinguish between scalar and vector quantities, and give examples of each

(h) add and subtract coplanar vectors

(i) represent a vector as two perpendicular components

<u>Errors and uncertainties:</u>

(j) show an understanding of the distinction between systematic errors (including zero error) and random errors

(k) show an understanding of the distinction between precision and accuracy

(l) assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required)

* Swinfen, T.C., & Association for Science Education. (200). *Signs, symbols and systematics: The ASE companion to 16 - 19 science*, Hatfield, Herts: Association for Science Education

Nature of Science

Physics is an experimental science. Precise measurements enable the collection of useful experimental data that can be tested against theoretical predictions to refine the development of physical theories. ***Experimental evidence is the ultimate authority in discriminating between competing physical theories. Scientific knowledge continues to evolve*** *as data from new or improved measurements helps us to understand and quantify the natural world.*

Relating Science and Society

Other sciences and society as a whole have ***benefited from the invention of many amazing measuring devices and techniques****. Examples include*

- *Modern engineering, in areas like design, construction,*
- *Medical industry, where sophisticated devices like magnetic resonance imaging (MRI) scanners provide important information that aids in diagnosis and treatment decisions.*

A Physical Quantities and SI Units

A.1 Physical Quantities

In Singapore, the A*Star National Metrology Centre is the custodian of the national physical measurement standards, responsible for establishing and maintaining the nation's highest level of physical measurement standards

https://www.a-star.edu.sg/nmc/About-NMC/National-Measurement-System

- Quantities that can be measured are known as physical quantities. A physical quantity consists of a numerical magnitude and a unit.

  For example,

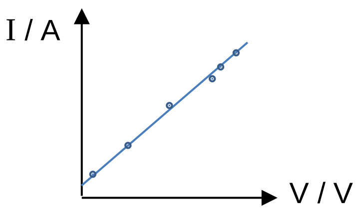
  mass, *m* = 65.0 kg

  physical quantity — symbol — numerical magnitude — unit

- A symbol may be used to represent a physical quantity.
  In the above example, the symbol "*m*" represents both the numerical magnitude and the unit. Hence, writing "*Let the mass be m kilograms*" is incorrect, and should be written as "*Let the mass be m*".

More details on the labelling of graphs and tables will be given in the upcoming Practical Lecture.

- A symbol divided by a unit represents a number. This is used in labelling of graph axes and for headings of table columns



I / A	V / V
12.5	2.50
16.2	3.24
…	…
…	…

Observe that the values in the columns do not have units as the units are in the column headings.

- Physical quantities can be classified into two categories: base quantities (see A.2) and derived quantities (see A.3). Consequently, units are also classified into two categories, base units and derived units.

Unit Conversion disaster:

http://mentalfloss.com/article/25845/quick-6-six-unit-conversion-disasters

- S.I. units was adopted to put an end to more than a hundred years of disarray with a great number of units and unit systems. The S.I. system (metric system) makes a distinction between two classes of units: S.I. base units and S.I. derived units.

Additional Notes

Definition of the SI base units

https://physics.nist.gov/cuu/Units/current.html

A.2 Base Quantities and Base Units

- A base quantity is one that is independent of other base quantities.

- A set of 7 base (or fundamental) quantities were chosen for convenience, instead of necessity and all other units are derived from them. Their corresponding units are called base units and are independent of each other.

- The seven base quantities in the S.I. convention and their corresponding base units are as below.

Base Quantity	Base unit	Symbol for unit
mass (*m*)	kilogram	kg
length (*l*)	metre	m
time (*t*)	second	s
electric current (*I*)	ampere	A
temperature (*T*)	kelvin	K
amount of substance (*n*)	mole	mol
luminous intensity (not in syllabus)	candela	cd

Memorize

Note the difference between quantities and units.

- A mole of any pure chemical element or compound contains a definite number of molecules, the same number for all elements and compounds.

- One mole is the amount of substance that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon−12.

- The number of molecules in a mole is given by the Avogadro number, N_A, and it is equal to 6.02×10^{23} mol^{-1}.

Check your understanding 1

Which of the following options contain all physical quantities?

A gram, pressure, second
B current, watt, hertz
C power, velocity, density
D ohm, ampere, kilogram

Check your understanding 2

Which of the following options contain all units?

A gram, watt, pressure
B current, seconds, hertz
C time, ampere, degrees
D ohm, kilogram, meter

A.3 Derived Quantities and Derived Units

- A derived quantity is a quantity obtained through a defining equation of base quantities. From the defining equation, a derived unit can be expressed as the product and/or quotient of the base units.

derived quantity	defining equation	derived unit
density	$\text{density} = \frac{\text{mass}}{\text{volume}}$	$\frac{\text{kg}}{\text{m}^3} = \text{kg m}^{-3}$
velocity	$\text{velocity} = \frac{\text{displacement}}{\text{time}}$	m s^{-1}

- Other examples of derived quantities are shown below.

S.I. base unit is a subset of S.I. unit.
e.g. Joule is a S.I. unit but not a S.I. *base* unit.

Derived quantity	S.I. base units	S.I. unit	
		Name	Symbol
Area	m^2	-	-
Momentum	kg m s^{-1}	-	-
Pressure	$\text{kg m}^{-1}\text{ s}^{-2}$	Pascal	Pa
Force	kg m s^{-2}	Newton	N
Work Done	$\text{kg m}^2\text{ s}^{-2}$	Joule	J
Power	$\text{kg m}^2\text{ s}^{-3}$	Watt	W
Electric Charge	A s	Coulomb	C
Potential Difference	$\text{kg m}^2\text{ A}^{-1}\text{ s}^{-3}$	Volt	V
Resistance	$\text{kg m}^2\text{ A}^{-2}\text{ s}^{-3}$	ohm	Ω
Frequency	s^{-1}	Hertz	Hz

- Note:
  - Units should be written in index form, e.g. m s^{-2} instead of m / s^2.
  - There is also a space break between different unit.
    For example, in "m s^{-1}" there is a space between "m" and "s^{-1}".
    Writing "ms^{-1}" means "per millisecond", and not "meter per second".

Example 1

Express the unit of force in terms of S.I. base units.

Never equate physical quantities to units.
A physical quantity is made up of a numerical value with accompanying unit. So for example, you can write "mass = 65.0 kg" but not "mass = kg"

Solution

Additional Notes

A.4 Homogeneity of Equations

- Base units can be used to check the homogeneity of equations.
- A homogeneous equation is one whereby every term in the equation has the same unit.
- An inhomogeneous equation is one whereby not every term has the same unit.
- A physical equation is one which is governed by the law of Physics. For an equation to be physically correct, it has to be confirmed experimentally.
- A physically correct equation must be homogeneous and the units of an unknown quantity can be determined by comparing the base units of both sides of the equation.
- However, a homogeneous equation may not necessarily be physically correct. There could be an incorrect coefficient, a missing or extra term, or simply a wrong positive or negative sign.

  For example, the equation $E_k = mv^2$ is homogeneous because the units on both sides of the equation are the same. However, it is not physically correct. The correct equation should be $E_k = \frac{1}{2}mv^2$.

- An inhomogeneous equation is definitely physically incorrect.

Example 2

Bernoulli's equation, which applies to fluid flow, states that

$$p + h\rho g + \tfrac{1}{2}\rho v^2 = k$$

where p is pressure, h is height, ρ is density, g is acceleration, v is velocity and k is a constant.

(a) Show that the left−hand side of the equation is homogeneous.
(b) State the SI base unit for k.

The Bernoulli's equation is a physically correct equation and is derived from the principle of conservation of energy in a fluid that is flowing. The quantities p, h and v refer respectively to the pressure, height and velocity of *a point* in the fluid.

A.5 Standard Prefixes

- Standard prefixes are used to indicate the decimal multiples or sub−multiples of both base and derived units. It is especially useful to state very small or large quantities using appropriate prefixes, removing the need to write many zeros.

A more comprehensive list (optional) is available at the following site: https://physics.nist.gov/cuu/Units/prefixes.html

Standard form / Scientific notation	Prefix	Symbol	Name	Decimal equivalent
10^{-15}	femto	f	Quadrillionth	0.000 000 000 000 001
10^{-12}	pico	p	Trillionth	0.000 000 000 001
10^{-9}	nano	n	Billionth	0.000 000 001
10^{-6}	micro	μ	Millionth	0.000 001
10^{-3}	milli	m	Thousandth	0.001
10^{-2}	centi	c	Hundredth	0.01
10^{-1}	deci	d	Tenth	0.1
10^{0}	−	−	One	1
10^{3}	kilo	k	Thousand	1,000
10^{6}	mega	M	Million	1,000,000
10^{9}	giga	G	Billion	1,000,000,000
10^{12}	tera	T	Trillion	1,000,000,000,000

For example, 0.000056 m = 56×10^{-6} m = 56 μm = 56 micrometer

A.6 Making Reasonable Estimates of Physical Quantities

- It is important to be aware of the typical order of magnitude of certain physical quantities and their respective units.
- The following is a compilation of some reasonable estimates.

The purpose of the table is to bring to your attention reasonable estimates of typical things around us. It is not intended for you to memorise them, nor is the list exhaustive.

In this course, you are required to make reasonable estimates of physical quantities learnt in this syllabus.

Physical Quantity	Approximate value	Scientific notation	Order of Magnitude
mass of a tennis ball	0.05 kg	5×10^{-2} kg	−2
mass of an apple	0.1 kg	1×10^{-1} kg	−1
mass of a car	1400 kg	1.4×10^{3} kg	3
mass of a bus	10,000 kg	1.0×10^{4} kg	4
mass of the Earth	6.0×10^{24} kg	6.0×10^{24} kg	24
diameter of hair	0.0002 m (0.2 mm)	2×10^{-4} m	−4
radius of Earth	6.4×10^{6} m	6.4×10^{6} m	6
speed of the fastest runner	10 m s^{-1}	1.0×10^{1} m s^{-1}	1
speed of sound in air	330 m s^{-1}	3.3×10^{2} m s^{-1}	2
speed of light in vacuum (*c*)	3.0×10^{8} m s^{-1}	3.0×10^{8} m s^{-1}	8
speed of an α-particle	3.0×10^{7} m s^{-1} (0.1 *c*)	3.0×10^{7} m s^{-1}	7
temperature of a room (Singapore)	300 K (27°C)	3.0×10^{2} K	2
temperature of a candle flame	1000 K − 1700 K	1.0×10^{3} K − 1.7×10^{3} K	3
power of a LED bulb	5 W	5 W	0
power of an incandescent bulb	40 W	4×10^{1} W	1
power of a hair dryer	1000 W − 2000 W	1.0×10^{3} W − 2.0×10^{3} W	3

Additional Notes

Example 3

Estimate the average kinetic energy of an ASRJC student competing in a 100 m race.

There is no one answer for estimation questions. The skill assessed for such questions is the ability to make reasonable estimates (i.e. the correct order of magnitude)

B Errors and Uncertainties

- Errors might arise in the process of taking a measurement, whether intentionally or unintentionally. They could arise from different sources.

- In general, errors can be classified into two main types: **SYSTEMATIC errors** or **RANDOM errors**.

B.1 Systematic Error and Random Error

- A **systematic** error will result in all readings having a <u>constant</u> error in <u>one direction</u>.

Memorize

- An error is systematic if repeating the measurement under the *same conditions* yields all measurements having a <u>constant</u> error in <u>one direction</u> i.e. being either all bigger or all smaller than the true value.

- Measurements with systematic errors change in a <u>predictable</u> manner depending on the conditions.

Examples of Systematic Errors:

Error Sources	Descriptions
Due to apparatus	• an instrument having a zero error, e.g. an voltmeter with a zero reading of −0.2 V will result in all readings taken to be 0.2 V smaller • stopwatch running fast, resulting in all time readings being too big • wrongly calibrated scale, e.g. an ammeter calibrated in Japan under different temperatures and earth's magnetic field from Singapore, where the ammeter is being used
Due to poor experimental technique	• consistent parallax error e.g. the experimenter always reads off a vertical meter rule from a position below eye level • bias of experimenter e.g. tendency of the experimenter to press a stopwatch too early / to always overestimate the fraction of a scale division
Due to external factors	• background radiation causes the count rate at the Geiger-Müller counter to be consistently higher than the true radiation from a radioactive source. (This concept will be covered in the last chapter of the syllabus, Nuclear Physics) • experimental conditions that were not accounted for e.g. temperature difference between object and surroundings, resulting in heat gained or lost to the surroundings, which affects the quantity measured but not taken into account • human reaction time of the experimenter using a stopwatch causes all readings obtained to be about 0.2 − 0.4 s larger than the true value

- Systematic errors can never be eliminated by taking average of repeated measurements with no change to the experimental setup or technique.

- They can be reduced or eliminated by using good experimental technique and by varying the instrumentation used.

  For example, check for proper calibration of instruments, check and account for zero error and verifying time taken by a stopwatch with another.

Memorize

A **random** error will result in a scatter of readings about a mean value.

- An error is random if repeating the measurement under the *same conditions* yields all measurements being scattered about a mean value. Measurements with random errors have an equal chance of being negative or positive.

Additional Notes

Examples of Random Errors:

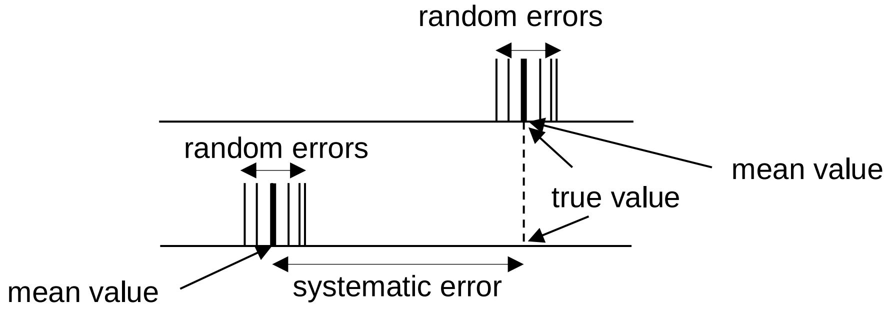
Error Sources	Descriptions
Due to inability of observer to repeat his action precisely	• inconsistent reaction time when using stopwatch to measure the period of an oscillation, at times stopping earlier and sometimes stopping later • random parallax error when the experimenter at times reads off a vertical meter rule from a position below eye level and sometimes at a position above eye level
Due to environmental conditions	• fluctuations in the measurement of the voltmeter or ammeter reading due to poor connection in electrical circuit • measuring the diameter of a wire of non-uniform thickness throughout
Due to the limited precision of instruments	• each division of the metre rule is 1 mm apart. The experimenter cannot read a point on the rule more precisely than half the smallest division.

Note that description of parallax error must be further elaborated with details because it can be classified as either systematic or random error, based on details provided.

- Random errors cannot be eliminated.
- They can be reduced by repeating a measurement and averaging the measurements and by plotting a graph and drawing a best fit line for the plotted points.
- Since positive and negative errors of this type are equally probable, a large number of measurements will give an average which is almost free from random error (as the positive and negative errors will tend to offset each other in the average).
- Hence quantities which are prone to random error can be made more reliable by repeating the measurement several times and obtaining the mean value.

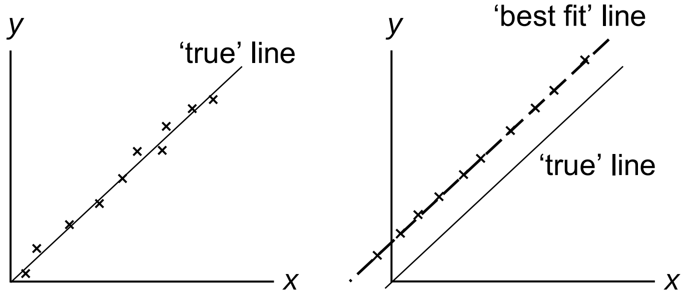
Distinguishing between Systematic and Random Errors through Illustrations

In the diagram below,
- random error causes a spread of readings approximately centred about a mean value.
- whereas systematic error shifts the mean value away from the true value.



Systematic and random errors usually occur concurrently.

- The effects of random errors and systematic errors appear in graphs as illustrated below.



Random error only

Systematic error only

<u>Check your understanding 3</u>

Errors associated with making experimental measurements may be described as the following:

P_1: error can possibly be eliminated
P_2: error cannot possibly be eliminated
Q_1: error is of constant sign and magnitude
Q_2: error is of varying sign and magnitude
R_1: error will be reduced by averaging repeated measurements
R_2: error will not be reduced by averaging repeated measurements

Which descriptions apply to random error?

A. P_1, Q_2, R_1
B. P_1, Q_1, R_2
C. P_2, Q_2, R_1`
D. P_2, Q_2, R_2

B.2 Precision and Accuracy

Memorize

Precision is the degree of agreement of repeated measurements of the same quantity.

- It is a measure of the <u>reproducibility</u> of results rather than their correctness.
- Good precision means that the readings are mostly <u>very close to their mean</u> and is <u>associated with small random error</u>.

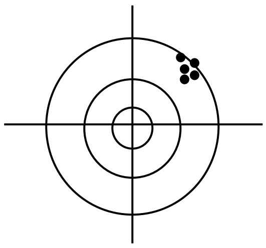
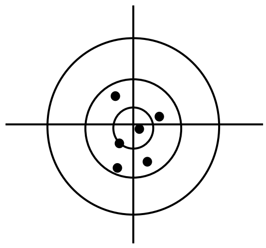
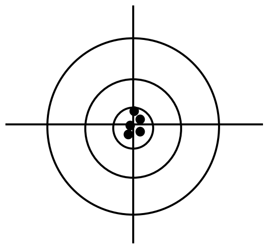
Memorize

Accuracy is the degree to which a measurement approaches the true value.

- It is a <u>measure of the correctness</u> of results.
- Good accuracy means that the reading or the mean of a set of readings is <u>very close to the true value</u> and is <u>associated with small systematic error</u>.

Additional Notes

Distinguishing between Precision and Accuracy through Illustrations

Diagram			
Precision	High	Low	High
Random Error	Small	Large	Small
Accuracy	Low	High	High
Systematic Error	Large	Small	Small

The dots represent measurements with the bullseye as the true value.

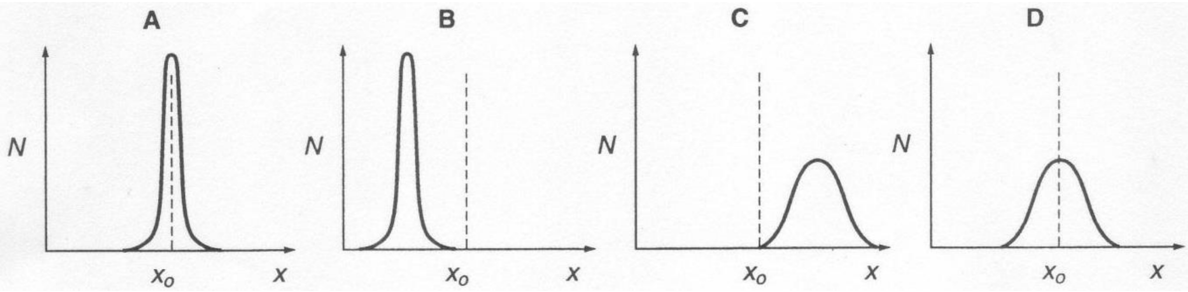
For high precision, the dots are close to one another (close to their mean position).

For high accuracy, their mean position is close to the bulleye (close to true value).

<u>Check your understanding 4</u>

A quantity that is measured repeatedly with the same instrument will exhibit some scattering around the true value x_o due to errors. When the number N of measurements is plotted against the measurements x, a bell-shape curve is obtained.

Determine which of the following set of results is precise and which of the following set of results is accurate.
(put a tick in the cell for those that apply and a cross for those that do not apply)



	A	B	C	D
Precise				
Accurate				

Example 4

The actual value of the acceleration of free fall near the Earth's surface is 9.81 m s^{-2}.
Four students each made a series of measurements of the acceleration of free fall *g*.
The table shows the results obtained.
Determine which student's results can be described as precise and accurate.

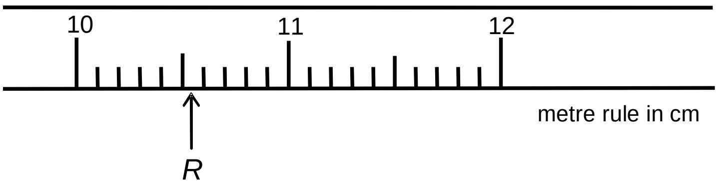
student	***results, g/*** **m s^{-2}**			
A	9.81	9.79	9.83	9.84
B	9.89	10.12	8.94	10.05
C	9.45	9.21	8.76	8.99
D	8.45	8.46	8.41	8.50

Additional Notes

B.3 Actual uncertainty

- In experimental Physics, one deals with measurements which involve the use of instruments. The accuracy is invariably limited by the sensitivity of the instrument; one can only read as precisely as the instrument can measure.

- For a reading R from a measuring instrument, its uncertainty, attributed to the measuring instrument only, is known as the actual uncertainty, ΔR.



- The reading R is between 10.5 cm to 10.6 cm, thus at best one can read it as 10.55 cm with an *actual uncertainty* ΔR, of ± 0.05 cm, generally taken to be half the smallest division (*here, the smallest division of the metre rule is 0.1 cm*).

- The standard way of expressing the reading R with its actual uncertainty in this example is R = (10.55 ± 0.05) cm.

- As actual uncertainties are estimated values, their numerical values are always quoted to one significant figure only.

- The reading R is expressed to the same place value as that of the uncertainty ΔR.

- The uncertainty of a reading is an indication of the range within which the reading is most likely to lie.
  e.g. (10.55 ± 0.05) cm simply means that the most likely value is 10.55 cm but it could be as low as 10.50 cm or as high as 10.60 cm.

- In general, the uncertainty of a reading is half the smallest division if the markings on the scaled instrument are close to one another. If the spacing between the markings is large (further apart than 1 mm), then the uncertainty of a reading may become one-quarter or one-fifth of the smallest division.

Actual uncertainty has to be stated with units.

B.4 Fractional and Percentage Uncertainty

Fractional uncertainty is unit-less as it is a ratio of 2 quantities with the same unit.

- Generally, all readings can be recorded in the form $R \pm \Delta R$ in which $\pm \Delta R$ is the actual uncertainty.

- However, the suitability of an instrument in relation to a measurement is reflected in the fractional or percentage uncertainty.

- fractional uncertainty of $R = \pm \dfrac{\Delta R}{R}$

- percentage uncertainty of $R = \pm \frac{\Delta R}{R} \times 100\%$

- Generally one has to choose an instrument which gives acceptably <u>low</u> fractional or percentage uncertainty.

Example 5
The thickness of a book was measured to be 2.5 ± 0.1 cm using a meter rule.
When measured with a vernier caliper, the thickness was measured to be 2.500 ± 0.005 cm.
Determine the percentage uncertainty of the thickness measurement with each of the instruments used.

Fractional uncertainty and percentage uncertainty is usually kept to 2 s.f unless otherwise stated by the question.

B.5 Uncertainty of a Derived Quantity

There are established statistical rules for analysis of uncertainty. However, at the A−level course, only <u>a simplified version</u> of the statistical treatment is required.

- Very often, we are interested in derived quantities that cannot be measured directly, but can be found through a certain mathematical function or expression of other measured quantities, e.g. density is found by taking mass divided by volume of an object, speed is found by taking distance travelled divided by time taken.

- In estimating the uncertainty of a derived quantity, we can use the first principle method. This is done by first determining the maximum and minimum values of the derived quantity, and the actual uncertainty is then taken as half the difference of the maximum and minimum values. Refer to Appendix A for how it is applied.

- It can be tedious/impossible to work from the first principle all the time. The following methods can be used to obtain the uncertainty (either actual or fractional/percentage) of the derived quantity, depending on the types of function or expression, without having to work from first principle.

Additional Notes

B.5.1 Uncertainty of a Derived Quantity from Addition and/or Subtraction of measured quantities (you can refer to Appendix B for the derivations from first principle)

Suppose:

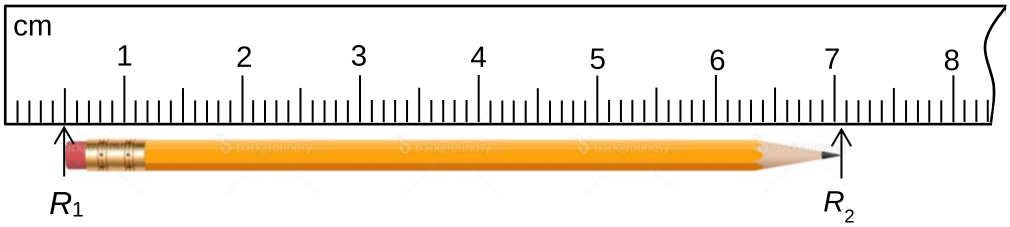
- A and B are measured quantities
- ΔA and ΔB are the corresponding actual uncertainties
- m and n are numerical constants
- R is related to A and B via the following addition and/or subtraction equations and the <u>actual uncertainty</u> can be obtained as per the table below.

Numerical constants have no uncertainty.

Physical Quantity	Actual Uncertainty
$R = A + B$	$\Delta R = \Delta A + \Delta B$
$R = A - B$	
$R = mA$	$\Delta R = m\Delta A$
$R = mA + nB$	$\Delta R = m\Delta A + n\Delta B$
$R = mA - nB$	

<u>Example 6</u>

A student wants to measure the length L of his pencil and places the pencil against a meter rule as shown in the diagram.



It is a bad practice to place the zero end of the meter rule against one end of the object to be measured, and to take the reading at the other end. This is because the end of the meter rule is usually worn out and this gives rise to a zero error.

He makes two readings, R_1 and R_2, from the meter rule.

(a) Write down the two readings with their actual uncertainty.

(b) Hence, determine the length L with its actual uncertainty.

Additional Notes

Even if one end of the object is placed at the zero mark of the meter rule, the measurement would still be given to the smallest division, that is, to 1 decimal place. This is because the measurement is made up of two readings on the meter rule: the reading at the zero mark and the reading on the rule that is at the other end of the object.

B.5.2 Uncertainty of a Derived Quantity from Multiplication and/or Division of measured quantities (you can refer to Appendix B for the derivations from first principle)

Suppose:

- A and B are measured quantities
- ΔA and ΔB are the corresponding actual uncertainties
- m and n are numerical constants
- R is related to A and B via the following multiplication and/or division equations and the fractional uncertainty can be obtained as per the table below.

Physical Quantity	FRACTIONAL Uncertainty
$R = AB$	$\frac{\Delta R}{R} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
$R = \frac{A}{B}$	
$R = A^m$	$\frac{\Delta R}{R} = m\frac{\Delta A}{A}$
$R = A^m B^n$	$\frac{\Delta R}{R} = m\frac{\Delta A}{A} + n\frac{\Delta B}{B}$
$R = \frac{A^m}{B^n}$	

Note:
Equations that consist of trigonometric or logarithmic functions would require an analysis from the first principle.

Example 7

The dimensions of a rectangle are recorded as (1.873 ± 0.005) mm and (1.580 ± 0.005) mm.

Determine the actual uncertainty and fractional uncertainty of
(a) its perimeter, and
(b) its area.

Additional Notes

B.5.3 Steps for finding the actual uncertainty of a derived quantity

1. Manipulate the equation to make the quantity x that you want to find its actual uncertainty as the subject.
2. Calculate the value of the quantity x.
3. Depending on the nature of the equation, do the following:
   i. the equation is a sum and/or difference of quantities
      - find the sum of the actual uncertainties of the quantities involved. This gives the actual uncertainty Δx.

   ii. the equation is a product and/or quotient of quantities
      - find the sum of fractional uncertainties. This gives the fractional uncertainty $\Delta x / x$.
      - calculate actual uncertainty Δx, by multiplying the fractional uncertainty with value of quantity x.

   iii. the equation is a combination of sum, difference, multiplication and/or division of quantities
      - break down equation into parts and work through by parts by applying i and ii above separately.

4. Keep actual uncertainty Δx to 1 significant figure.
5. Round off the value of the quantity x found in part 2 to the same place value as the actual uncertainty Δx and express in scientific notation.

Example 8

The thickness of 100 pages of a book is measured to be 9.00 mm using a micrometer screwgauge with an uncertainty of 0.01 mm.
Determine the average thickness of a page with its uncertainty.

Why is it better to determine the thickness of a page by finding average thickness from 100 pages instead of measuring just one page, even with a precise instrument?

C Scalars and Vectors

C.1 Scalars

Memorize

A scalar quantity is a quantity that has magnitude only, not direction.

- e.g. distance, speed, time, volume, mass, density, temperature, pressure, energy
- To specify a scalar, only its value (magnitude) and its unit need to be given.
- Scalar quantities can be added or subtracted as per the normal rules of algebra.
  e.g. 3 kg of rice + 4 kg of rice = 7 kg of rice

C.2 Vectors

Memorize

A vector quantity is a quantity having both magnitude and direction.

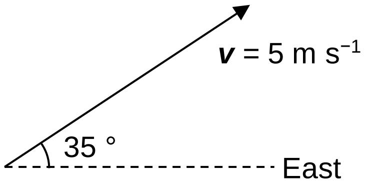
e.g. 2.5 m s^{-1} due East

magnitude direction

There is a chapter on Vectors in H2 Mathematics that is more comprehensive, and some are required for H3 Physics. The focus of this section is on relevant skills in manipulating vectors with reference the H2 Physics context.

- e.g. displacement, velocity, acceleration, force, weight, momentum
- To specify a vector, the value (magnitude), unit and direction must be given.

- A vector quantity may be represented in the following forms:
  1. pictorially by the use of an arrow

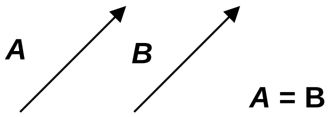


  - the *length of the arrow* represents the magnitude of the vector quantity
  - the *direction of the arrow* indicates the direction of the vector quantity
  - besides using the symbol $\boldsymbol{v}$ to accompany the arrow, we can also use the symbols $\vec{v}$ or $\underset{\sim}{v}$.

  2. described qualitatively
  - the velocity vector of 5 m s^{-1} in a direction 55 ° East of North

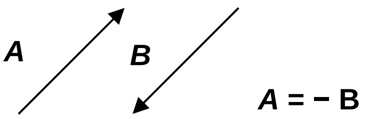
Equal vectors

- Vectors are equal if they have the same magnitude and direction.



Negative vector

- The negative of a vector has the same magnitude but *opposite* direction.



Coplanar vectors

- Vectors can be in any direction in a 3−dimensional space.

- Coplanar vectors are vectors that lie on the same plane (2−dimensional).

Check your understanding 5

Which of the following set contains vector quantities only?

A current, force
B acceleration, temperature
C energy, length
D weight, displacement

Additional Notes

Vector addition is different from scalar addition. When adding vectors, besides the magnitude, the direction of the vectors must be *taken together* into consideration. A vector diagram is therefore necessary. For example, in adding vectors of magnitude 3 and 2 may not be give a resultant vector of magnitude 5.

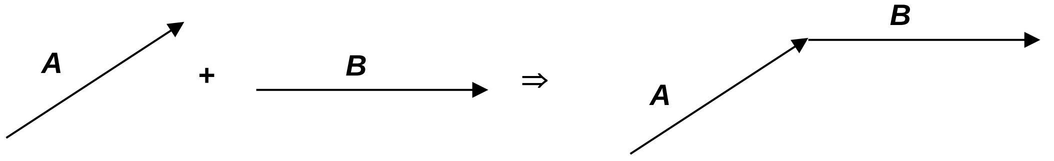
C.3 Vector Addition

- Unlike scalars, vector quantities cannot be added and subtracted using algebra.

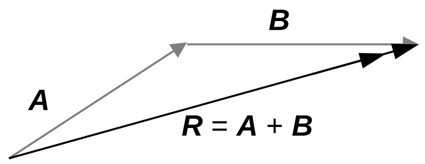
- Instead, one of the following vector diagrams can be used to perform vector addition (and subtraction).

1. Vector Triangle Method

   - Rearrange the vectors ***A*** and ***B*** such that one vector (***B***) has its tail placed at the tip of the other vector (***A***).

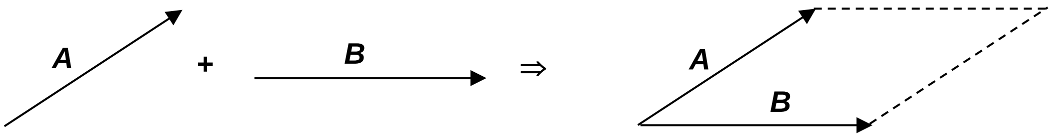


   - Complete the vector triangle by drawing the line directed from the tail of vector ***A*** to the tip of vector ***B***, with a double arrow.
   - This is the resultant vector, ***R***, which is equal to the addition of the two vectors ***A*** and ***B***.

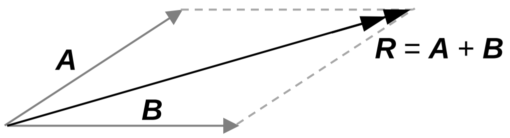


2. Parallelogram method

   - Rearrange the vectors ***A*** and ***B*** such that the tails of both vectors are placed together.
     - Using the two vectors as adjacent sides of a parallelogram, draw the remaining two sides (parallel to the two vectors) in dotted lines.



     - The resultant vector, ***R***, is the line drawn along the diagonal of the parallelogram from the tail of the two vectors to the other vertex, with a double arrow.
     - This resultant vector, ***R***, is also equal to the addition of the two vectors ***A*** and ***B***.



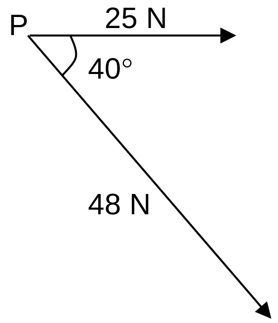
- Both methods will give the same resultant vector.

Additional Notes

- The <u>magnitude</u> and <u>direction</u> of the resultant vector, ***R***, can then be found using one of the following methods:
  i. resolving of vectors (see C.5) into specific perpendicular components,
  ii. sine and/or cosine rules (using triangular geometry) or
  iii. scale drawings (<u>not to be used</u> unless specified by the question)

<u>Worked Example</u>

Two forces act at a point P as shown below. Determine the resultant (magnitude and direction) of these two forces.



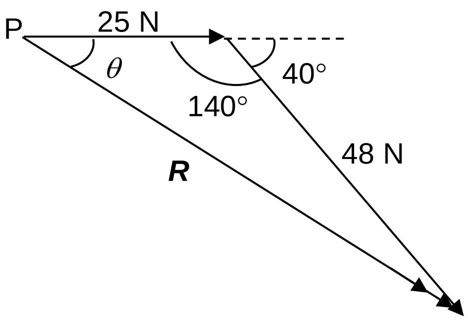
Take note that the cardinal directions (North, South, East and West) can only be used in defining directions, if the vectors are in the horizontal plane.

In this question, we are not told which plane the forces are acting in and hence, cannot use the cardinal directions.

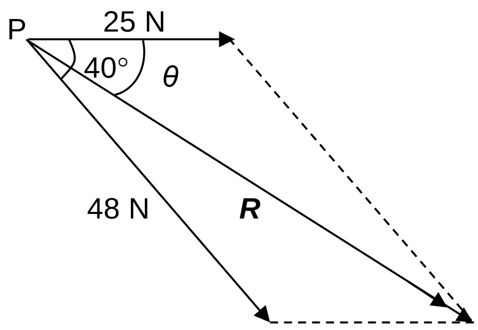
Solution

Draw the vector diagram to obtain the resultant.

<u>Vector triangle method:</u>



<u>Parallelogram method:</u>



(Using appropriate triangles in the vector diagrams, we apply cosine rule to solve for the length of the side of the triangle which gives the magnitude of the resultant vector.)

By cosine rule,
$R^2 = 25^2 + 48^2 - 2(25)(48)\cos140°$
$R = 69.047 = 69.0$ N (3 s.f.)

Apply cosine rule,
$48^2 = 25^2 + 69.047^2 - 2(25)(69.047)\cos\theta$
$\theta = 26.542 = 26.5°$ (3 s.f.)

Thus the resultant force is 69.0 N with θ = 26.5° as shown in the diagram.

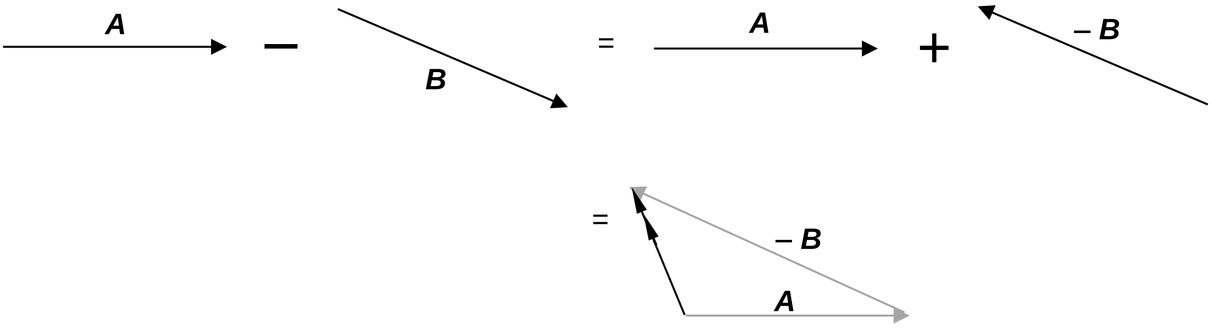
C.4 Vector Subtraction

- Reversing the direction of a vector ***B*** gives vector – ***B***.

- Hence, to <u>subtract</u> vector ***A*** by vector ***B*** i.e. ***A*** – ***B***, we simply <u>reverse</u> the direction of ***B*** and then do <u>vector addition</u>.

$$\text{i.e. } \boldsymbol{A} - \boldsymbol{B} = \boldsymbol{A} + (-\boldsymbol{B}).$$

For example,



Finding *change* in a physical quantity

- When finding the *change* in a physical quantity, ΔQ, it is always the <u>final value</u> of the physical quantity, Q_{final}, <u>minus</u> the <u>initial value</u> of the physical quantity, Q_{initial}.
  i.e. $\Delta Q = Q_{\text{final}} - Q_{\text{initial}}$

- If the physical quantity is a vector quantity, a <u>vector diagram</u> must be used to find the *change* in the physical quantity.
  e.g. *change* in velocity, $\Delta\vec{v} = \overrightarrow{v_{\text{final}}} - \overrightarrow{v_{\text{initial}}}$

<u>Example 9</u>
An object is moving at 5.0 m s^{-1} due East. It changes direction to travel due South with a speed of 7.5 m s^{-1}. Determine the change in the object's velocity.

One way of showing direction is to draw a vector diagram and identify the angle of interest in the diagram.

There is no need to use the cardinal directions to describe the direction, unless specified by the question.

Additional Notes

C.5 Resolving Vectors

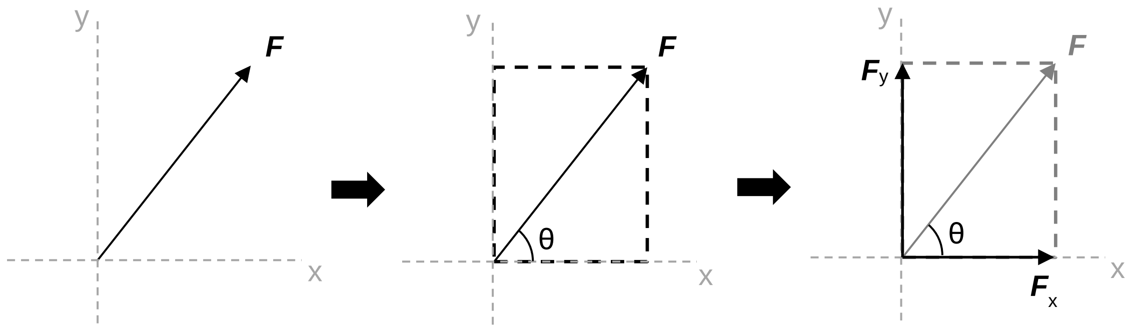
- Since two vectors can be added to give a resultant vector, any vector can be resolved (or separated) into two vectors or components.

- It is more convenient to resolve a vector into two mutually−perpendicular components through the use of trigonometry and Pythagoras' theorem.

- Mutually−perpendicular vectors are independent of each other.

- Method of resolving a vector into 2 perpendicular components

    - Identify the 2 perpendicular directions that you want to resolve the vector ***F*** into.
    - Draw a rectangle using the 2 perpendicular directions as reference. Ensure that the vector ***F*** to be resolved lies on the diagonal of the rectangle.
    - The components of vector ***F***, $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$, are on the adjacent sides of the rectangle.
    - The arrows of the component vectors $\boldsymbol{F}_x$ and $\boldsymbol{F}_y$ are drawn with their tails at the same position as the tail of the resolved vector ***F*** and the arrowheads are at ends of the respective length of the rectangle.

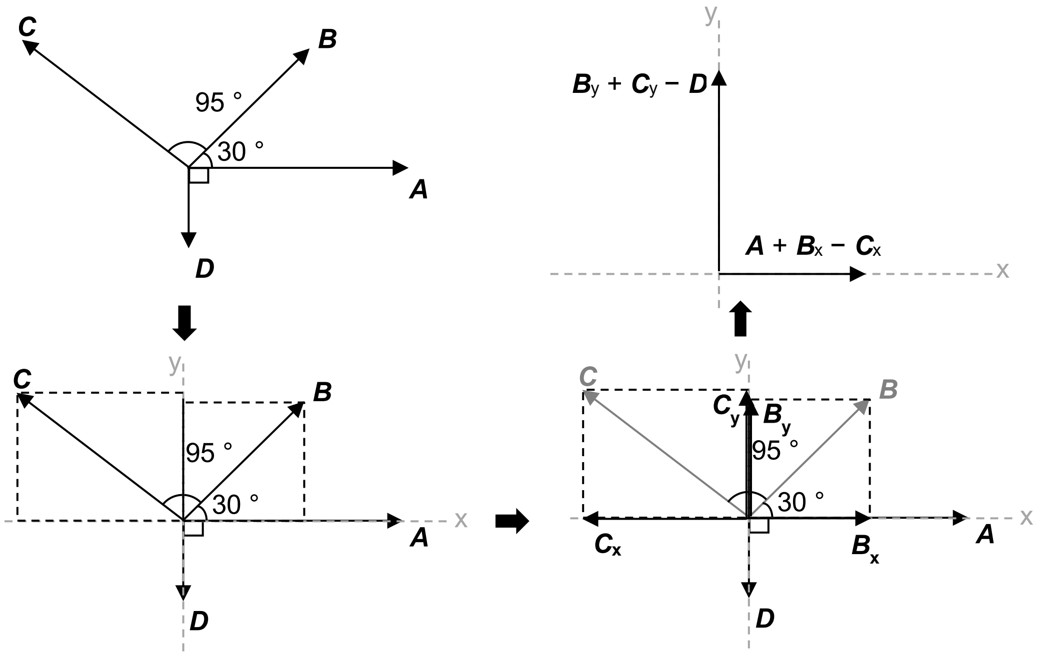


    - The magnitude of the component vectors can be found using trigonometric identities, i.e. $F_y = F \sin \theta$ and $F_x = F \cos \theta$.

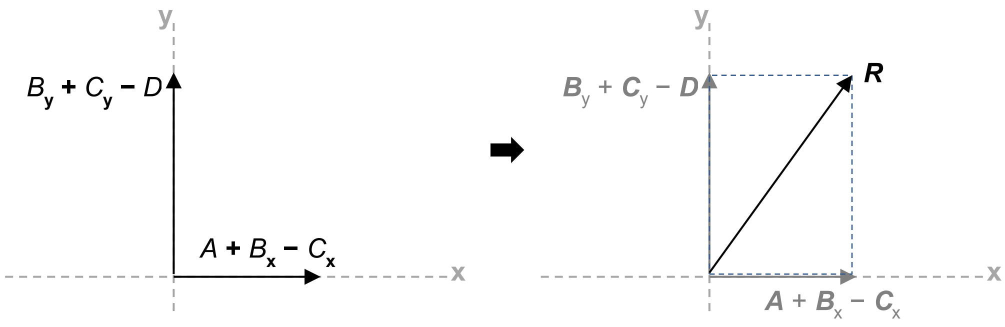
- Adding and/or subtracting more than 2 vectors can be easily handled without having to adopt the vector triangle method or parallelogram method repeatedly.

- We can resolve all the vectors into the identified two perpendicular components, and apply vector addition and/or subtraction in the two directions separately. The result would give the components of the resultant vector.

For example, to find the sum of vectors ***A***, ***B***, ***C*** and ***D***, we resolve all the vectors into the two chosen perpendicular directions and perform vector sum in the respective directions.



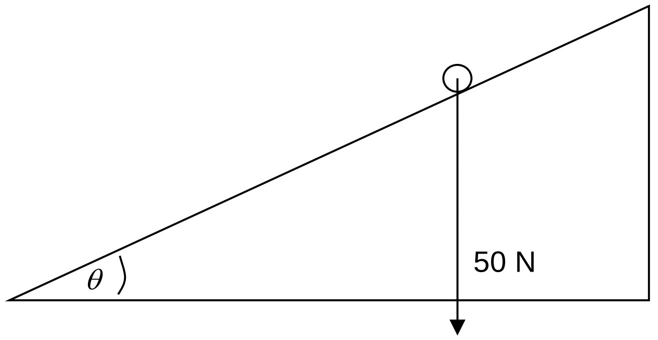
- To find the resultant vector ***R***, we apply trigonometric identities using the components found.



Additional Notes

Example 10

An object of weight 50 N rests on the plane of an inclined slope as shown.



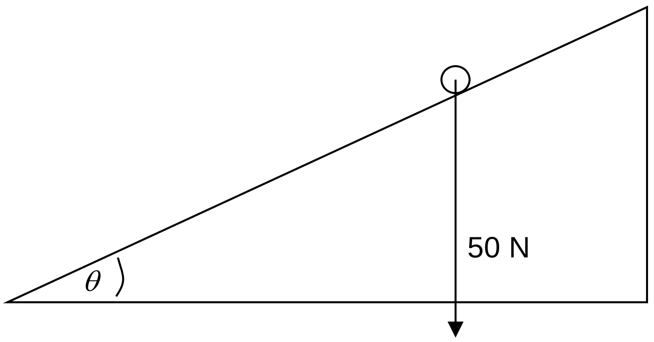
(a) Draw components of the weight (in the diagram below)

i. parallel to the slope

ii. perpendicular (normal) to the slope.

(b) Label on the diagram the magnitude of the two components in terms of θ.

Solution

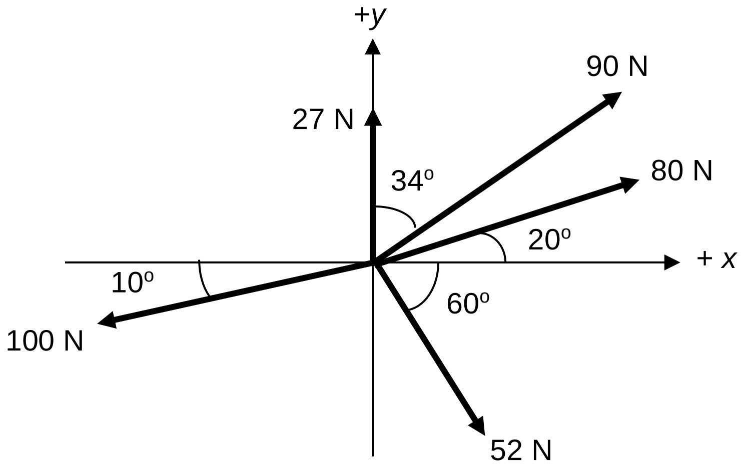


We don't always resolve vectors into the vertical and horizontal components.

It is more suitable to resolve vectors into perpendicular components, where one of the components is along the direction of motion of the object.

Example 11

The 5 forces shown act on an object. Determine the resultant force due to them.



Do remember to determine the direction of the resultant force.

SUMMARY

Term	Definition
Random errors	a random error will result in a scatter of readings about a mean value.
Systematic errors	a systematic error will result in all readings having a constant error in one direction.
Accuracy	accuracy is the degree to which a measurement approaches the true value.
Precision	precision is the degree of agreement of repeated measurements of the same quantity.
Scalar	a quantity that has magnitude only, not direction.
Vector	a quantity having both magnitude and direction.

Additional Notes

APPENDIX A

Calculation of actual uncertainty from first principle

For all the following parts, we will use

R_1 = 41.2 ± 0.1 cm
R_2 = 20.1 ± 0.1 cm

(a) Addition

Let $R = R_1 + R_2$ and ΔR be the actual uncertainty of R.

R = 41.2 + 20.1 = 61.3 cm
Then R can be written as R = (61.3 ± ΔR) cm

From first principle,
$R_{min} = (R_1)_{min} + (R_2)_{min}$ = 41.1 + 20.0 = 61.1 cm
$R_{max} = (R_1)_{max} + (R_2)_{max}$ = 41.3 + 20.2 = 61.5 cm

Hence, $\Delta R = \frac{R_{max} - R_{min}}{2}$ = 0.4 / 2 = 0.2
Therefore, R = (61.3 ± 0.2) cm since ΔR = ± 0.2 cm

Thus for addition $\Delta R = \Delta R_1 + \Delta R_2$.

(b) Subtraction

Let $R = R_1 - R_2$ and ΔR be the actual uncertainty of R.

R = 41.2 − 20.1 = 21.1 cm
Then R can be written as R = (21.1 ± ΔR) cm

From first principle,
$R_{min} = (R_1)_{min} - (R_2)_{max}$ = 41.1 − 20.2 = 20.9 cm
$R_{max} = (R_1)_{max} - (R_2)_{min}$ = 41.3 − 20.0 = 21.3 cm

Hence, $\Delta R = \frac{R_{max} - R_{min}}{2}$ = 0.4 / 2 = 0.2
Therefore, R = (21.1 ± 0.2) cm since ΔR = ± 0.2 cm

Thus for subtraction $\Delta R = \Delta R_1 + \Delta R_2$.

(c) Product

Let $R = R_1 \times R_2$ and ΔR be the actual uncertainty of R.

R = 41.2 × 20.1 = 828.12 cm^2
Then R can be written as R = (828.12 ± ΔR) cm^2

From first principle,
$R_{min} = (R_1)_{min} \times (R_2)_{min}$ = 41.1 × 20.0 = 822.00 cm^2
$R_{max} = (R_1)_{max} \times (R_2)_{max}$ = 41.3 × 20.2 = 834.26 cm^2

Hence, $\Delta R = \frac{R_{max} - R_{min}}{2}$ = 12.26 / 2 = 6 cm^2 (to 1 s.f.)

Therefore, R = (828 ± 6) cm^2 since ΔR = ± 6 cm^2

Obviously $\Delta R \neq \Delta R_1 + \Delta R_2$.

If one works out the fractional uncertainties, we get

$\frac{\Delta R_1}{R_1} = \frac{0.1}{41.2} = 0.0024$ and $\frac{\Delta R_2}{R_2} = \frac{0.1}{20.1} = 0.0050$

And consider $\left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2}\right) \times R = (0.0024 + 0.0050) \times 828.12$

$= 6.1 = 6$ (to 1 s.f) $= \Delta R$

Thus, $\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2}$.

(d) **<u>Quotient</u>**

Let $R = R_1 / R_2$ and ΔR be the actual uncertainty of R.

R = 41.2 / 20.1 = 2.0498
Then R can be written as R = (2.0498 ± ΔR)

From first principle,
$R_{min} = (R_1)_{min} / (R_2)_{max}$ = 41.1 / 20.2 = 2.0347
$R_{max} = (R_1)_{max} / (R_2)_{min}$ = 41.3 / 20.0 = 2.0650

Hence, $\Delta R = \frac{R_{max} - R_{min}}{2}$ = 0.0303 / 2 = 0.02 (to 1 s.f.)

Therefore, R = (2.05 ± 0.02) since ΔR = ± 0.02

Obviously $\Delta R \neq \Delta R_1 + \Delta R_2$.

If one works out the fractional uncertainties, we get

$\frac{\Delta R_1}{R_1} = \frac{0.1}{41.2} = 0.0024$ and $\frac{\Delta R_2}{R_2} = \frac{0.1}{20.1} = 0.0050$

And consider $\left(\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2}\right) \times R = (0.0024 + 0.0050) \times 2.0498$

$= 0.015 = 0.02$ (to 1 s.f) $= \Delta R$

Thus, $\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2}$.

APPENDIX B

Derivation of uncertainty formulae

Consider the following quantities with their actual uncertainty:

$$R_1 = A \pm \Delta A \quad \text{and} \quad R_2 = B \pm \Delta B$$

(a) Addition

Let $W = R_1 + R_2$.

maximum $W = (A + \Delta A) + (B + \Delta B) = (A + B) + (\Delta A + \Delta B)$

minimum $W = (A - \Delta A) + (B - \Delta B) = (A + B) - (\Delta A + \Delta B)$

$\Delta W = ½$ (maximum W − minimum W) $= \Delta A + \Delta B$

Hence, $W \pm \Delta W = (A + B) \pm (\Delta A + \Delta B)$

(b) Subtraction

Let $X = R_1 - R_2$.

maximum $X = (A + \Delta A) - (B - \Delta B) = (A - B) + (\Delta A + \Delta B)$

minimum $X = (A - \Delta A) - (B + \Delta B) = (A - B) - (\Delta A + \Delta B)$

$\Delta X = ½$ (maximum X − minimum X) $= \Delta A + \Delta B$

Hence, $X \pm \Delta X = (A - B) \pm (\Delta A + \Delta B)$

From **(a)** and **(b)**, we can see that when quantities are added and/or subtracted, the *actual uncertainty is the sum of the individual actual uncertainties*.

(c) Multiplication

Let $Y = R_1 \times R_2$.

maximum $Y = (A + \Delta A) \times (B + \Delta B) = AB + B\Delta A + A\Delta B + \Delta A\Delta B$

minimum $Y = (A - \Delta A) \times (B - \Delta B) = AB - B\Delta A - A\Delta B + \Delta A\Delta B$

$\Delta Y = ½$ (maximum Y − minimum Y) $= B\Delta A + A\Delta B$

Hence, $Y \pm \Delta Y = AB \pm (B\Delta A + A\Delta B)$.

But this is not an easy form to remember, especially in cases with more than 2 quantities in the equation. Instead, we divide the actual uncertainty, ΔY, with Y throughout.

$$\frac{\Delta Y}{Y} = \frac{B\Delta A}{Y} + \frac{A\Delta B}{Y}$$

Since $Y = AB$, we have

$$\frac{\Delta Y}{Y} = \frac{B\Delta A}{AB} + \frac{A\Delta B}{AB}$$

$$\frac{\Delta Y}{Y} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence, for multiplication of quantities, the *fractional uncertainty is given by the sum of the individual fractional uncertainties of the quantities*.

(d) Division

Let $Z = R_1 / R_2$.

maximum $Z = (A + \Delta A) / (B - \Delta B) = \dfrac{A(1+\frac{\Delta A}{A})}{B(1-\frac{\Delta B}{B})} = \dfrac{A}{B}(1+\dfrac{\Delta A}{A})(\dfrac{1}{1-\frac{\Delta B}{B}})$

The expression $(\dfrac{1}{1-\frac{\Delta B}{B}})$ can be written as $(1 - \dfrac{\Delta B}{B})^{-1}$ and we can apply the Binomial expansion^ to obtain the following:

$$(1 - \frac{\Delta B}{B})^{-1} = 1 + \frac{\Delta B}{B} + (\frac{\Delta B}{B})^2 + (\frac{\Delta B}{B})^3 + \ldots$$

Since usually, $\left|\dfrac{\Delta B}{B}\right| \ll 1$, the terms $(\dfrac{\Delta B}{B})^2$, $(\dfrac{\Delta B}{B})^3$, …, will all be << 1. These terms can be neglected and the expression converges to become

$$(1 - \frac{\Delta B}{B})^{-1} \approx 1 + \frac{\Delta B}{B}$$

Hence, maximum $Z \approx \dfrac{A}{B}(1+\dfrac{\Delta A}{A})(1+\dfrac{\Delta B}{B}) = \dfrac{A}{B}(1+\dfrac{\Delta A}{A}+\dfrac{\Delta B}{B}+\dfrac{\Delta A}{A}\dfrac{\Delta B}{B})$

Similarly,

minimum $Z = (A - \Delta A) / (B + \Delta B) = \dfrac{A(1-\frac{\Delta A}{A})}{B(1+\frac{\Delta B}{B})} = \dfrac{A}{B}(1-\dfrac{\Delta A}{A})(\dfrac{1}{1+\frac{\Delta B}{B}})$

$$(1 + \frac{\Delta B}{B})^{-1} = 1 - \frac{\Delta B}{B} + (\frac{\Delta B}{B})^2 - (\frac{\Delta B}{B})^3 + \ldots \approx 1 - \frac{\Delta B}{B}$$

Hence, minimum $Z \approx \dfrac{A}{B}(1-\dfrac{\Delta A}{A})(1-\dfrac{\Delta B}{B}) = \dfrac{A}{B}(1-\dfrac{\Delta A}{A}-\dfrac{\Delta B}{B}+\dfrac{\Delta A}{A}\dfrac{\Delta B}{B})$

ΔZ = ½ (maximum Z − minimum Z) $= \dfrac{A}{B}(\dfrac{\Delta A}{A}+\dfrac{\Delta B}{B})$

Dividing throughout by Z, we have

$$\frac{\Delta Z}{Z} = \frac{\frac{A}{B}(\frac{\Delta A}{A}+\frac{\Delta B}{B})}{Z} = \frac{\frac{A}{B}(\frac{\Delta A}{A}+\frac{\Delta B}{B})}{\frac{A}{B}}$$

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Hence, for division of quantities, the fractional uncertainty is also given by the *sum* of the fractional uncertainties of the quantities.

^The binomial expansion is part of the H2 Mathematics syllabus. Here is one form of the binomial expansion used in the above derivation.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$