Additional Notes

Topic 11: Wave Motion

Content:

- Progressive waves
- Transverse and longitudinal waves
- Polarisation
- Determination of frequency and wavelength

Learning Outcomes:

Candidates should be able to:

(a) show an understanding of and use the terms displacement, amplitude, period, frequency, phase difference, wavelength and speed.

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.

(c) recall and use the equation $v = f\lambda$.

(d) show an understanding that energy is transferred due to a progressive wave.

(e) recall and use the relationship, $intensity \propto (amplitude)^2$.

(f) show an understanding of and apply the concept that a wave from a point source and travelling without loss of energy obeys an inverse square law to solve problems.

(g) analyse and interpret graphical representations of transverse and longitudinal waves.

(h) show an understanding that polarisation is a phenomenon associated with transverse waves.

(i) recall and use Malus' law (intensity $\propto \cos^2 \theta$) to calculate the amplitude and intensity of a plane polarised electromagnetic wave after transmission through a polarising filter.

(j) determine the frequency of sound using a calibrated oscilloscope.

(k) determine the wavelength of sound using stationary waves. *(To be discussed in Superposition)*

Introduction

- The propagation of light and sound are examples of wave phenomena.
- Waves originate from a source which is oscillating. These oscillations then spread and bring energy away from the source in the form of waves – the disturbance travels from one region of space to another.
- A wave is a means by which energy may be transferred from one place to another as a result of oscillations. This takes place without the physical transfer of any material between the points.

A Classification of Waves

Waves can be classified using:

- Medium through which the wave travels through: Mechanical or electromagnetic
- Motion of waves: Progressive or stationary
- Mode of transport: Transverse or longitudinal

A.1 Medium through which the wave travels through – Mechanical waves and electromagnetic waves

For this section, it is sufficient to understand the nature of these waves

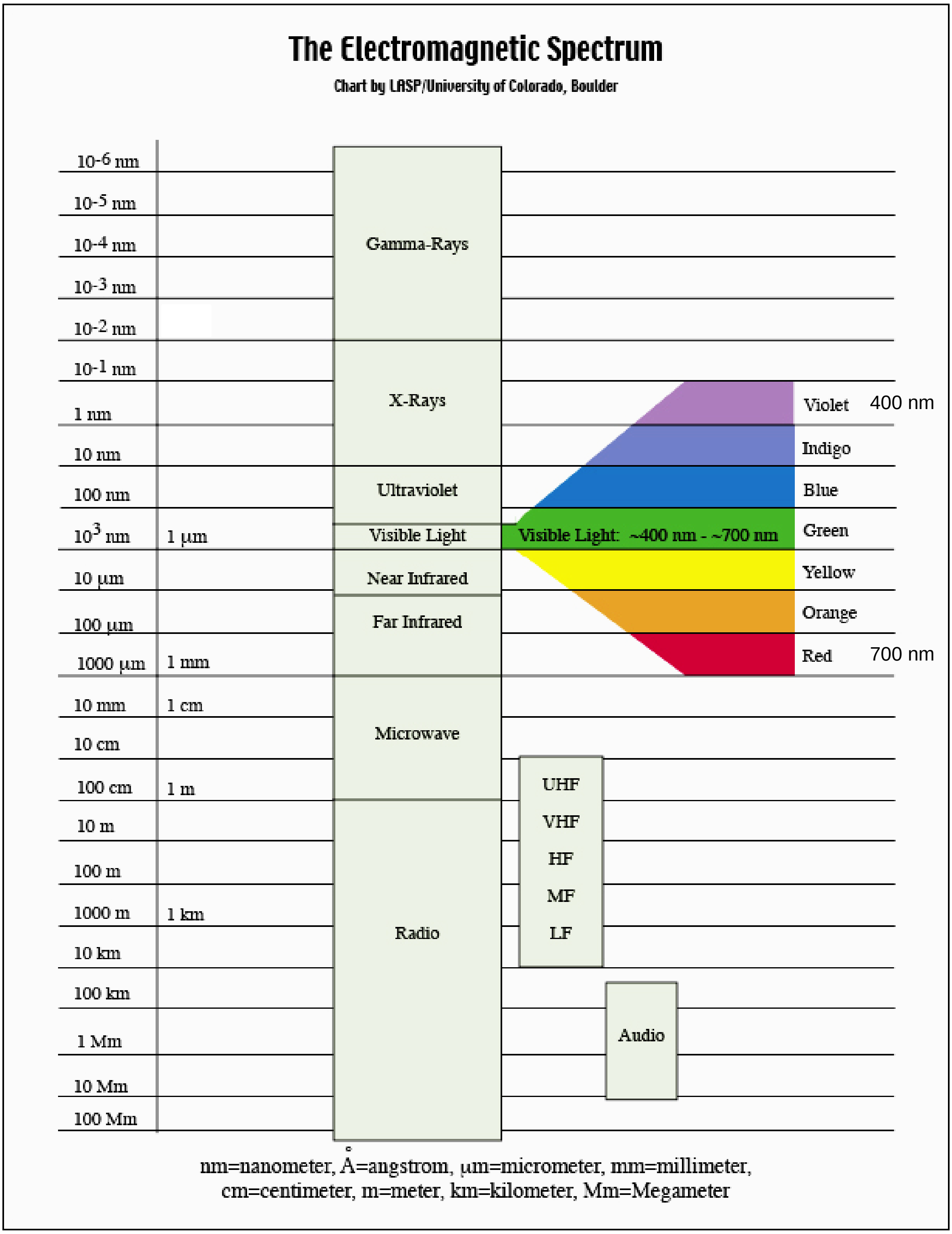
Mechanical waves:

- Mechanical waves require a medium to propagate. Examples include:
  - waves in a slinky coil or a rope
  - water waves
  - sound waves
  - seismic waves which travel along the Earth's crust during an earthquake

Electromagnetic waves

- Electromagnetic waves are generally termed "light" but this term includes the entire electromagnetic spectrum (gamma rays, X-rays, ultraviolet rays, visible light, infrared rays, microwaves and radio waves).
- All these forms of light travel at a constant speed in a vacuum (c = 3.00 x 10^8 m s^{-1}) but have different frequencies (different wavelengths).
- Electromagnetic waves do not require a medium to travel.

Additional Notes



Note: There is really no absolute value where a particular EM radiation (e.g. X-rays) start or end e.g. 10^{-1} nm may be considered as gamma rays.

A.2 Motion of waves: Progressive waves vs Stationary waves

Progressive waves:

- Progressive waves (or travelling waves) are waves in which energy is carried from one point to another by means of vibrations or oscillations within the wave.
- The profile (the wavefronts) of the wave appears to be moving although the particles in the medium do not get transported along with the wave (a wavefront is a line or surface which marks out all those points on a wave that have the same phase).

  (refer to Appendix for more details on wavefront)

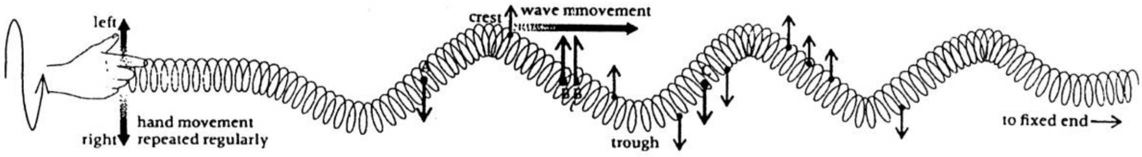
Stationary waves:

- Stationary waves or standing waves are waves in which the vibrational energy (associated with the waves) is stored (i.e. the energy is not transferred).
- The profile of the wave does not appear to move. Energy within the wave is said to be localised (the energy at a specific location maintains a constant value).
- This will be covered in greater detail in the Topic "Superposition".

A.3 Mode of transport (Direction of Oscillation): Transverse vs Longitudinal Waves

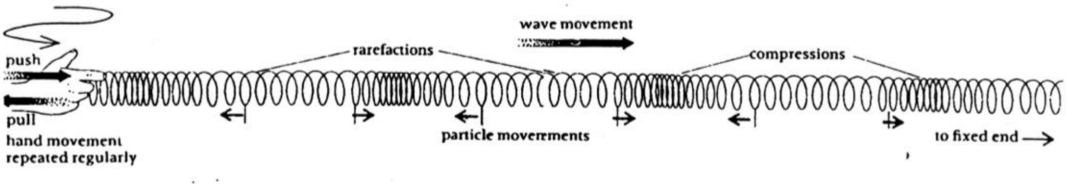
Transverse wave:

- A transverse wave is a wave in which the oscillations of the particles in the wave are at right angles to the direction of transfer of energy of the wave.



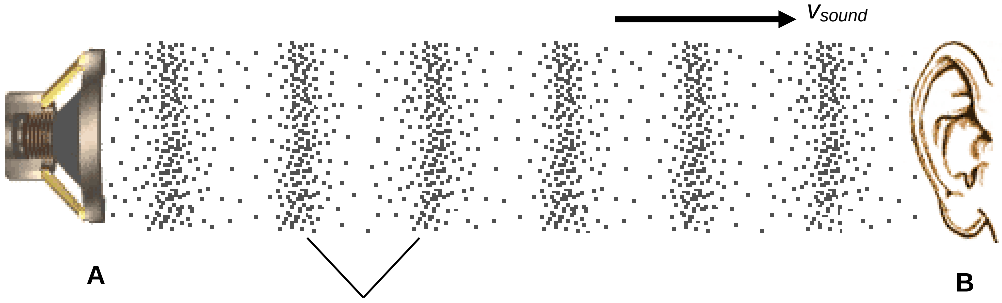
Longitudinal wave:

- A longitudinal wave is a wave in which the oscillations of the particles in the wave are along the direction of transfer of energy of the wave.
- Within a longitudinal wave, there are regions of compression where the particles are closer to each other, and regions of rarefaction where the particles are further from each other.



Additional Notes

Sound waves as longitudinal waves



This compression is a disturbance.
It travels at the wave speed, v_{sound}.

- When the loudspeaker cone at A moves forward, it compresses the air in front of it. As the sound wave passes through air, air molecules vibrate left and right.
- Energy is transferred from A to B but there is no transfer of matter from A to B.

Note:

A wave can transfer energy in different dimensions. Wave on rope is a 1-D wave, wave on water surface is a 2-D wave, while sound and light waves from point sources are 3-D waves.

Check Your Understanding 1

1) Radiowave is a longitudinal wave. (True / False)
2) When sound travels from the vocal chords to the ears of a listener, the molecules at the vocal chords must have travelled to the listener's ear. (True / False)

B Graphical Representation of Waves

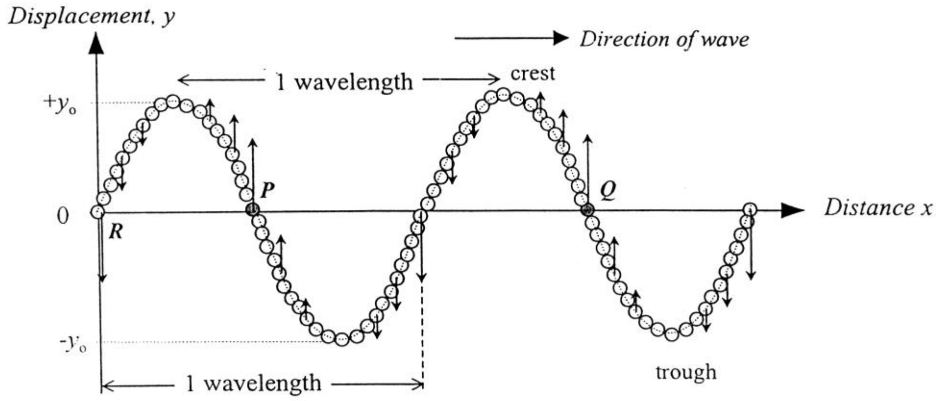
There are two types of graphs associated with waves. They complement each other in representing a particular wave, as waves have both a spatial component as well as a time component.

B.1 Displacement-Distance Graph

Displacement-distance (or displacement-position) graph

- A wave consists of not just one particle, but a series of particles all connected together (like a string of beads).
- Each particle oscillates about its own equilibrium position with the same frequency (i.e. same period).
- This collective string of particles form the wave profile or the shape of the wave.
- For each particle, we can define:
    - Displacement y, its distance from its equilibrium position. Displacement is a vector quantity, so it can have a positive or negative value.
    - Amplitude y_0: its maximum displacement. Amplitude is always positive (magnitude only).
    - Period of oscillation T: The time taken for the particle to complete one cycle.

- A displacement-distance graph shows the displacements of many particles at a given instant in time. (a snapshot of the wave)



- The crest of the wave is where there is maximum positive displacement; the trough of the wave is where there is maximum negative displacement.
- The wavelength λ is the shortest distance between any two successive points on a progressive wave which are vibrating in phase, or equivalently, the distance between two successive crests (or troughs).

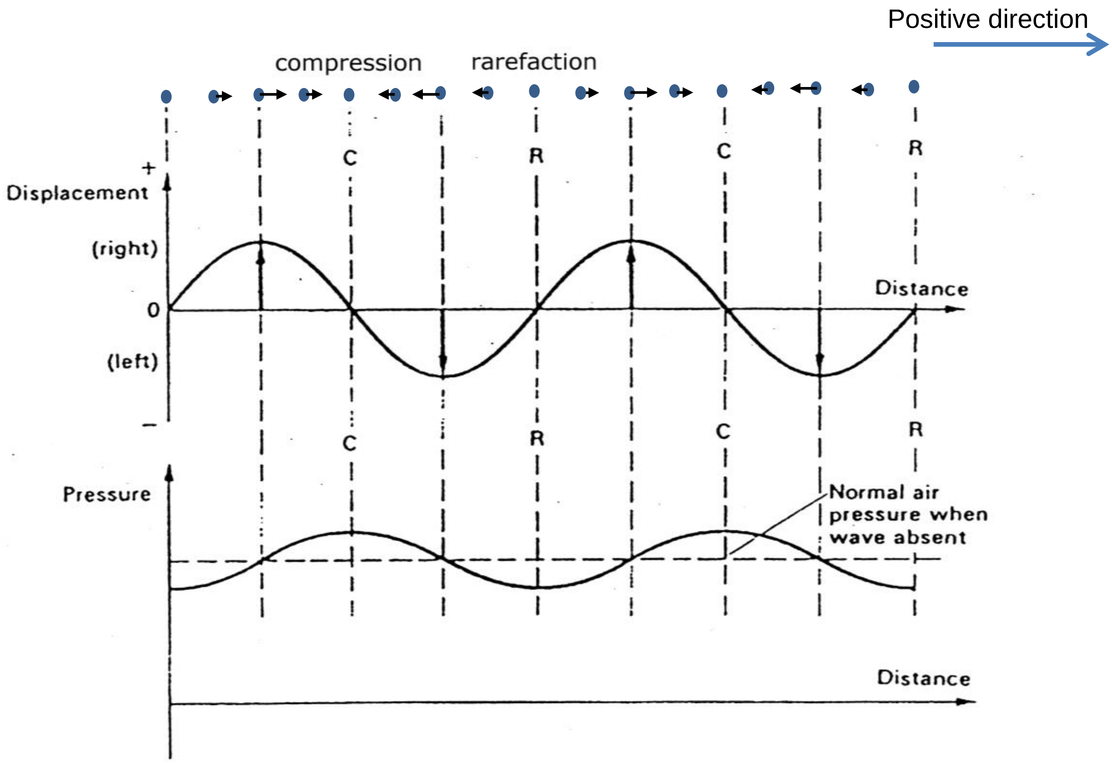
Displacement-position graph for longitudinal waves (e.g. sound waves)

- For longitudinal waves the displacement of the particles are in the same direction as the transfer of energy of the wave (e.g. for a sound wave travelling to the right, the particles oscillate left and right). As a result, the representation of the displacement-distance graph does not resemble the actual movement of the particles.

- For a longitudinal wave, the displacement-distance graph also allows the identification of points of compression and rarefaction, which are accompanied by the corresponding variations in air pressure.

Note:

Even though the wave profile is moving to the right, none of the particles which make up the wave are carried along with it. They merely create an "illusion" of a progressive motion (like the "twisting" display tube outside a barber's shop, or the swaying motion of long grass in a field).



Additional Notes

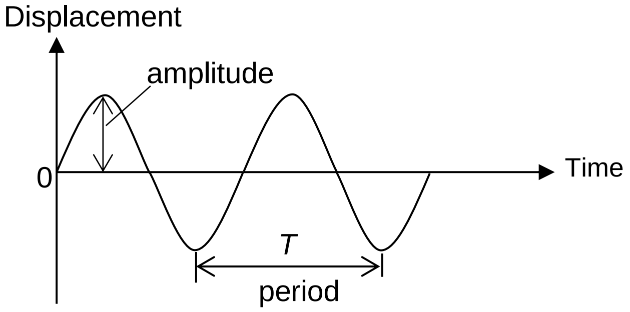
- At compression, the pressure is above normal pressure. At rarefaction, the pressure is below normal pressure

B.2 Displacement-Time Graphs

- When a particle on the wave oscillates, its displacement from the equilibrium position varies sinusoidally with time.
- The oscillation is one dimensional (restricted to along a single line) and is a function of time only.
- This can be plotted on a displacement-time graph, to show how the displacement of a **single particle** on the wave varies with time.
- One cycle on a displacement-time graph represents one period of oscillation – the time taken to generate one cycle of the wave.
- The frequency f of the oscillation (and thus the wave) can be obtained using $f = \frac{1}{T}$.

Particles along a wave oscillate in s.h.m.

This means that the particle will have maximum velocity at the equilibrium position and zero velocity at the amplitudes.



B.3 Derivation of $v = f\lambda$

- In one cycle, the crest of the wave travels one wavelength λ, and the time taken is one period T.
- **Wave speed v**, which is the distance travelled by the wave energy per unit time, is

$$v = \frac{\lambda}{T}$$

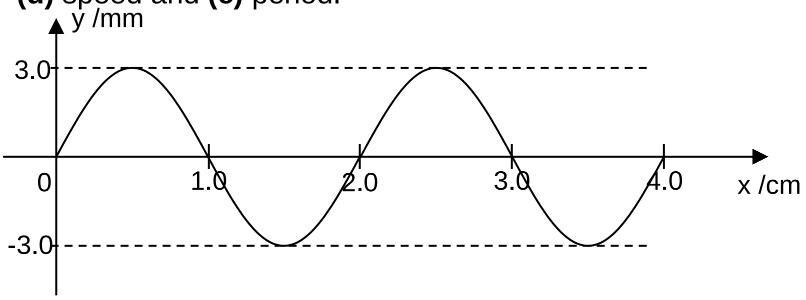
- Since frequency f is the number of cycles per unit time, hence wave speed as

$$v = \frac{\lambda}{T} = f\lambda$$

Worked Example 1

The wave shown in the following graph is being sent out by a 60 Hz vibrator.

Determine the following for the wave: **(a)** amplitude, **(b)** frequency, **(c)** wavelength, **(d)** speed and **(e)** period.

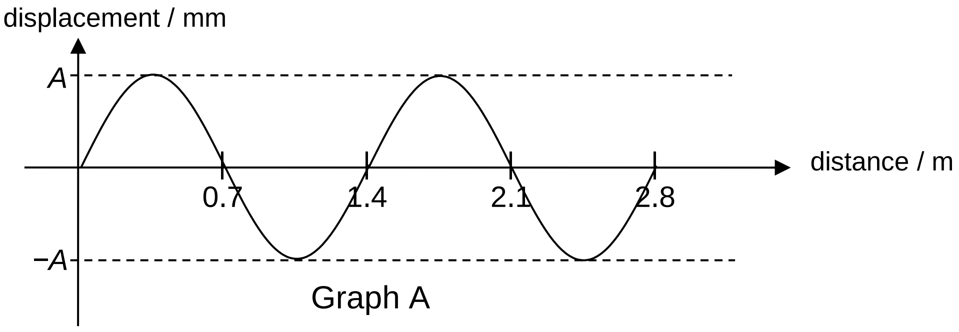


Solution

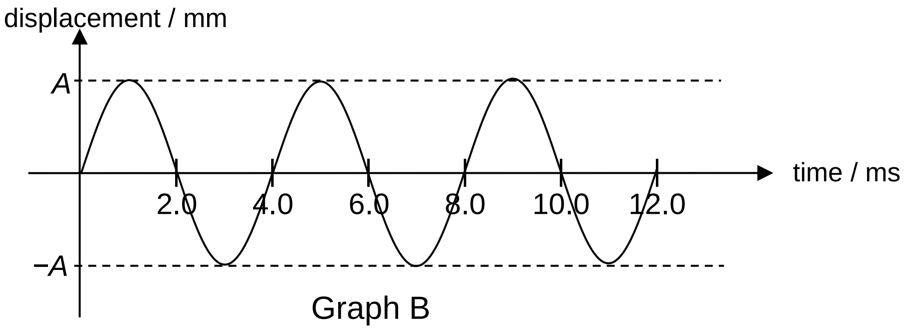
(a) 3.0 mm, **(b)** 60 Hz, **(c)** 2.0 cm,
(d) Using $v = f\lambda$,
$v = 60 \times 0.020 = 1.2$ m s^{-1}
(e) Using $T = 1/f$, $T = 1/60 = 0.017$ s

Example 1

A tuning fork is used to produce a sound wave that propagates through air. This wave is represented in the two graphs below.



Graph A



Graph B

(a) Calculate the speed of the wave.

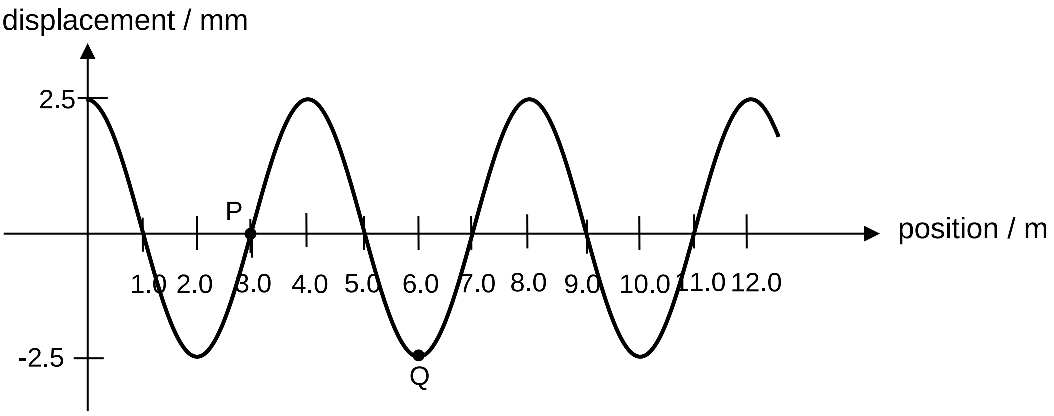
(b) The sound wave now travels through a solid, where its speed is 10 times that in air. Calculate the wavelength of the sound wave in the solid.

Solution

Additional Notes

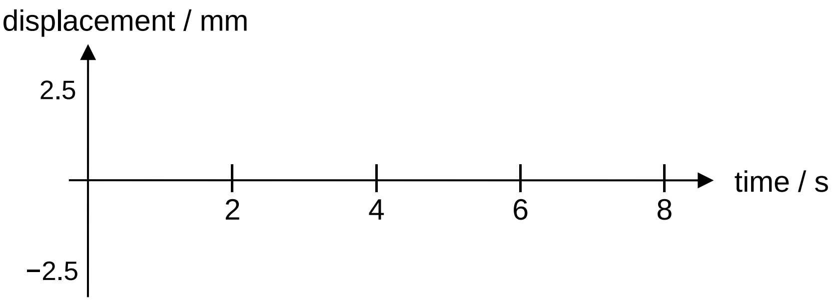
Example 2

A wave is represented by the following displacement-position graph. The wave is moving to the right with a speed of 1 m s^{-1}.



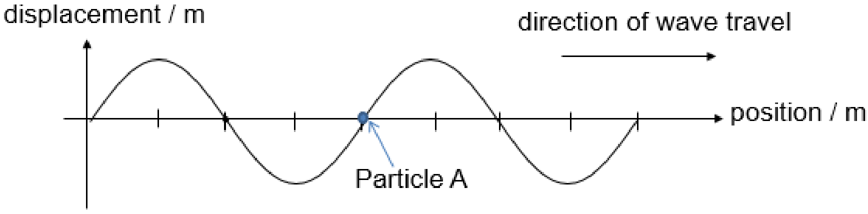
(a) Sketch using the axes above, a displacement position graph for the wave in the next second.

(b) Sketch on the same axes below, displacement-time graphs for particles P and Q for the next 8 s.

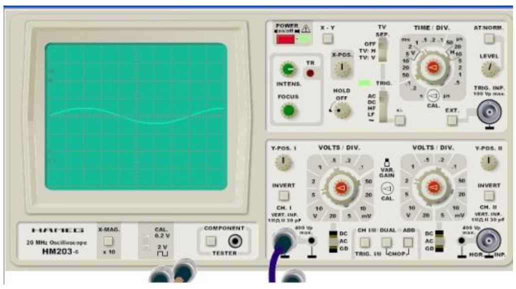


Check Your Understanding 2

A transverse wave is represented by the following graph. Particle A will be going up in the next instant. (True / False)



B.4 Determination of Frequency of a Wave using a Calibrated Oscilloscope



- A calibrated oscilloscope is widely used for displaying waveforms.
- These waves are recorded from instruments that measure oscillations (e.g. microphone). The measured oscillations – converted to an oscillating voltage signal – are then displayed on the screen.
- The voltage signal is represented on the y-axis. The scale (in volts per division) can be changed by adjusting the *sensitivity* or *gain control*. An example of a possible setting is 5 V cm^{-1}.
- The x-axis of the display is the time axis. The scale can be changed by adjusting the *time* base. An example of a possible setting is 2 μs cm^{-1}.
- The frequency of the signal can be thus be determined from the display along the time axis.

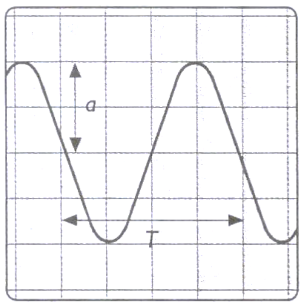
Worked Example 2

Given:

Sensitivity (y-plates) set at: 10 V cm^{-1}

Time base (x-plates) set at: 0.5 ms cm^{-1}

What is the frequency of the wave?



Solution

Period T = 4 x 0.5 x 10^{-3} = 0.0020 s
Frequency f = 1 / T = 500 Hz

Additional Notes

C Phase and Phase Difference

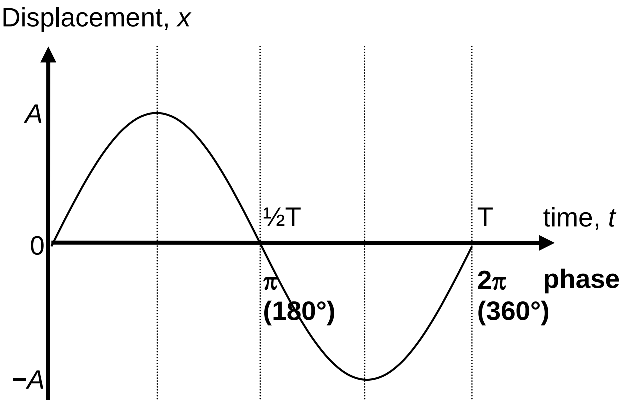
Phase

- The **phase angle** ϕ (from 0 to 2π rad; or 0° to 360°) gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave.

  e.g. at t = ¼T, ϕ of the particle is given by

  $\boxed{\frac{\phi}{2\pi}=\frac{t}{T}} \Rightarrow$

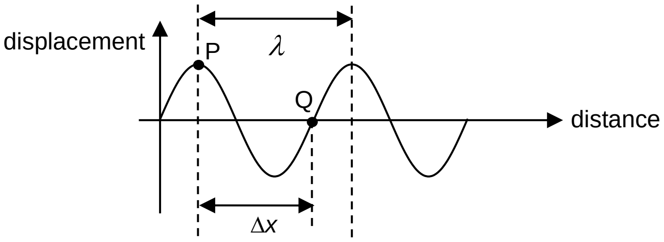
  $\phi=\frac{\frac{1}{4}T}{T}\times 2\pi=\frac{1}{2}\pi=1.57\ rad$



Phase Difference

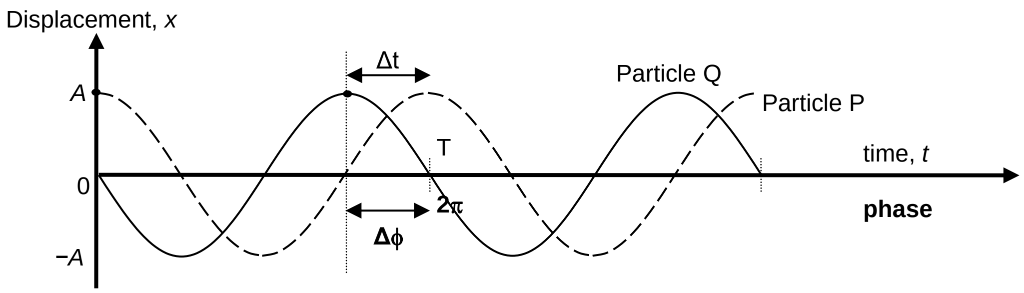
- **Phase difference** is a measure of how much one wave is out of step with another. A phase difference of one cycle corresponds to 360° or 2π rad.

- For a displacement-distance graph, we can calculate phase difference, Δϕ by looking at the distance between the two particles, Δ*x*.



- The phase difference between P and Q is $\boxed{\Delta\phi=\frac{\Delta x}{\lambda}\times 2\pi}$

- For a displacement-time graph, we can calculate phase difference, Δϕ by looking at the time lag, Δ*t* between the two particles.



- $\frac{\Delta t}{T}=\frac{\Delta\phi}{2\pi}$ so the phase difference is $\boxed{\Delta\phi=\frac{\Delta t}{T}\times 2\pi}$

Note:

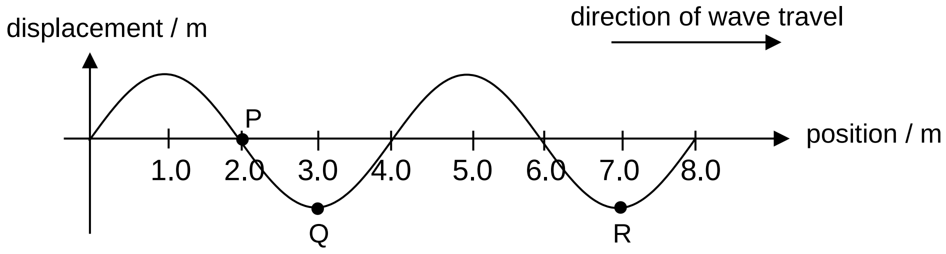
Phase difference can only be calculated for particles on waves of the **same** frequency.

Generally,

- If the phase difference is 0 rad (or equivalently 2π rad), the particles are in phase.
- If the phase difference is π rad or 180°, the particles are in anti-phase.
- Phase difference is normally quoted such that $0 \leq \Delta\phi < 2\pi$ rad.
  (refer to Appendix for more details)

Example 3

The figure below shows the displacement-position graph of a wave.



(a) Calculate the phase difference between particles P and Q.

(b) Calculate the phase difference between particles P and R.

Solution

Additional Notes

D Energy Associated with a Wave

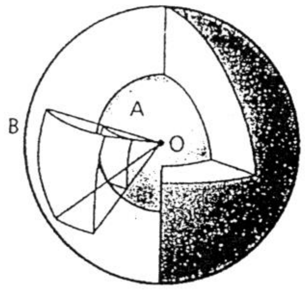
- A progressive wave transfers energy – the intensity of a wave describes how much energy is transferred by the wave at a particular point.

<u>Intensity</u> of a wave is the rate of energy flow per unit cross-sectional area perpendicular to the direction of wave propagation.

$$\text{Intensity} = \frac{\text{Power}}{\text{Area}}$$

Energy flow per unit time is also referred to as <u>power</u>.

- A uniform point source is one where energy is propagated away from the source uniformly in all directions.
- If the source provides energy at a rate (i.e. power) of P, then this energy must also flow through the surface of an (imaginary) sphere with the source at its centre and radius *r*.
- So all the energy flows perpendicularly through a total surface area of $4\pi r^2$.

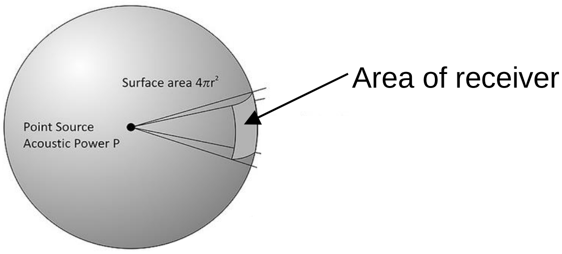


- The expression for intensity at a distance *r* from the source is given by

$$I = \frac{\text{Power of Source}}{4\pi r^2}$$

Note: This formula is applicable only if the point source radiates energy <u>uniformly in all directions</u>.

- Thus the Intensity due to a point source without energy loss follows an inverse square law.



- When a receiver or detector is placed in the path of the wave, the energy passing through can be absorbed by the receiver.
- The power received by the **receiver** is proportional to the area of the **receiver** i.e.

Power received by receiver = (Intensity at distance *r* from the source) × (Area of receiver)

Example 4

A point source emits waves uniformly in all directions. Initially, the intensity received by a detector placed at a certain distance is 80 W m^{-2}. The cross-sectional area of the detector is *A*. The power of the source is then halved and a second detector of cross-sectional area 2*A* is placed twice the distance away from the source.

(a) Determine the new intensity detected by the second detector.

(b) The rate of energy received by the second detector is 0.40 W. Determine the rate of energy received by the first detector initially.

Solution

D.2 Relationship between Intensity and Amplitude of a Wave

Frequency of the wave has to be constant when considering these relationships.

- As the wave moves away from the source, its intensity and thus its amplitude decreases.
- The amount of energy associated with a wave is proportional to the square of its amplitude *A.* (Recall from SHM that energy of an oscillating particle $E = 2\pi^2 m f^2 A^2$, where f = frequency, and A = amplitude)
- Thus the intensity I of a wave is proportional to the square of its amplitude A.

$$\boxed{I \propto A^2}$$

Worked Example 3

A sound wave of amplitude 0.20 mm has an intensity of 3.0 W m^{-2} at a point in space. Calculate the intensity of another sound wave of amplitude 0.40 mm but the same frequency at that point.

Solution

Use the relationship intensity $\propto$ (amplitude)^2

$$\frac{I_2}{I_1} = \left(\frac{A_2}{A_1}\right)^2 \Rightarrow \frac{I_2}{3.0} = \left(\frac{0.40}{0.20}\right)^2$$

I_2 = 12 W m^{-2}

Additional Notes

Example 5

A point source radiates energy uniformly in all directions. The intensity of the wave at a point 20 m from the source is 0.2 W m^{-2}.

(a) Calculate the power of the source.

(b) Calculate the intensity of the wave 40 m from the source.

(c) Calculate the energy received by a receiver of a cross-sectional area of 0.4 m^2 placed 40 m from the source, over a period of 2 s.

(d) The amplitude of the wave at a point 20 m from the source is 1.5 mm. Calculate the amplitude of the wave at a point 60 m from the source.

Solution

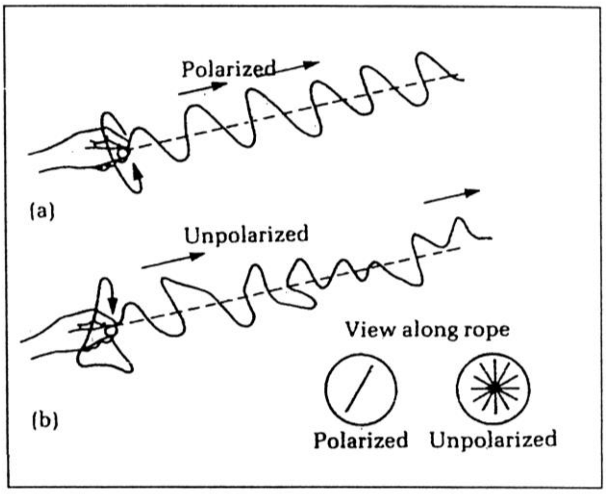
E Polarisation of Transverse Waves

Relating Science and Society

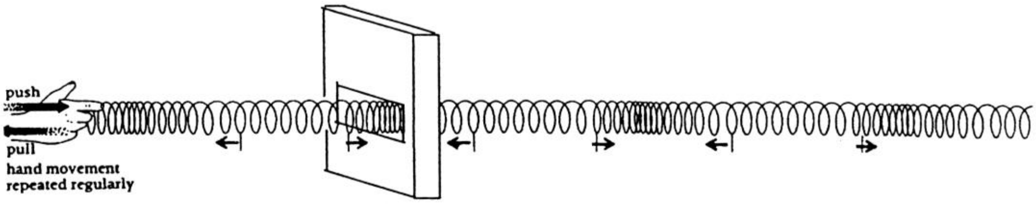
There are many uses of polarisation in real life such as in glare-reducing sunglasses due to light being polarised after reflection off some surfaces, and in polarised lenses to view 3D movies. Let's understand how polarisation work.

E.1 Polarisation

- A wave is said to be polarised when the oscillations in a wave are confined to one direction only in a plane normal to the direction of transfer of energy of the wave.



- **Only transverse waves can be polarised**; longitudinal waves cannot be polarised because its axis of vibration is parallel to its direction of energy transfer.

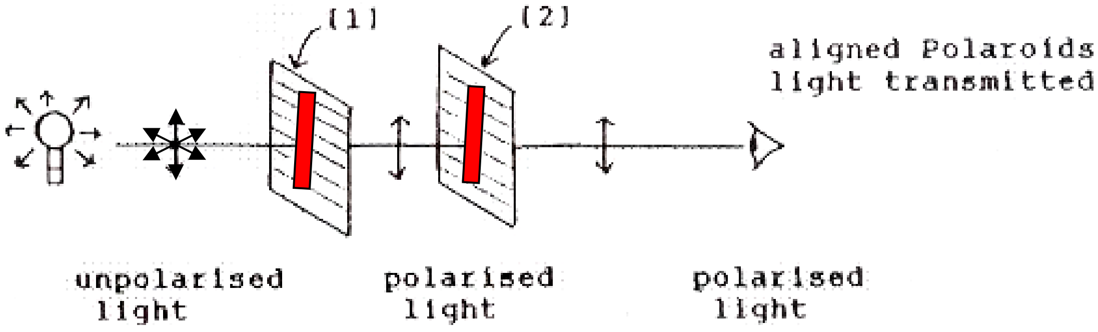


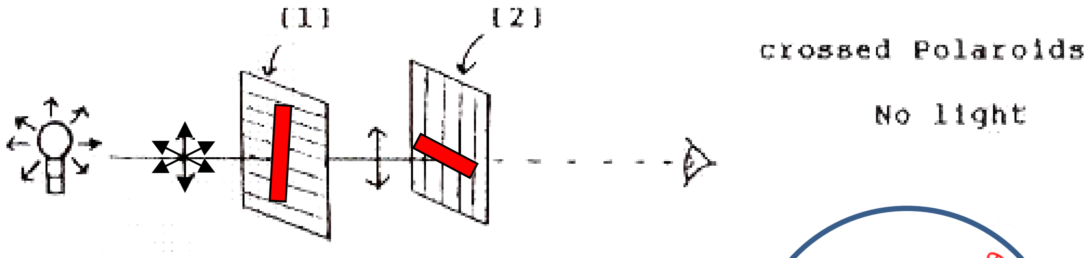
Additional Notes

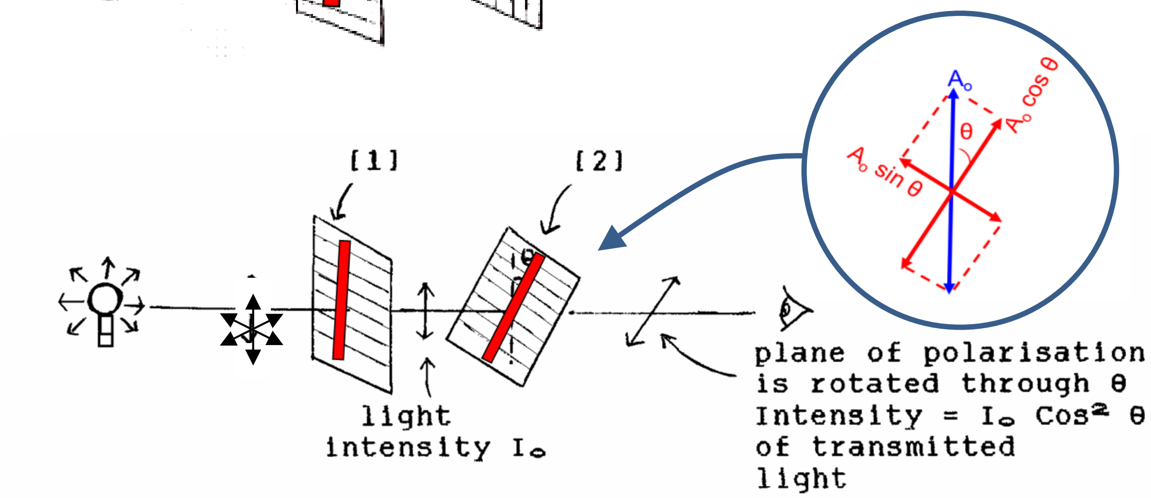
Consider a source which is emitting unpolarised transverse waves, which then pass through two polarisers 1 and 2.

- After passing through polariser 1, the wave is polarised in a vertical plane.
- If polariser 2 is in the same orientation (direction), then the wave will continue through polariser 2 with no decrease in intensity.
- If polariser 2 is oriented perpendicular to polariser 1, then the wave cannot propagate.

A polariser (polaroid) is an optical filter that is used to convert a beam of unpolarised EM radiation to a beam of polarised EM radiation.





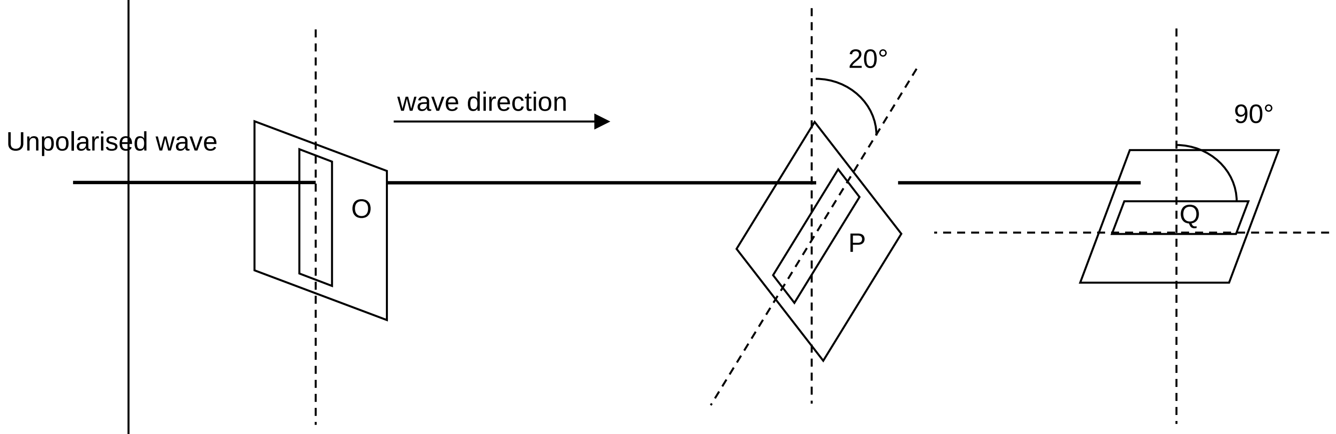


- When polariser 2 is rotated at a particular angle, θ, a variation in intensity of light is observed.
- Transmitted amplitude A = $\mathbf{A_o \cos\theta}$.

Malus' law (intensity $\propto \cos^2\theta$) is due to intensity $\propto A^2$.

Example 6

As shown below, an unpolarised wave passes through polariser O such that the emerging wave is plane-polarised with an amplitude of A_o. A second polariser P is placed further such that the plane-polarised wave is incident normally on it. Polariser P is rotated clockwise by an angle of 20°. A third Polariser Q is placed after Polariser P and it is rotated clockwise by an angle of 90°.



(a) Determine the amplitude of the wave after passing through polariser P in terms of A_o.

(b) Determine the amplitude of the wave after passing through polariser Q in terms of A_o.

Solution

Additional Notes

E.2 Malus' Law

- As mentioned in page 17, when light passes through a second polariser which has its axis of polarisation at an angle of θ different from the first polariser, the transmitted amplitude $A = A_o \cos\theta$.
- Since $I \propto A^2$, it is clear that the transmitted intensity $\boxed{I \propto \cos^2\theta.}$

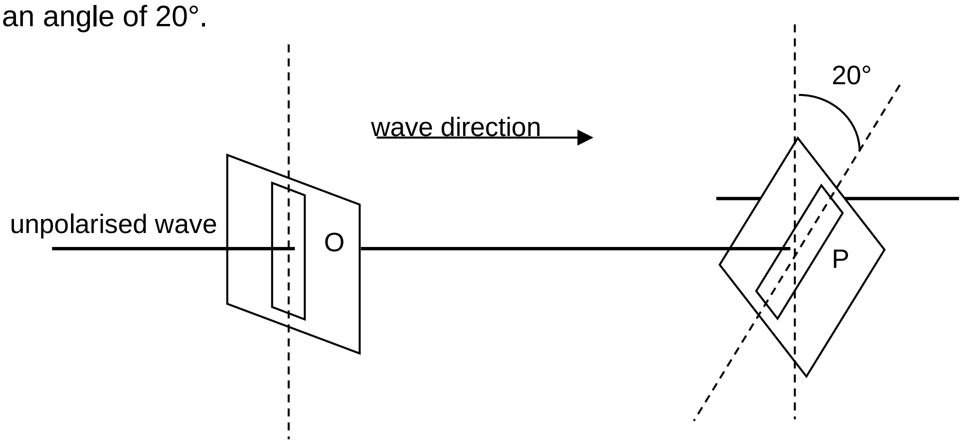
  This is known as Malus' Law.

Frequency of the wave has to be constant when considering these relationships.

As waves travel through a medium, energy, E, is transmitted as vibrational energy from particle to particle of the medium.
$E = 2\pi^2 m f^2 A^2$, where
f = frequency,
A = amplitude.

Example 7

As shown below, an unpolarised wave passes through polariser O such that the emerging wave is plane-polarised with an intensity of 4.0 W m^{-2}. A second polariser P is placed further such that the plane-polarised wave is incident normally on it. Polariser P is rotated clockwise by an angle of 20°.



Determine the intensity of the wave after passing through polariser P.

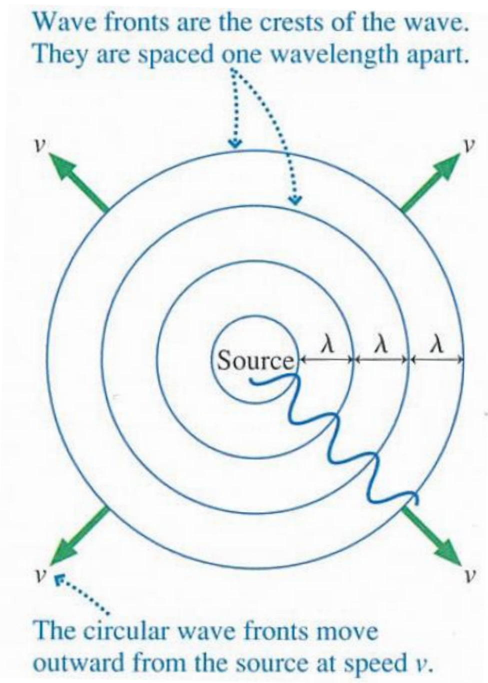
Solution

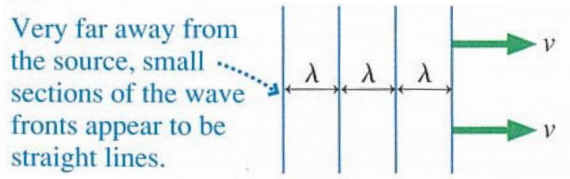
Note: When an unpolarised wave of intensity I_O passes through a polariser, the intensity I of the emerged wave is **half of the original intensity** (refer to Appendix 3 for more details).

Appendix

1. Wavefront (refer to A.2)

Suppose you were to take a photograph of ripples spreading on a pond. If you mark the location of the crests on the photo, your picture will look like Fig. 1(a). A wave like this is called a circular wave. It is a two-dimensional wave that spreads across a surface. The lines that locate the crests are called wavefronts, and they are spaced precisely one wavelength apart. All wavefronts are perpendicular to the velocity, *v* of the wave.





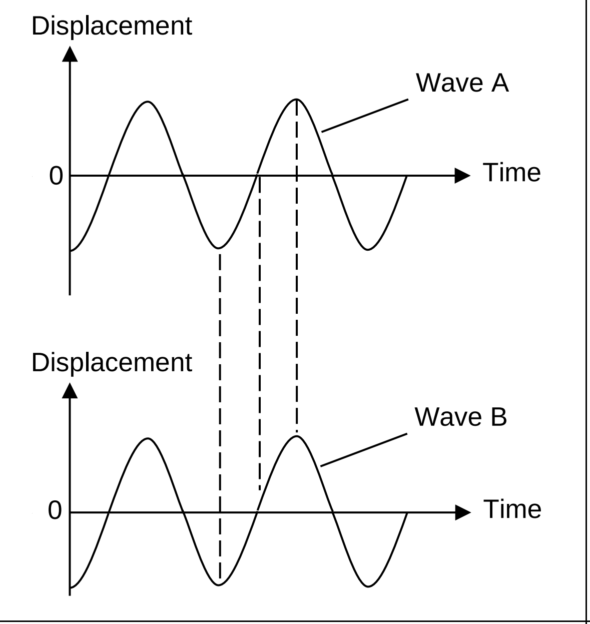
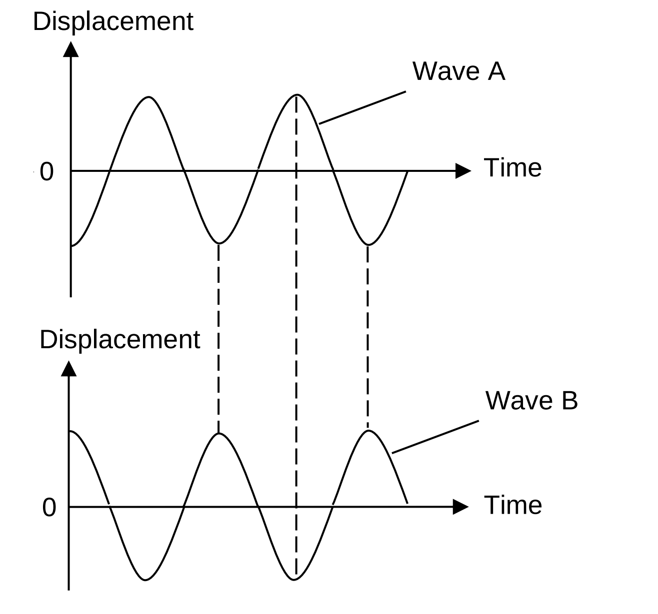
(b) Although the wavefronts are circles, you would hardly notice the curvature if you observe a small section of the wavefronts very far away from the source. The wavefronts would appear to be parallel lines, still spaced one wavelength apart and travelling at speed *v*.

Fig. 1(a). The wavefronts of a circular wave.

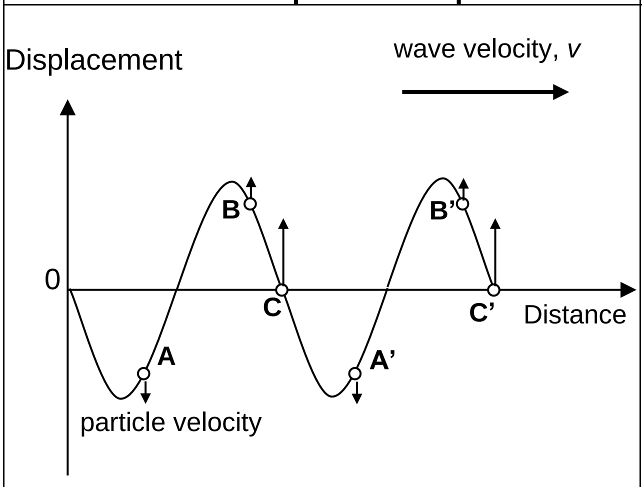
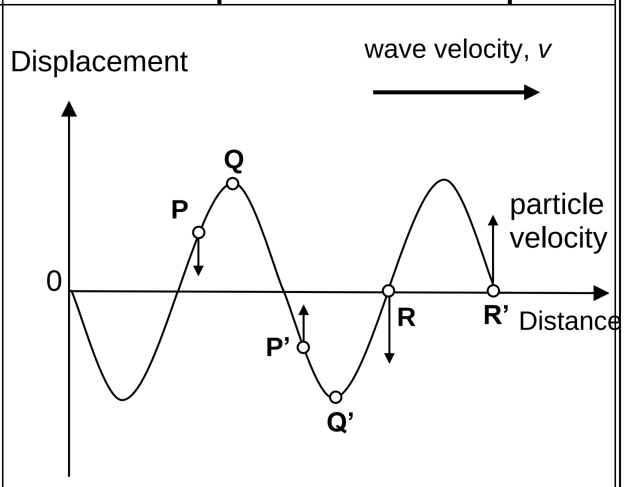
A **wavefront** is a line (or surface) which marks out all those points on a wave that have the same phase.

2. Phase Difference (refer to C; additional details)

Phase difference is a measure of how much one wave is out of step with another.

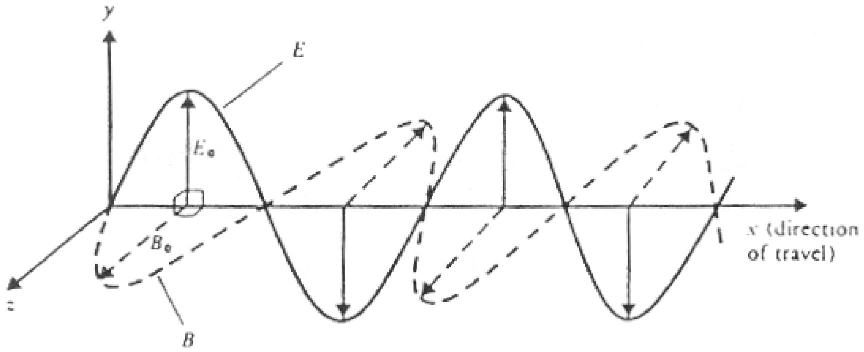
Case 1: Two waves in phase	Case 2: Two waves 180° out-of-phase
	
In Case 1, all the crests of A are aligned with crests of B, while the troughs of A are aligned with the troughs of B. The waves are said to be **in phase** with each other. The **phase difference in this case is zero.**	In Case 2, all the crests of A are aligned with troughs of B, while the troughs of A are aligned with the crests of B. The waves are said to be in **anti-phase** (or exactly out of phase). One wave is half a cycle behind the other. Since one cycle is equivalent to 360° or 2π radians, the **phase difference in this case is 180° or π radians.**

We can also compare the relative state of disturbance of **two particles in the same wave** using phase difference.

Case 1 : Two particles in phase	Case 2 : Two particles 180° out-of-phase
	
When two particles in the same wave are in the **same phase,** they are precisely in the **same state of disturbance** at the same time (e.g. the points A & A', B and B' and C & C').	A crest (Q) and a trough (Q') are **180º or π radians out of phase** with each other. Similarly, P & P' and R & R' have a phase difference of 180º or π radians.

3. Nature of Electromagnetic Waves & Polarisation

Electromagnetic (EM) waves are produced by oscillating electric charges. All EM waves consist of electric and magnetic fields oscillating at right angles to one another and to the direction of travel. The two fields oscillate sinusoidally, always in phase with one another. They carry no charge.



An ideal polarising filter (polariser/polaroid) passes 100% of the incident light that is polarised parallel to the filter's polarising axis but completely blocks all light that is polarised perpendicular to this axis.

In the above figure, unpolarised light is incident on a flat polarising filter. The electric-field vector, *E* of the incident wave can be represented in terms of components parallel and perpendicular to the polarising axis; only the component of *E* parallel to the polarising axis is transmitted. Hence the light emerging from the polariser is linearly polarised parallel to the polarising axis. For this ideal polariser, the intensity of the transmitted light is *exactly half* that of the incident unpolarised light, no matter how the polarising axis is oriented.

Qualitative Explanation: We can resolve *E* into a component parallel to the polarising axis and a component perpendicular to it. Because the incident light is a random mixture of all states of polarisation, these two components are, on average, equal. The ideal polariser transmits only the component that is parallel to the polarising axis, so half the incident intensity is transmitted.

Mathematical Proof:

$$I = \frac{1}{2\pi}\int_0^{2\pi} I_o \cos(\theta)^2 \, d\theta = I_o/2$$