Additional Notes

Topic 8: Temperature and Ideal Gases

Content:

- Thermal equilibrium
- Temperature scales
- Equation of state
- Kinetic theory of gases
- Kinetic energy of a molecule

Learning Outcomes:

Candidates should be able to:

(a) show an understanding that regions of equal temperature are in thermal equilibrium

(b) explain how empirical evidence leads to the gas laws and to the idea of an absolute scale of temperature (i.e. the thermodynamic scale that is independent of the property of any particular substance and has an absolute zero)

(c) convert temperatures measured in Kelvin to degrees Celsius: T/ K = T/ °C + 273.15

(d) recall and use the equation of state for an ideal gas expressed as $pV = nRT$, where n is the amount of gas in moles

(e) state that one mole of any substance contains 6.02×10^{23} particles and use the Avogadro number $N_A = 6.02 \times 10^{23}$ mol^{-1}

(f) state the basic assumptions of the kinetic theory of gases

(g) explain how molecular movement causes the pressure exerted by a gas and hence derive the relationship $pV = \frac{1}{3} Nm\langle c^2 \rangle$, where N is the number of gas molecules (a simple model considering one-dimensional collisions and then extending to three dimensions using $\frac{1}{3} \langle c^2 \rangle = \langle c_x^2 \rangle$ is sufficient)

(h) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature (i.e. $\frac{1}{2} m\langle c^2 \rangle = \frac{3}{2} kT$) to new situations or to solve related problems

Nature of Science

Heat and temperature are often used interchangeably in everyday language. However, these terms have different and specific meanings in physics. Macroscopically, temperature can be defined in terms of its measurement, while heat refers to the energy transferred between two systems at different temperatures.

Understanding thermal physics requires us to approach the concepts from both the macroscopic and microscopic lenses.

Relating Science and Society
Many researchers still actively investigate the behaviour of gases using computer simulation and other techniques. Environmental problems connected with Earth's atmosphere, such as the depletion of the ozone layer and the unwanted discharge of thermal energy into the environment known as thermal pollution, are motivations for such work. A better understanding of the behaviour of gases might lead to an improved characterisation and possible solutions to these problems.

A Temperature Scales

A.1 Temperature, Heat and Internal Energy

- The temperature of a system is a measure of its average molecular kinetic energy. A higher temperature implies that the molecules in the system are moving faster on average.

Note: Thermal energy is not internal energy.

Refer to Topic 9 "First Law of Thermodynamics" for more information about thermal energy and internal energy.

- Heating (thermal energy supplied) is a transfer of energy to an object resulting in an increase in the random kinetic and/or potential energies of the atoms or molecules of the object.

- Internal energy is the sum of a random distribution of kinetic and potential energies associated with the molecules of a system.

- When a body absorbs heat, two situations may arise:
    1. The particles move further apart (increased molecular PE), causing an expansion. They also move more quickly on the average (increased molecular KE), causing a rise in temperature.
    2. The particles move further apart (increased molecular PE), causing an expansion. However, they move with the same speed on average (same molecular KE), hence there is no change in temperature.
        - In other words, it is possible to supply heat to a body without causing a change in its temperature. This happens when the heat added to a body increases its molecular potential energy (PE) while leaving molecular kinetic energy (KE) unchanged. E.g. when water is boiling, when ice is melting.
        - Similarly, it is possible to remove heat from a body without causing a change in its temperature. This happens when the heat removed from a body decreases its molecular potential energy (PE) while leaving molecular kinetic energy (KE) unchanged. E.g. when water is freezing.

Check Your Understanding

1. Temperature of a system is a measure of its average molecular kinetic energy and molecular potential energy. (True / False)

2. It is possible for a body to absorb heat without causing a change in its temperature. (True / False)

Additional Notes

A.2 Thermal Equilibrium

- When objects or substances are in contact such that heat is able to flow from one object or substance to another, they are said to be in thermal contact.
- When two bodies in thermal contact have the same temperature and there is no net flow of heat between them, the two bodies are said to be in thermal equilibrium.

Memorise

- When two bodies are in thermal equilibrium, their temperatures are the same.
- Therefore, the temperature of a body is the physical property which determines whether the body will be in thermal equilibrium with another body.

Zeroth Law of Thermodynamics

If two bodies *X* and *Y* are separately in thermal equilibrium with a third body *T*, then *X* and *Y* are in thermal equilibrium with each other.

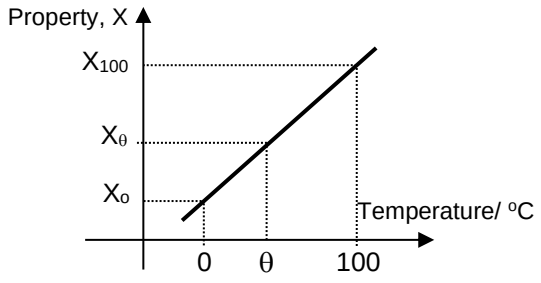
A.3 Temperature Scales

- A thermometer is calibrated according to a temperature scale.
- Two temperature scales, the empirical scale and the thermodynamic scale have been established.
- They are established by making use of reference or fixed points.

The Empirical Scale

- The empirical scale of temperature is a scale of temperature that is based on the variation with temperature of a property of a substance, assuming that the property varies linearly with temperature although it may not necessarily be so.
- An example is the Centigrade scale used in a mercury thermometer. This empirical scale of temperature is based on the expansion of mercury's volume with rising temperature.
- At melting point of ice (0 ºC) and the steam point (100 ºC) which are fixed points, the respective values of thermometric property X are measured as X_o and X_{100}.
- To calculate an unknown temperature θ, the corresponding thermometric property X_θ is measured. θ can be calculated as

$$\theta = \frac{X_\theta - X_o}{X_{100} - X_o} \times 100$$



Worked Example

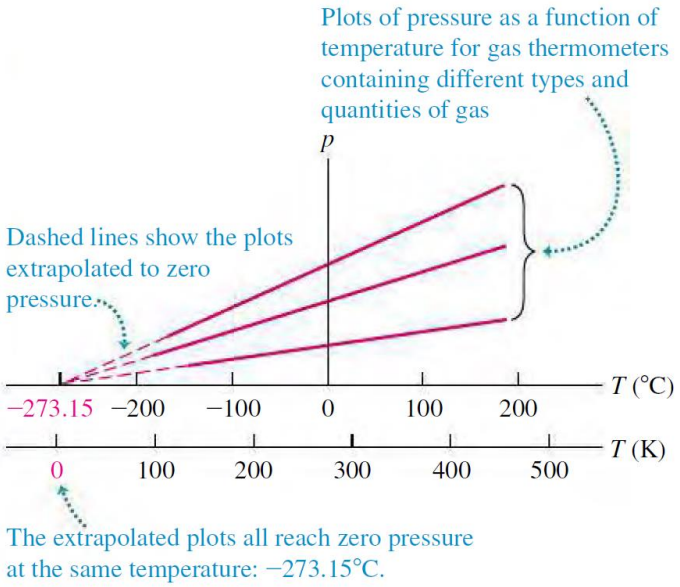
A resistance thermometer has a resistance of 9.97 Ω at the ice point and 14.04 Ω at the steam point. Find the temperature on the centigrade scale of this thermometer when its resistance is 11.51 Ω.

Solution

$$\theta = \frac{X_\theta - X_o}{X_{100} - X_o} \times 100 = \frac{11.51 - 9.97}{14.04 - 9.97} \times 100 = 37.8\ ^\circ\text{C}$$

Thermodynamic Scale of Temperature

- Empirical scale may agree at the calibration points of, for example 0 ºC and 100 ºC but not at intermediate temperatures.
- Hence, we want an ideal temperature scale that does not depend on the properties of a particular material.
- The gas thermometer comes closest to this ideal temperature scale.
- The principle of a gas thermometer is that the pressure of a gas at constant volume increases with temperature. A quantity of gas is placed in a constant volume container and its pressure is measured at 0 °C and 100 °C, say; and a straight line drawn.
- Graphs of pressure versus temperature at constant volume for three different types and quantities of gas are plotted.



- Extrapolation of the graph shows a hypothetical temperature –273.15 ºC at which absolute pressure of gas would become zero for different quantities of gas used as well as different gases used.

- We use this extrapolated zero-pressure temperature as the basis for a temperature scale with its zero at this temperature. This is the thermodynamic (or Kelvin) temperature scale.

- This acts as the basis for a temperature scale independent of properties of a material.

Additional Notes

Absolute Scale of temperature (thermodynamic scale of temperature) is a scale that is independent of the property of any particular substance and has an absolute zero.

Memorise

- It is not an empirical scale of temperature.
- The thermodynamic scale of temperature is an absolute scale of temperature which is based on the properties of an ideal gas.
- An ideal gas is a theoretical gas that follows the Ideal Gas Equation (see Section C).

$$pV = nRT$$

where p is the pressure of the gas
V is the volume of the gas
n is the amount of gas (measured in moles)
R is the molar gas constant (8.31 J mol^{-1} K^{-1})
T is the thermodynamic temperature

- In the thermodynamic scale of temperature:

1. The absolute zero is the temperature at which the pressure of an ideal gas becomes zero, and is given the value 0 K (zero kelvin). This is the temperature at which all substances have a minimum internal energy.

Memorise

2. The triple point of water is a state where ice, water and water vapour coexist in thermal equilibrium at a temperature of 0.01 ºC and a fixed pressure. It is chosen as a fixed point due to its precision and reproducibility. The triple point of water can be obtained by using a triple point cell.

Triple Point is the temperature at which a substance can exist in equilibrium in the solid, liquid and gaseous states.

- The thermodynamic temperature scale is defined by the equation:

$$T = \frac{p_T}{p_{tr}} \times 273.16\,K$$

where p_T = pressure for a fixed mass of ideal gas at temperature T K and
p_{tr} = pressure of the gas at the triple point of water.

- The Celsius scale (renamed from Centigrade scale in 1948) of temperature (θ / °C) is related to the thermodynamic temperature (T / K) by the relationship

$$T/\text{K} = \theta/°\text{C} + 273.15$$

- Comparison of some temperature values:

Reference point	Celsius	Kelvin
Steam point	100.00	373.15
Triple point	0.01	273.16
Ice point	0.00	273.15
Absolute zero	-273.15	0.00

Example 1

A fixed mass of an ideal gas is maintained at constant volume. The pressure of the gas at the triple point of water is 1.00×10^5 Pa.

Determine the thermodynamic temperature of the gas when its pressure is 1.20×10^5 Pa.

Additional Notes

B Avogadro Constant and Molar Quantities

- The Avogadro constant N_A is the number of atoms in 0.012 kg of carbon-12. Memorise
- This number was found experimentally to be 6.02×10^{23}.
- The mole is a unit for the amount of substance.
- One mole of any substance is the amount containing 6.02×10^{23} particles. Memorise
- For instance, one mole of argon gas contains 6.02×10^{23} Ar atoms and one mole of water contains 6.02×10^{23} H_2O molecules.
- The mass of one mole of a substance is known as the molar mass M. The unit for molar mass is kg mol^{-1}.
- The mass of one atom or molecule m can be found by dividing the molar mass of the substance by the Avogadro constant.

$$m = M / N_A$$

$$M = m N_A$$

- Sometimes, it is convenient to express ideal gas in terms of the total number of molecules N in the gas.
- For N molecules, the number of moles n is given by $n = \frac{N}{N_A}$ where N_A is the Avogadro constant.
- Hence, the ideal gas equation becomes

$$pV = \frac{N}{N_A}RT = NkT$$

where p is the pressure of the gas
V is the volume of the gas
N is the number of molecules
k is the Boltzmann constant (1.38×10^{-23} JK^{-1})
T is the thermodynamic temperature

Note:
$k = \frac{R}{N_A}$, where
R is the molar gas constant.
R = 8.31 J K^{-1} mol^{-1}

Check Your Understanding 1

1. One mole of hydrogen has the same number of atoms as one mole of neon. (True / False)

Example 2

(a) A sample of carbon-12 has a mass of 3.0 g. Write down an expression for the number of atoms in the sample in terms of the Avogadro constant N_A.

Monatomic gas is one in which atoms are not bound to each other. i.e. gas consisting of single type of atom.

(b) Argon and neon are monatomic gases. One mole of argon has mass 40 g and one mole of neon has mass 20 g.

What is the ratio $\frac{\text{number of atoms in 1 mole of argon}}{\text{number of atoms in 1 mole of neon}}$?

Additional Notes

C The Ideal Gas Equation

C.1 State of a Gas

- The state of a gas is the condition in which a particular material exists.
- It can be described using the following macroscopic properties that can be easily measured:

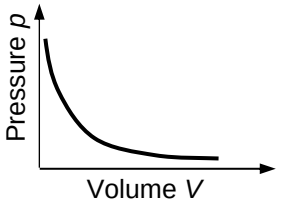
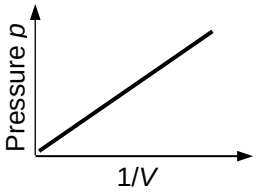
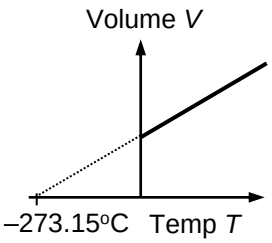
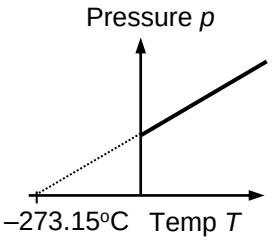
  1. pressure
  2. volume
  3. temperature

- Any changes in the state of a gas can be brought about by changing any of the above properties. For e.g. if pressure is changed, then either volume, temperature or both will change.

Try this Ideal Gas Model simulation based on Kinetic Theory of Gas

C.2 The Experimental Laws

- Based on experimental observations, the following laws were formulated:

Experimental Law	Formulation	Process	Variation of property
Boyle's Law	$p \propto \frac{1}{V}$	Isothermal (temp constant)	 
Charles's Law	$V \propto T$	Isobaric (pressure constant)	
Pressure Law	$p \propto T$	Isochoric (volume constant)	

Watch the following videos for the demonstrations.

Boyle's Law

Charle's Law

- Combining the gas laws, we have

$$\frac{pV}{T} = \text{constant}$$

- This constant is proportional to the amount of gas (measured in moles).
- Hence, the equation of state is $pV = nRT$ or $pV = NkT$

where

n = amount of substance (measured in moles)
R = molar gas constant 8.31 J mol^{-1} K^{-1}
N = number of molecules
k is the Boltzmann constant (1.38 x 10^{-23} JK^{-1})

- Experimental evidence shows that the equation of state is only true for an ideal gas. An ideal gas is one where there is no potential energy (due to negligible electrostatic forces) between molecules.

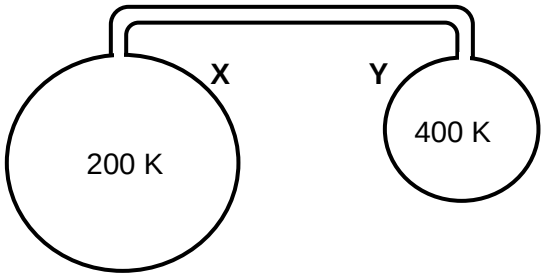
Memorise

- An ideal gas is one which obeys the equation of state pV = nRT at all pressures, volumes and temperatures.

- Real gases approximate the behaviour of an ideal gas at
  - low pressure and
  - high temperature

Example 3

In the figure, the volume of flask X is twice that of flask Y. The system is filled with an ideal gas. If the mass of gas in X is *m*, what is the mass of gas in Y?



Thinking Process

What is the significance of the tube linking the two flasks?

- The tube equalizes the pressure in the two flasks as there is no net flow of gas molecules in the tube when equilibrium is established.

Additional Notes

Worked Example

After the pressure of the air in a bicycle tyre has been increased slightly by pumping air into the tyre, the amount of air in the tyre increased by 2%, the thermodynamics temperature by 1%, and the internal volume of the tyre by 0.2%. By what percentage has the pressure of the air in the tyre increased? [2.8 %]

Solution

$$pV = nRT \quad \Rightarrow \quad \frac{pV}{nT} = R = \text{ constant}$$

$$\frac{p_1V_1}{n_1T_1} = \frac{p_2V_2}{n_2T_2} \Rightarrow \frac{p_2}{p_1} = \left(\frac{n_2}{n_1}\right)\left(\frac{T_2}{T_1}\right)\left(\frac{V_1}{V_2}\right)$$

$$= \left(\frac{102}{100}\right)\left(\frac{101}{100}\right)\left(\frac{100}{100.2}\right)$$

$$= 1.028$$

The final pressure is 2.8% higher than the initial pressure.

Example 4

An ideal gas exerts a pressure of 60 Pa in a container of volume *V*, when its temperature is 400 K and the number of molecules present is *N*. Another sample of the same gas exerts a pressure of 30 Pa in the same container when its temperature is 300 K.
How many molecules are present in a unit volume of this second sample?

Thinking Process

What are the variables in this question?

- *p*, *N* and *T*
- *V* and k are constant.

C.3 Kinetic Theory of Gases

- In the kinetic theory, the pressure of an ideal gas is related to the impact of molecules with the walls of the container; its temperature to the sum of kinetic energies of the molecules; its volume with the space in which the molecules are free to move.

- The kinetic theory of gases applies the laws of mechanics (Conservation of momentum, Newton's laws of motion, etc) to molecular movement so that expressions for the pressure and temperature of a gas can be expressed in terms of the molecular speed, mass, number of molecules, etc.

- The assumptions of the kinetic theory include:

Using mnemonics, remember **large rivet**.

1. A gas consists of a **large** number of identical molecules.
2. The gas molecules are in **r**apid, random motion.
3. There are no **i**nter-molecular forces between the molecules, except during a collision.
4. The **v**olume of the gas molecules is negligible compared with the volume of the container.
5. Collisions between gas molecules and with the walls of the container are **e**lastic.
6. The **t**ime taken for a collision is negligible compared with the time interval between collisions.

Additional Notes

D Mean Kinetic Energy of a Molecule

- Here, we find an expression for the mean kinetic energy of a molecule of an ideal gas. This is a microscopic quantity which is nearly impossible to measure even with sophisticated equipment.

- The approach is to relate this microscopic quantity to the macroscopic properties of the system.

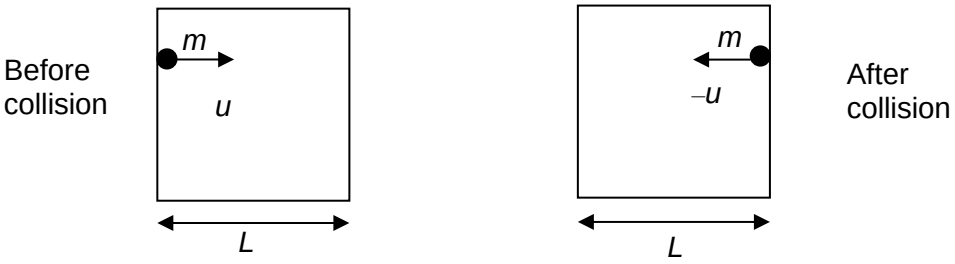
D.1 Microscopic and Macroscopic properties of ideal gas

D.1.1 (a) Derivation of $pV = \frac{1}{3}Nm\langle c^2 \rangle$

(b) Show that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature

- The pressure exerted by a gas can be explained by the kinetic theory. When randomly moving gas molecules hit the wall of the container, they exert a force on the wall. Since pressure is defined as force per unit area, a gas exerts pressure on the wall of the container during collisions with the container.

- Consider a single molecule of mass m in a cubical box with sides of length L. At this instant after it has just collided with the left-hand side wall, it is moving with velocity u directly towards the right-hand side wall of the box.



- Initial momentum before collision with the wall = mu ($\rightarrow$ positive direction)

- Final momentum just after collision with the wall = $-mu$
  Note: This is because the collisions are elastic. Refer to assumption 4.

- Change in momentum = final momentum – initial momentum
  $= -mu - mu$
  $= -2mu$

- Before making another collision with the left-hand side wall, the molecule has to travel a distance $2L$.

- As the speed is u, the time between collisions with the same wall will be $\frac{2L}{u}$.

- By Newton's 2nd law, the force exerted by the wall on the molecule is equal to the rate of change of momentum of the molecule,

  Force = rate of change of momentum of the molecule
  = change in momentum ÷ time taken
  $= (-2mu) \times \frac{u}{2L} = -\frac{mu^2}{L}$

- By Newton's 3rd Law, the molecule exerts an equal but opposite force on the wall,

  i.e. force exerted by molecule on wall = $\frac{mu^2}{L}$

- Since pressure is the force per unit area,

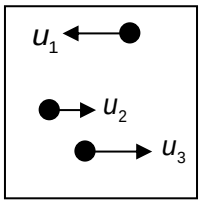
  Pressure on wall of the container = force on wall / area of wall

  $$= \frac{mu^2}{L} \div L^2$$
  $$= \frac{mu^2}{L^3}$$
  $$= \frac{mu^2}{V}$$

  where V = volume of container

- This is the pressure on the wall due to ONE molecule.

- If there are N molecules all moving back and forth along the same axis, each with their own velocities of u_1, u_2, u_3, etc , the pressure here will be an average over a period of time.



- The total pressure on a wall
  $$= \frac{mu_1^2}{V} + \frac{mu_2^2}{V} + \frac{mu_3^2}{V} + ... + \frac{mu_N^2}{V}$$
  $$= \frac{m}{V}\left(u_1^2 + u_2^2 + u_3^2 + ... + u_N^2\right)$$
  $$= \frac{m}{V} N \left\langle u^2 \right\rangle = \frac{Nm}{V}\left\langle u^2 \right\rangle \quad \text{------ (*)}$$

  where the average of u^2 is written as $\left\langle u^2 \right\rangle$, more often called the mean-square speed of the molecules.

Mean-square value
Given that x = {3, 5, 6}, the 'mean-square' of this set of x-values can be calculated by squaring each number individually and then dividing the sum of their squares by 3.

That is, $\langle x^2 \rangle = (3^2 + 5^2 + 6^2) / 3 = 23.3$

Compare with mean value, $\langle x \rangle = (3 + 5 + 6) / 3 = 4.67$

Additional Notes

Example 5

Six particles have the following speeds (in m s^{-1}): 3.0 5.0 5.0 6.0 6.0 9.0

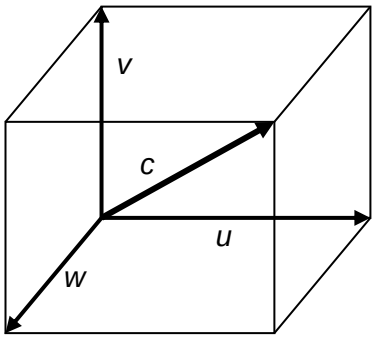
Calculate

(a) the average or mean speed.

(b) the root mean square (r.m.s.) speed.

- The above analysis is confined to ONLY one dimension. In a gas, the molecules will move in three dimensions.
- Using the Pythagoras' Theorem for the velocity c_1 of the first molecule gives

$$c_1^2 = u_1^2 + v_1^2 + w_1^2$$



Note
The directions of the 3 velocities *u, v* and *w* are perpendicular to one another.

- Applying this to all molecules of the gas,

$$c_2^2 = u_2^2 + v_2^2 + w_2^2$$
$$c_3^2 = u_3^2 + v_3^2 + w_3^2$$
$$c_4^2 = u_4^2 + v_4^2 + w_4^2$$
$$\ldots\ldots\ldots\ldots\ldots$$
$$c_N^2 = u_N^2 + v_N^2 + w_N^2$$

- Adding all the equations together, we get

$$N\langle c^2\rangle = N\langle u^2\rangle + N\langle v^2\rangle + N\langle w^2\rangle$$

$$\langle c^2\rangle = \langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle$$

- Since the gas molecules are in random motion (assumption 2), there is no tendency for the gas to exert a greater pressure in any one direction. It follows that

$$\langle u^2\rangle = \langle v^2\rangle = \langle w^2\rangle$$

$$\langle c^2\rangle = 3\langle u^2\rangle$$

$$\langle u^2\rangle = \frac{1}{3}\langle c^2\rangle$$

- From equation (*), the pressure p exerted by the gas on the wall of the container is

$$\boxed{p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle}$$

- The term Nm is equal to the total mass of the gas sample (M). Therefore,

$$\frac{Nm}{V} = \frac{M}{V} = \rho$$

where ρ = density of the gas

- Hence, another equivalent expression for pressure is

$$\boxed{p = \frac{1}{3}\rho\langle c^2\rangle}$$

Additional Notes

- We consider the product of pressure and volume (pV) for an ideal gas from two perspectives.

Microscopic Level	Macroscopic Level
By considering the collisions of the gas molecules with the walls of an enclosed space such as a box, it can be deduced that:	Earlier, it has been mentioned that experimentally, ideal gases are found to obey the following equation:
$pV = \frac{1}{3}Nm\langle c^2\rangle$ ……(1)	$pV = NkT$ ……(2)
where N = no. of molecules in the sample m = mass of one molecule $\langle c^2\rangle$ = mean-square speed of the molecules	where N = no. of molecules in the sample k = Boltzmann constant T = thermodynamic temperature

- Equating the right-hand-side of equations (1) and (2), we obtain

$$\frac{1}{3}Nm\langle c^2\rangle = NkT$$ only for monatomic gases

If gas is not monatomic, the equation would then be an *estimation* of the KE of molecule.

- Multiplying both sides by 3/2,

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

- On the left of the equation, the term $\frac{1}{2}m\langle c^2\rangle$ represents the mean kinetic energy of a molecule of an ideal gas.

- This equation implies that for an ideal gas, the mean kinetic energy of a molecule is proportional to the thermodynamic temperature.

- $\frac{1}{2}m\langle c^2\rangle$ multiplied by N gives the total kinetic energy E_k of all the molecules. That is, the internal energy U of the ideal gas since there is no potential energy (due to negligible electrostatic forces) between molecules (assumption 3).

$$\boldsymbol{U = E_k = \tfrac{1}{2}M\langle c^2\rangle = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT}$$

Note
The total mass of the gas, M is calculated by taking the sum of masses of all the gas molecules.

i.e. $M = Nm$

$\boldsymbol{M}$ is the total mass of the gas

- This equation implies that for an ideal gas, the internal energy of the gas is proportional to its thermodynamic temperature.

D.1.2 Root-Mean Square Speed of Molecules in an Ideal Gas

From the above derivation, $\frac{1}{2} Nm\langle c^2 \rangle = \frac{3}{2} nRT$

Root-Mean-Square speed = $\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3nRT}{Nm}}$

where N = number of gas molecules in sample
m = mass of one molecule
n = number of moles in sample

- The quantity $\sqrt{\langle c^2 \rangle}$, which is the square root of the mean-square speed of the molecules, is called the 'root-mean-square speed' c_{rms}.
- Since N is the number of gas molecules and m is the mass of one molecule, the product Nm gives the mass of the sample.
- If for n = 1 mole, then the number of molecules $N = 6.02 \times 10^{23}$; and Nm = molar mass M
- Root-mean-square speed $c_{rms} = \sqrt{\langle c^2 \rangle} = \sqrt{\frac{3RT}{M}}$
- This expression implies that the root-mean-square speed of molecules in an ideal gas is proportional to the square root of its thermodynamic temperature and inversely proportional to the square root of its molar mass.
- For a given gas (fixed M), the molecules are moving about with a larger average speed when the temperature is higher. Also, at a fixed temperature, the gas with a smaller molar mass has molecules with higher speeds on the average.

Additional Notes

Example 6

Argon and helium are monatomic gases. Argon atoms in the Earth's atmosphere have a root-mean-square speed of about 500 m s^{-1}. If the relative molecular masses of argon and helium are 39.9 and 4.0 respectively, what is the best approximation to the root-mean-square speed of helium atoms in the atmosphere?

Thinking Process

What quantities do the root-mean-square speed depend on?

- Molecular mass and temperature.
- In this case, since both gases are found in the atmosphere, it may be assumed that they are at the same temperature.

Worked Example

A mixture of two monatomic gases X and Y is in thermal equilibrium. The molecules of Y have twice the mass of those of X. The mean translational kinetic energy of a molecule of Y is 6.0 x 10^{-21} J.

What is the mean translational kinetic energy of a molecule of X? [6.0 x 10^{-21} J]

Solution

Since the system is in thermal equilibrium, the two gases are at the same temperature.

For ideal gases, the mean translational kinetic energy of a molecule is proportional to T since k is a constant.

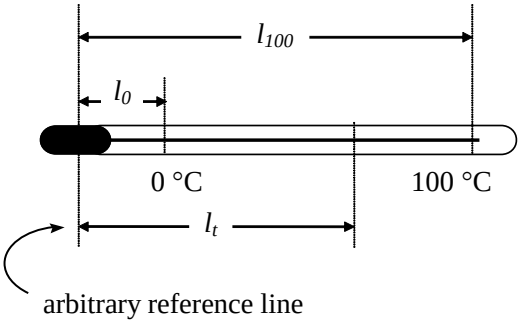
$$\frac{1}{2}mc_{rms}^{\ 2} = \frac{3}{2}kT$$

Hence, the mean translational kinetic energy of a molecule of X is the same as that of Y. i.e. 6.0 x 10^{-21} J.

Annex A: Types of thermometer (Empirical Scale)

Mercury-in-glass Thermometer

Thermometric property: Expansion of a liquid, usually mercury or alcohol coloured with dye.

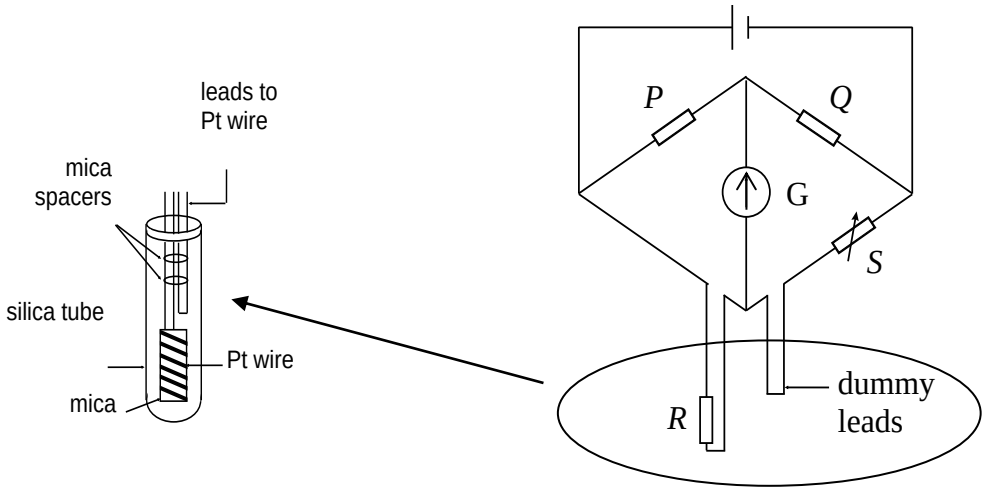


$$T = \frac{l_T - l_0}{l_{100} - l_0} \times 100$$

Platinum Resistance Thermometer

Thermometric property: Resistance thermometers require a small current to be passed through in order to determine the resistance. The resistance of a pure platinum wire increases with temperature.

For the Wheatstone bridge circuit, when the millivoltmeter (G) = 0

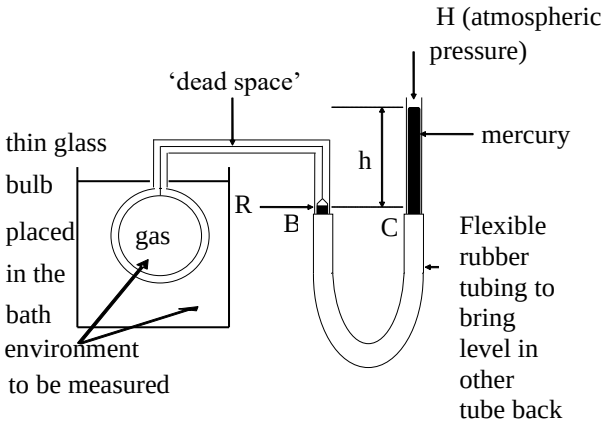


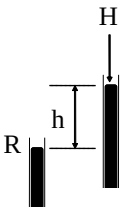
$$T = \frac{R_T - R_0}{R_{100} - R_0} \times 100$$

Additional Notes

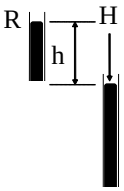
Constant Volume Gas Thermometer

Thermometric property: Variation in the pressure of a gas kept at constant volume.
As temperature of bath of environment increases, the gas expands pushing mercury down in B and up in tube C. Raise C to restore mercury level in B to reference mark R, hence, keeping volume of gas constant.





gas pressure $P_T = H + h$ (in mmHg)

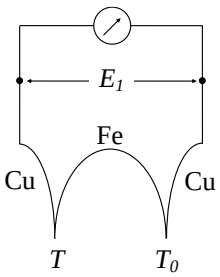


gas pressure $P_T = H - h$ (in mmHg)

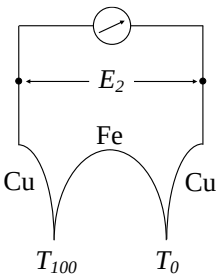
$$T = \frac{P_T - P_0}{P_{100} - P_0} \times 100$$

Thermoelectric Thermometer (Thermocouple)

Thermometric property: When 2 different metal wires are joined in a circuit and their junctions are kept at different temperatures, a small (order of mV) electromotive force (e.m.f.) is set up between the junctions. The magnitude of the e.m.f. is proportional to the temperature difference ΔT at the junctions.



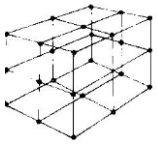
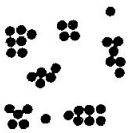
$E_1 \propto (T - T_0)$



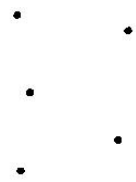
$E_2 \propto (T_{100} - T_0)$

$$\frac{E_1}{E_2} = \frac{T - T_0}{T_{100} - T_0}$$

Annex B: Three Phases of Matter

PHASES OF MATTER	STRUCTURE AND CHARACTERISTICS	MOVEMENT OF MOLECULES – A KINETIC MODEL
Solid 	1. Has a regular, geometrical structure in which the atoms or molecules are closely packed in a regular pattern called a space lattice. This accounts for the fact that solids have a high density. 2. Has a fixed volume and a fixed shape. To a certain extent, solids return to their original shape when stretched or compressed. 3. A typical interatomic distance in a crystal is about 3×10^{-10} m.	1. Due to strong intermolecular attractive and repulsive forces, motion is limited to vibrations of the molecules/atoms about their mean positions in the space-lattice. 2. The kinetic energy of the vibrations increases with temperature. At melting point, the molecules vibrate so violently that they collide with one another. When the attractive forces can no longer hold them together, the space lattice collapses and the solid melts or liquefies. The work required to overcome the forces between the molecules, i.e., to break up the space lattice is called the latent heat of fusion.
Liquid 	1. The molecules are slightly further apart than in a solid, resulting also in high density. They occur in clusters and the molecules can move between them. 2. Fixed in volume but not in shape. Liquids take the shape of the container and are almost incompressible. 3. The order of separation of molecules in liquids is about the same as that for solids.	Random motion throughout the liquid. The average kinetic energy of the molecules increases with temperature. The presence of attractive, cohesive forces between the molecules are responsible for the pulling back of molecules near the surface of the liquid, thus opposing their escape. However, the more energetic ones can still escape by overcoming the attraction of others. This results in evaporation, and cooling then occurs.

Additional Notes

Gas 	1. The molecules are very far apart as compare to other state, thus giving rise to a low density. 2. No fixed volume or shape. Gases fill up the entire space/ container in which they are placed. They are easily compressible. 3. At s.t.p. (standard temperature and pressure), the average separation of gas molecules is about 33 x 10^{-10} m.	1. Random motion at high speeds throughout the space the atoms/ molecules occupy. The average kinetic energy increases proportionately with temperature. 2. Negligible attractive or repulsive forces between the atoms or molecules, as they are much further apart. 3. The molecules exert pressure on the walls of the container by colliding with the walls.

Annex C: Some Useful Equations

Temperature Conversion	T / K = θ / °C + 273.15
Ideal Gas Equation or Equation of State	$pV = nRT$ $pV = NkT$
Avogadro constant N_A	$N_A = N / n$
Boltzmann constant k	$k = R / N_A$
Mean Kinetic energy of a molecule in an ideal gas	$E_K = (3/2)\,kT$
Internal energy U of an ideal gas (with N particles)	$U = (3/2)\,NkT = (3/2)\,nRT$