ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749

Additional Notes

Name: ________________________________ (   ) **Class:** 25 / ___

Topic 13 Electric Fields

Content
A Concept of an electric field
B Electric force between point charges
C Electric field of a point charge
D Uniform electric fields
E Electric potential

Learning Outcomes

Candidates should be able to:

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point.

(b) represent an electric field by means of field lines.

(c) recognise the analogy between certain qualitative and quantitative aspects of electric and gravitational fields.

(d) recall and use Coulomb's law in the form $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{r^2}$ for the force between two point charges in free space or air.

(e) recall and use $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ for the field strength of a point charge in free space or air.

(f) calculate the electric field strength of the uniform field between charged parallel plates in terms of potential difference and plate separation.

(g) calculate the forces on charges in uniform electric field.

(h) describe the effect of a uniform electric field on the motion of charged particles.

(i) define electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point.

(j) state that the field strength of the field at a point is numerically equal to the potential gradient at that point.

(k) use the equation $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ for the electric potential in the field of a point charge, in free space or air.

A Concept of an electric field

Candidates should be able to:

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength as force per unit positive charge.
(b) represent an electric field by means of field lines.

A.1 Concept of an electric field:

Many everyday forces are pushes or pulls between bodies in contact.

In other cases forces arise between bodies that are separated from one another (action-at-a-distance forces).

An action-at-a-distance effect in which one body A exerts a force on another body B not in contact with it, can be regarded as due to a 'field of force' (or simply field) in the region around A.

Field of force is a region of space where a body experiences a force.

A.2 Definition of electric field:

Later you will learn that a magnetic field can cause a force to act on a moving charge.

An electric field is a region of space where a stationary charge experiences a force.
The direction of the electric field is the direction of the force on a positive charge.

Note: The definition is in terms of the force on a stationary charge. A force on a moving charge could indicate that a magnetic field is present.

A.3 Electric field strength (intensity), *E*:

memorize

Electric field strength at a point is defined as the *force per unit positive charge* acting on a small stationary charge placed at that point.

$$\boxed{E = \frac{F}{q}}$$

where F is the force acting on a small stationary positive charge, q.

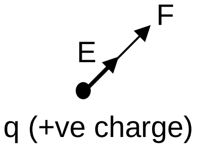
The test charge must be small so that the E-field will not be distorted.

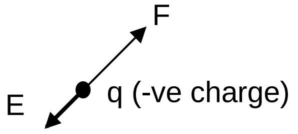
Units: N C^{-1} or V m^{-1}

E is a vector quantity.

Compare *E* with gravitational field strength

To standardise: the direction of field to be drawn as the direction of force on a *positive* test charge.

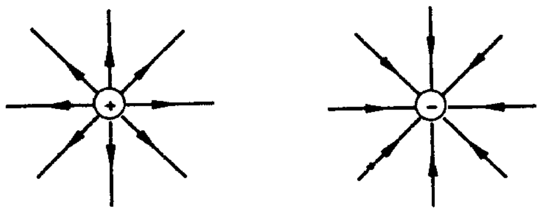




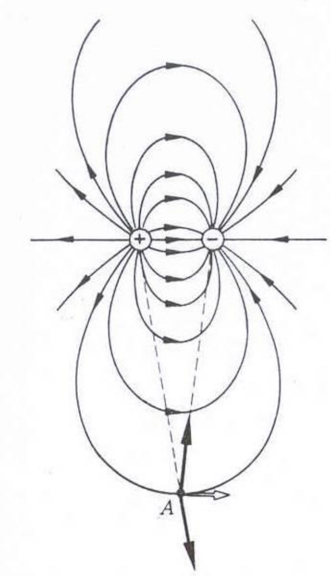
A.4 Representing an electric field:

An electric field can be represented and so visualised by electric field lines (lines of force).
Electric field lines are lines that show the direction of the force acting on a stationary positive point charge in the field.
The field line is imaginary but the field it represents is real.

Examples:
(a) Isolated point charge:



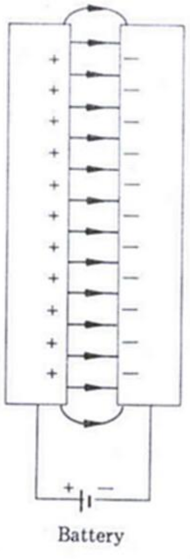
(b) Electric dipole:



The electric field lines originate on the positive charge and end on the negative charge.

A positive test charge will experience two forces when placed at point A. The resultant of these two forces is tangent to the line of the force at A.

(c) Uniform electric field:

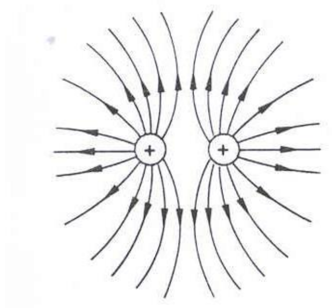


The battery places equal charges of opposite sign on the two metal plates.

Although some “fringing” of the lines occurs near the edges, the field lines are mostly directed straight across the gap between the plates.

The electric field between the plates is uniform. This is indicated in the diagram by the *equal spacing* of the field lines.

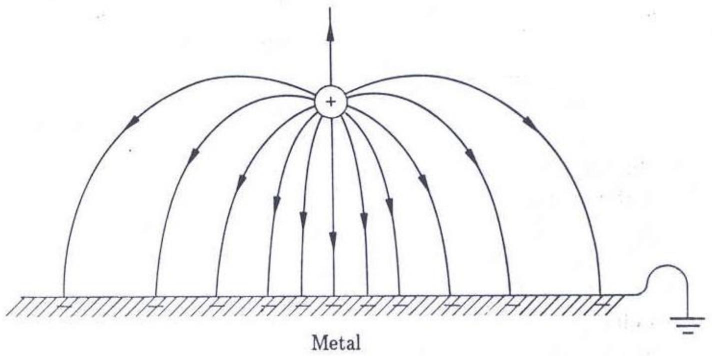
(d) Two like charges:



The stronger the electric field is, the closer the lines of force.

The field is weak midway between the charges.

(e) Metal conductors:



The electric field just outside the surface of a metal is *perpendicular* to the surface.

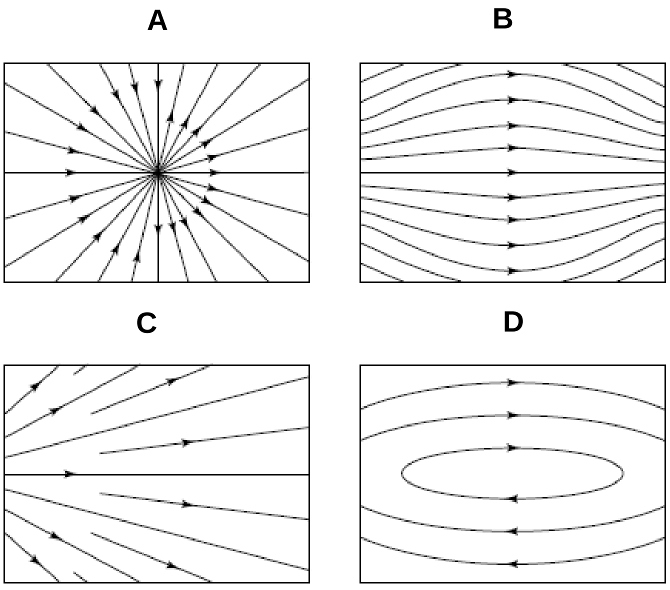
A.5 Summary of the rules for drawing electric lines of force around electrically charge bodies:

	Rule	Diagram
1	Electric lines of force always leave +ve charges and end on -ve charges.	A.4 (b)
2	Field lines always leave (or end on) conducting surfaces at right angles to the point of contact.	A.4 (e)
3	A uniform electric field has equally spaced field lines.	A.4 (c)
4	The stronger the electric field around a charged body, the closer the lines of force.	A.4 (d)
5	Field lines never cross.	A.4 (a) to (e)

Additional Notes

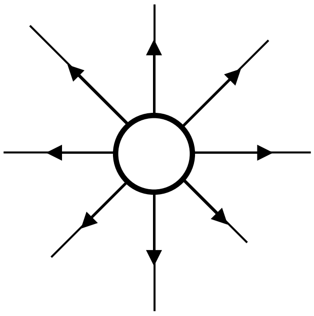
Check Your Understanding 1

Consider the four field patterns shown. Assuming there are no charges in the regions shown, which of the pattern represents a possible electrostatic field?



Check Your Understanding 2:

A charged sphere has both a gravitational field and an electric field around it. The diagram represents the field around such a sphere.



Which field could this diagram represent ?

A both electric and gravitational
B electric but not gravitational
C gravitational but not electric
D neither electric nor gravitational [2013 P1 Q24]

A.6 Electric field in daily life: Lightning

Explore the cause of lightning and the use of lightning conductors, which serve to protect lives, properties and equipment.

B Electric force between point charges:

Candidates should be able to:

(d) recall and use Coulomb's law in the form $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ *for the force between two point charges in free space or air.*

B.1 Coulomb's law:

The force between two point charges Q_1 and Q_2 is directly proportional to the product of the charges and inversely proportional to the square of the distance r between them.

Compare with Newton's Law of Gravitation.

$$F = k\frac{Q_1Q_2}{r^2}$$ where k is a constant

or

$$\boxed{F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}}$$

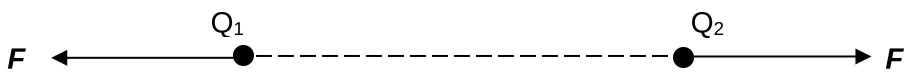
where ε_0 (epsilon naught): permittivity of free space (vacuum or air)=8.85×10^{-12} C^2 N^{-1} m^{-2}

B.2 Direction of electrostatic force:

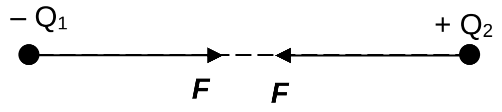
The **direction** of the electrostatic force is along the line joining the two charges.

(a) The force is **repulsive** if Q_1 and Q_2 are charges of the same signs.

Compare with gravitational force



(b) The force is **attractive** if Q_1 and Q_2 are charges of opposite signs.



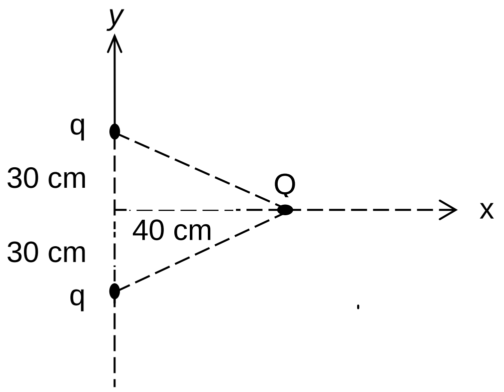
Additional Notes

Worked Example

(a) Find the force of the electrostatic repulsion between 2 protons, a distance of 1×10^{-15} m apart in a nucleus? What is the value of the acceleration of the protons? The mass of a proton is 1.67×10^{-27} kg.

(b) Find the force of gravitational attraction between 2 protons.

(c) In the given figure, 2 equal +ve charges q of magnitude 2.0×10^{-6} C interact with a third charge Q of magnitude 4.0×10^{-6} C. Find the magnitude and direction of the total force on Q.



Solution:

(a)

$$F_e = \frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2}$$

$$= \frac{1}{4\pi(8.85 \times 10^{-12})}\frac{(1.6 \times 10^{-19})^2}{(1 \times 10^{-15})^2}$$

$$= 230 \text{ N}$$

$$F = ma$$

thus $$a = \frac{F}{m} = \frac{230}{1.67\times10^{-27}} = 1.38 \times 10^{29} \text{ m s}^{-2} \quad \text{(very large)}$$

(b)

$$F_g = G\frac{m^2}{r^2}$$

$$= 6.67 \times 10^{-11} \text{ x } \frac{(1.67 \times 10^{-27})^2}{(1 \times 10^{-15})^2}$$

$$= 1.86 \times 10^{-34} \text{ N}$$

$$<< F_e$$

F_g is <u>insignificant</u> as compared to F_e

Note that Gravitational force << Electric force. Hence can ignore the effect of gravity when calculating force on charges in electric field.

(c) From Coulomb's law:

$$F_1 = F_2 = \frac{1}{4\pi\varepsilon_o}\frac{Qq}{r^2}$$

$$= \frac{1}{4\pi(8.85 \times 10^{-12})}\frac{(4.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.5)^2} = 0.29 \text{ N}$$

$$F_{1x} = F_1 \cos\theta = 0.29 \times \frac{0.4}{0.5} = 0.23 \text{ N}$$

$F_{1y} = F_1 \sin\theta = 0.29 \times \frac{0.3}{0.5} = 0.17\text{ N}$

$F_{2x} = 0.23\text{ N}$
$F_{2y} = -0.17\text{ N}$

$F_x = F_{1x} + F_{2x} = 2(0.23) = 0.46\text{ N}$
$F_y = F_{1y} + F_{2y} = 0$

Hence total force is horizontal with magnitude 0.46 N.

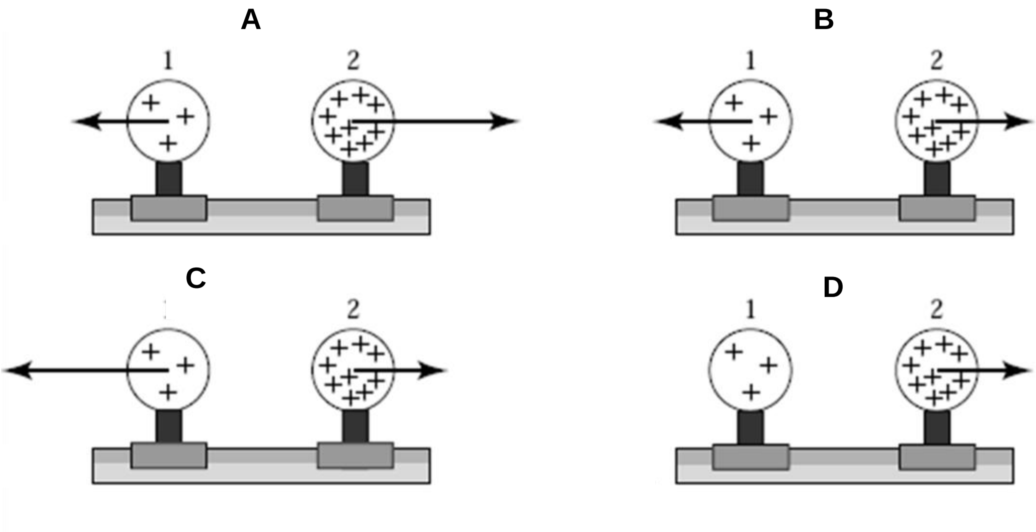
Check Your Understanding 3

Two positive charges q and Q are held in place a distance s apart. By what factor would the magnitude of the electric force on the q charge change if the charges were separated by a distance 2s?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 2 **D** 4

Check Your Understanding 4

Two uniformly charged spheres are firmly fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces?



Additional Notes

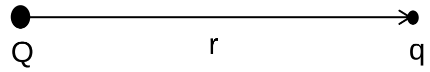
C Electric field strength

Candidates should be able to:

(e) recall and use $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ *for the field strength of a point charge in free space or air.*

C.1 Electric field strength of a point charge:

point charge Q — small +ve test charge q



A small positive test charge q is placed a distance r from a point charge Q.

By Coulomb's law, the force acting on q:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

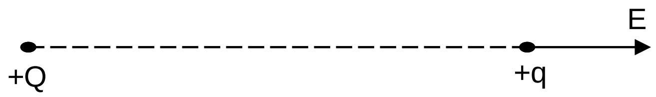
<u>magnitude</u> of the electric field strength is

$$\boxed{E = \frac{F}{q} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}}$$

Compare with gravitational field strength due to a point mass

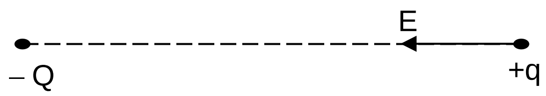
(The formula applies to a charged spherical conductor when the distance r is greater than the radius of the sphere.)

<u>Direction</u>:



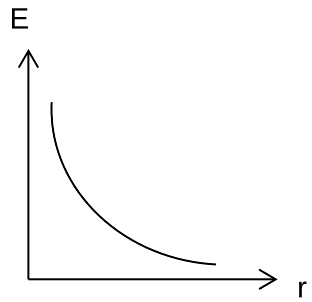
E is directed <u>away</u> from +Q.

If a point charge -Q replaced +Q, E would be directed towards -Q.



<u>Note:</u>

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

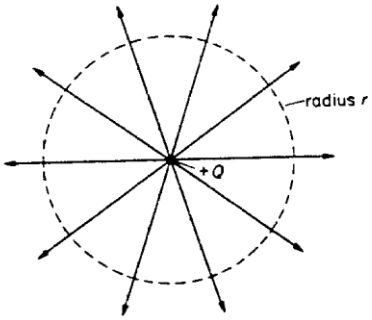


E decreases with distance from the point charge according to an <u>inverse square law.</u>

i.e. $E \propto \dfrac{1}{r^2}$

Additional Notes

The field due to an isolated point charge is thus non-uniform but it has the same values at equal distances from the charge and so has spherical symmetry.



Eg: The electric field in the space surrounding a point, positive charge.

Worked Example

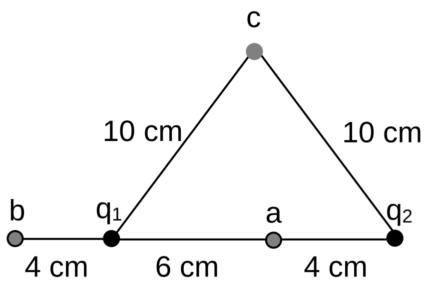
Point charges q_1 and q_2 of $+12 \times 10^{-9}$ C and -12×10^{-9} C respectively are placed 0.10 m apart, as in the figure shown. Compute the electric fields due to these charges at points a, b and c.

Solution:

Hint:
Draw arrows to represent the electric field due to the 2 point charges at point a.
Find the magnitude of the electric field using
$E = \dfrac{q_1}{4\pi\varepsilon_0 r^2}$
Then use vector addition to find resultant E
Repeat for point b and c.

a : $E_{1a} = \dfrac{q_1}{4\pi\varepsilon_0 (6 \times 10^{-2})^2}$

$= 3.00 \times 10^4$ N C^{-1}



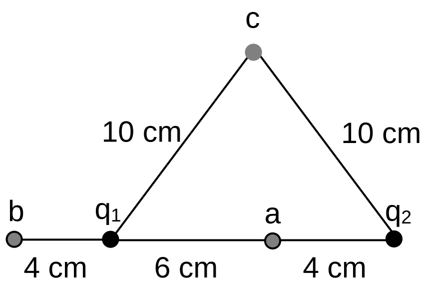
$E_{2a} = \dfrac{q_2}{4\pi\varepsilon_0 (4 \times 10^{-2})^2}$

$= \dfrac{12 \times 10^{-9}}{4\pi\varepsilon_0 (4 \times 10^{-2})^2} = 6.75 \times 10^4$ N C^{-1}

Thus $E_a = E_{1a} + E_{2a} = 9.75 \times 10^4$ N C^{-1} towards the right.

b : $E_{1b} = \dfrac{q_1}{4\pi\varepsilon_0 (4 \times 10^{-2})^2}$

$= 6.75 \times 10^4$ N C^{-1}



Compare with electric potential on p18.

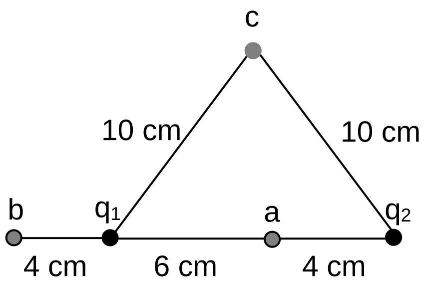
$E_{2b} = \dfrac{q_2}{4\pi\varepsilon_0 (14 \times 10^{-2})^2}$

$= 0.55 \times 10^4$ N C^{-1}

Thus $E_b = E_{1b} - E_{2b} = 6.20 \times 10^4$ N C^{-1} towards the left.

c : $E_{1c} = \dfrac{q_1}{4\pi\varepsilon_0 (10 \times 10^{-2})^2}$

$= 1.08 \times 10^4$ N C^{-1}



$E_{2c} = \dfrac{q_2}{4\pi\varepsilon_0 (10 \times 10^{-2})^2}$

$= 1.08 \times 10^4$ N C^{-1}

Thus $E_c = 2E_{1c} \cos 60° = 1.08 \times 10^4$ N C^{-1} towards the right.

Additional Notes

C.2 Electric field strength of a conducting sphere:

A charge +Q on an isolated conducting sphere is uniformly distributed over its surface (due to the repulsion of like charges) and has a radial electric field pattern.

The electric field within conductor = 0

The field at any point outside the sphere is exactly the same as if the whole charge were concentrated at a point charge +Q at the centre of the sphere.
From outside the sphere, all the lines appear to be diverging from its centre, i.e. a radial field.

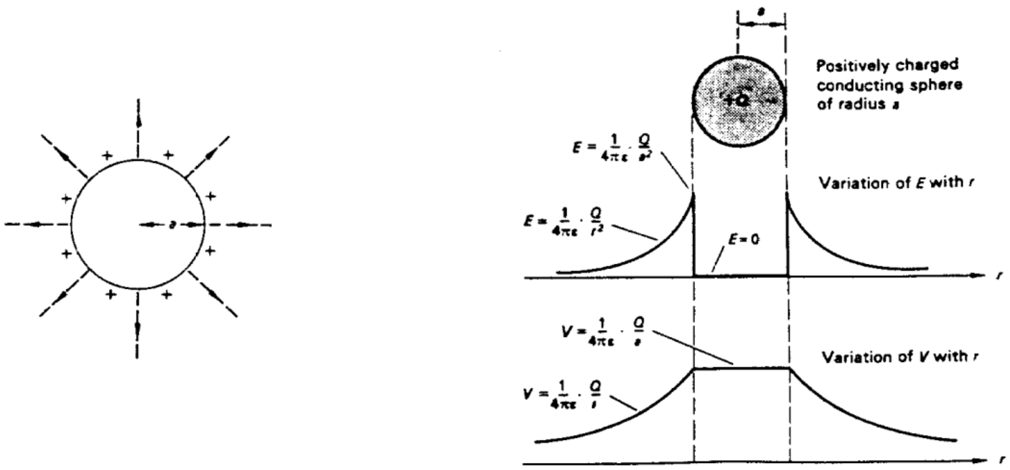


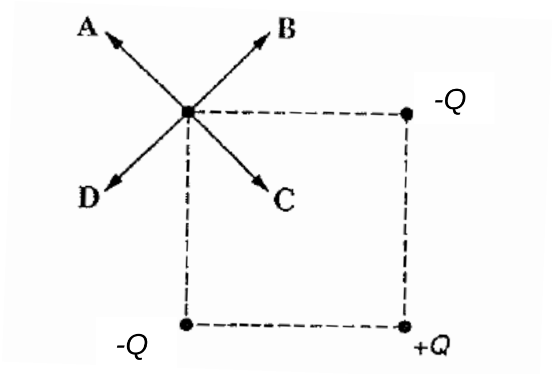
Fig. C.2

C.3 Electric field inside a hollow charge conductor:

Zero charge on the inside surface of the sphere and hence zero field within the hollow space.

Check Your Understanding 5

Point charges, each of magnitude Q, are placed at three corners of a square as shown in the diagram.



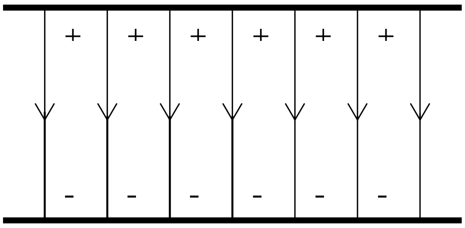
What is the direction of the resultant electric field at the fourth corner?

Additional Notes

D Uniform electric fields

Candidates should be able to:
(g) calculate the forces on charges in uniform electric fields.
(h) describe the effect of a uniform electric field on the motion of charged particles.

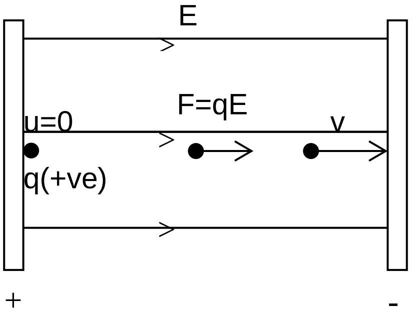
D.1 Field between two uniformly charged plates:



Field is <u>uniform</u> throughout the space provided the edge of the plates is avoided.

D.2 Forces on charges and their motion in a Uniform electric field:

Case 1: Stationary charge, u = 0

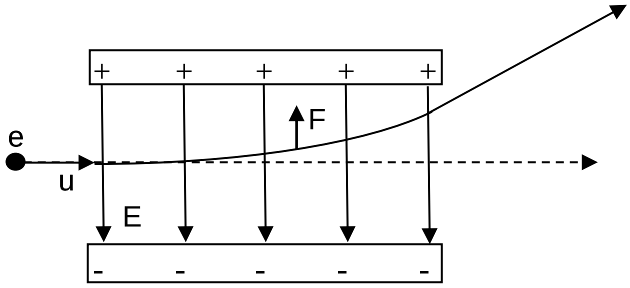


The force on a charged particle in a uniform electric field is *constant*.

The velocity of the charged particle will change under the influence of an electric field. Its path in the field is *straight*.

$$\boxed{\begin{aligned} F &= qE = ma \\ a &= \frac{qE}{m} \end{aligned}}$$

Case 2: Moving electron projected perpendicularly to E-field.



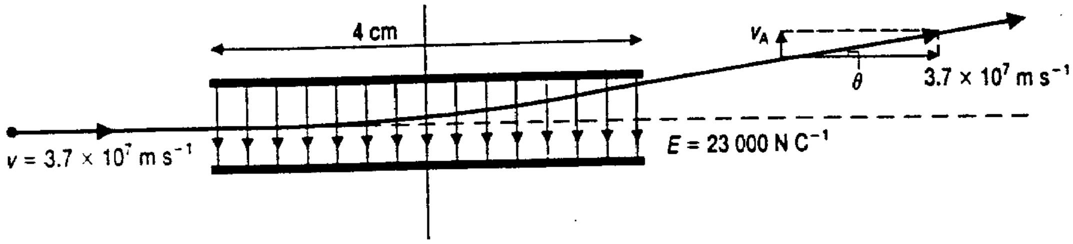
If a charged particle enters a uniform electric field at right angles to the field lines, its path will be parabolic.

Compare with the path of the mass projected at right angles to the gravitational field

Additional Notes

Example 1:

The electric field strength between a pair of plates of length 4.0 cm in a cathode ray tube is 2.3×10^4 N C^{-1}. An electron enters the field at right angles to it with a velocity of 3.7×10^7 m s^{-1} as shown in the diagram. Find the velocity of the electron when it leaves the electric field. (3.73×10^7 m s^{-1})



Hint:
Find the force on the electron in the electric field.
Recognize that the force causes the path of the electron to be parabolic in the uniform electric field.
Find the velocity using horizontal velocity and vertical velocity at the point of leaving.

Solution:

Additional Notes

E Electric Potential and the Relationship between potential and field

Candidates should be able to:

(i) define potential at a point in terms of the work done in bringing unit positive charge from infinity to the point.

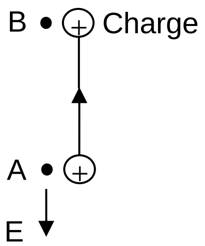
(k) use the equation $V = Q/4\pi\varepsilon_0 r$ for the electric potential in the field of a point charge, in free space or air.

Information about the field may be given by stating the field strength at any point; alternatively the potential can be quoted.

E.1 Electric potential

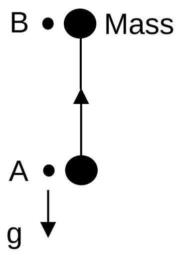
Meaning of potential

A charge in an electric field experiences a force and if it moves work will be done. If a positive charge is moved from A to B in a direction opposite to that of the field E, an external agent has to do work against the force of the field and energy has to be supplied. As a result, the system (of the charge in the field) gains an amount of electrical potential energy equal to the work done. When the charge is allowed to return from B to A, work is done by the forces of the field and the electrical potential energy previously gain by the system is lost. If, for example, the motion is in a vacuum, an equivalent amount of kinetic energy is transferred to the charge.



Electrical potential energy greater at B than at A

Fig. E.1 (a)



Gravitational potential energy greater at B than at A

Fig. E.1 (b)

In general, the potential energy associated with a charge at a point in an electric field depends on the location of the point and the magnitude of the charge. Therefore if a unit positive charge is chosen and the change of potential energy which occurs when such a charge is moved from one point to another is called the change of the potential of the field itself.

Hence the potential at B exceeds that at A by the work which must be done against the electric force to take unit positive charge from A to B.

If we select the potential at an infinite distance from the any electric charges as zero of potential, then the potential at a point in a field can be defined as follow.

E.1.1 Definition of Electric potential:

memorize

The electric potential, V, at a point is defined as the work done per unit positive charge in moving a point charge from infinity to the point.

$$V = \frac{W}{q}$$

Units: J C^{-1} or V (volt)
Electric potential is a scalar quantity.

memorize

Note:
(a) Potential at infinity is chosen as zero.
(b) Electric potential may be +ve or -ve depending on whether the potential is due to a positive or a negative charge. Work must be done on a positive charge in moving it from infinity towards a fixed positive charge and thus the potential is positive. Work is done by a positive charge in moving it towards a negative charge, resulting in a negative potential.
(c) Potential is a property of the field and not the individual charge placed there.

E.1.2 Electric potential difference:

Electric potential difference between 2 points B & A is defined as work done per unit positive charge when a very small charge moves from A to B.

$$V_{ba} = V_b - V_a = \frac{W_{ab}}{q}$$

Units: J C^{-1} or V (volt)

If the p.d between 2 points in an electric field is 10 volts then 10 joules of work are done per coulomb of charge moving from one point to the other and an energy change of 10 joules per coulomb occurs.

E.1.3 Potential and field compared:

When describing a field, potential is usually a more useful quantity than field strength because, being a scalar, it can be added directly, when more than one field is concerned. Field strength is a vector and addition by the parallelogram law is more complex. Also, it is often more important to know what energy changes occur (rather than what forces act) when charges move in a field and these are readily calculated if potentials are known

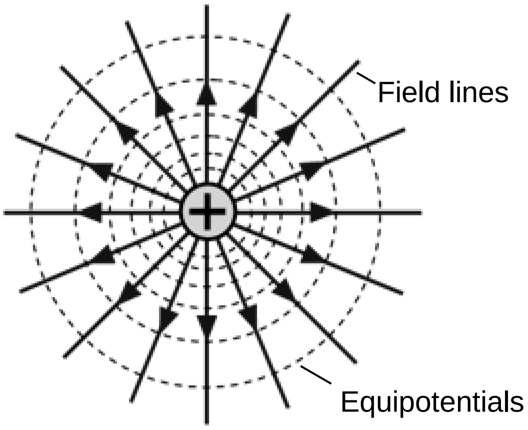
Additional Notes

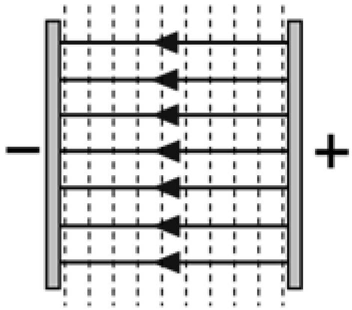
E.1.4 Equipotentials:

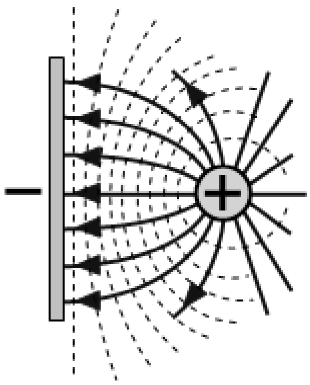
(i) All points in a field which have the same potential can be imagined as lying on a surface (or plane) called an equipotential surface (or line in two dimensional diagram).

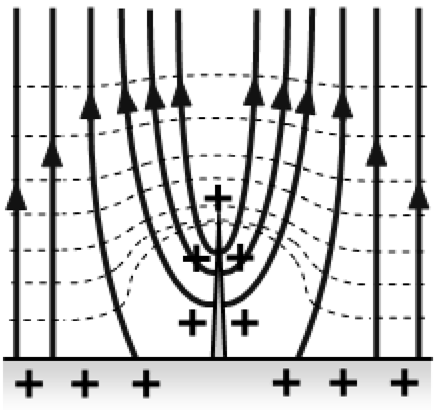
(ii) When a charge moves on such a surface (or line) no energy charge occurs and no work is done. The force due to field must therefore act at right angles to the equipotential surface at any point and so equipotential surfaces and fields line always intersect at right angles.

(iii) Equipotential surfaces for a point charge are concentric spheres (circles in two dimensions). There is spherical symmetry.









E.1.5 Electric potential energy:

The **electric potential energy U** of a small test charge q at a point P a distance r from a point charge Q is defined as the work done on the charge q by some external system in moving the charge q from infinity to point P.

See Appendix 1 for the derivation

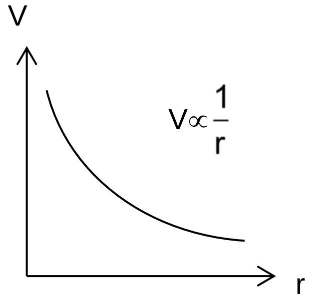
It can be shown that $\boxed{U = \dfrac{Q\,q}{4\,\pi\,\varepsilon_o\,r}}$

Unit: J

E.1.6 Electric potential due to a point charge:

Continue from above, the **potential at P (V_p)**:

$$\boxed{V_P = \frac{W}{q} = \frac{Q}{4\pi\varepsilon_0 r}}$$



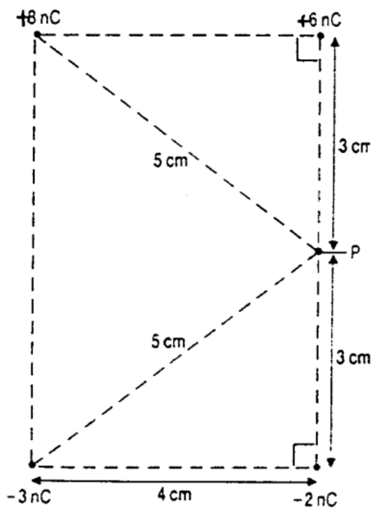
Units: $J\ C^{-1}$ or V (volt)

<u>Note:</u>

V_p can be positive or negative values depending upon whether Q is positive or negative.

The formula applies to a charged spherical conductor when the distance r is greater than the radius of the sphere.

<u>Worked Example</u>:
Find the potential at the point P due to the four charges.



Hint:
To find the potential at a point due to a series of charges, all that is necessary is to find the potential due to each charge separately and add the results together because potential is a scalar quantity.

Solution:

+6 nC: $V_p = \frac{Q}{4\pi\varepsilon_o r}$

$= \frac{6 \times 10^{-9}}{4\pi(8.9\text{x}10^{-12})(3\text{x}10^{-2})}$

$=$ 1790 V

-2 nC: $V_p = \frac{-2 \times 10^{-9}}{4\pi(8.9\text{x}10^{-12})(3\text{x}10^{-2})}$

$=$ -600 V

-3 nC: $V_p = \frac{-3 \times 10^{-9}}{4\pi(8.9\text{x}10^{-12})(5\text{x}10^{-2})}$

$=$ -540 V

+8 nC: $V_p = \frac{8 \times 10^{-9}}{4\pi(8.9\text{x}10^{-12})(5\text{x}10^{-2})}$

$=$ 1433 V

Hence Total potential at P = 1790 – 600 – 540 + 1433

= 2100 V

Additional Notes

Check your understanding 6

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to point A a distance r from $+Q$. Next, $+q$ is removed and a test charge $+2q$ is brought to point B a distance $2r$ from $+Q$. How is the electrostatic potential of the charge at B compared with the **electrostatic potential** of the charge at A?

A greater **B** smaller **C** the same

Check your understanding 7

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to a point a distance r from $+Q$. Then this charge is removed and test charge $-q$ is brought to the same point. Which test charge has greater **electrostatic potential energy?**

A +q **B** -q **C** It is the same for both

E.1.7 The electric potential of a *charged spherical conductor*:

(a) Inside the sphere: No p.d between any point inside and the surface of the sphere. All points inside the sphere have the *same* potential.

(b) Outside the sphere: The potential outside the sphere carrying a charge Q is the same as if the whole charge were concentrated at its centre. (Refer to Fig.C.2 in page 11)

E.2 The relationship between potential and field

Candidates should be able to:

(j) state that the field strength of the field at a point is numerically equal to the potential gradient at that point.

(f) calculate the field strength of the uniform field between charged parallel plates in terms of potential difference and separation.

Relation between E and V:

memorize

$$\boxed{E = -\frac{dV}{dx}}$$

(see appendix 2)

$\frac{dV}{dx}$ is called the *potential gradient*.

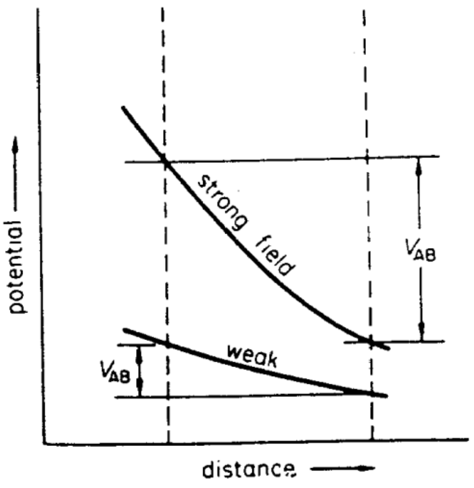
Potential gradient is the rate of change of electric potential with distance.

Note:

(a) The field strength at a point equals the negative of the potential gradient there.
i.e. the electric potential gradient is the electric field;
the direction of the field is the same as the direction of decreasing potential.

(b) Potential gradient ($\frac{dV}{dx}$) is a vector quantity.
Units: V m^{-1}
It follows that unit of E is also V m^{-1}. ($\equiv$ N C^{-1})

(c)



Additional Notes

(d) In a <u>uniform field</u> E is constant in magnitude and direction at all points,

$$\Rightarrow \quad \frac{dV}{dx} = \text{constant}$$

i.e. the potential changes steadily with distance.

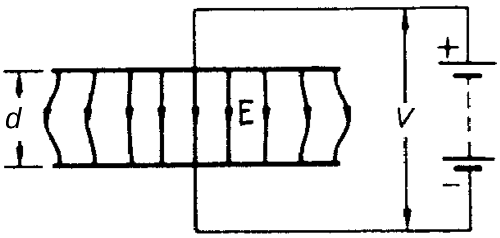
Hence the potential difference per unit distance is the electric field for a uniform field.

$$\boxed{E = \frac{V}{x}} \text{ (numerically)}$$

The electric field is in a direction in which the potential falls.

(e) The electric field between 2 parallel charged metal plates is uniform. (not at the edges).

$$\boxed{E = \frac{V}{d}}$$

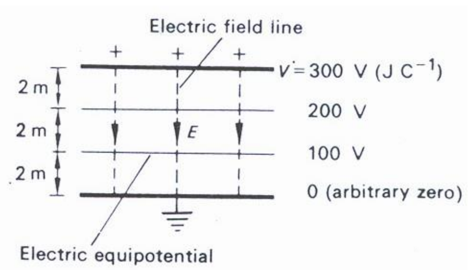


memorize

where V: potential difference between the plates.
d: separation between the plates.

If V = 300 V and d = 6.0 m, then the field strength E at any point in the uniform region is:

$E = \frac{V}{d}$ (numerically)
$= 300/6 = 50 \text{ V m}^{-1}$.



Thus, in a uniform electric field:

(i) E is constant in magnitude and direction at all points, hence $\frac{dV}{dx}$ is constant, i.e. the potential changes steadily with distance. (See diagram).

(ii) The direction of the field is the same as the direction of <u>decreasing</u> potential.

<u>Example 2</u>

Do Q4 on p7 of PbI worksheet.

F. Analogy between Gravitational and Electrostatic field

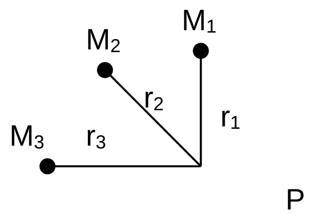
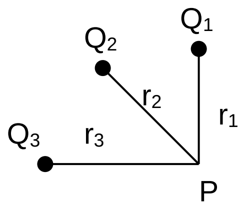
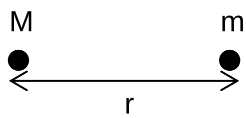
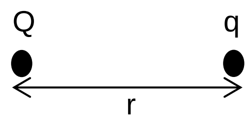
Candidates should be able to:

(c) recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational fields.

Comparison between Gravitational and Electrostatic field:

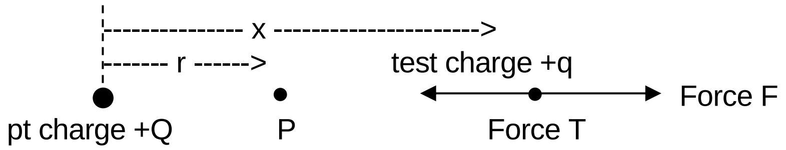
		Gravitational	Electrostatic
1.	Origin of force	Due to mass interaction	Due to charge interaction
2.	Nature of force	Only attractive M F_g F_g m r	Either attractive (bet. opp. charges) or repulsive (bet. similar charges) *Repulsive:* F_E +Q +q F_E r *Attractive:* -Q F_E F_E +q r
3.	Quantitative law	Newton's gravitational law: $F = G\dfrac{Mm}{r^2}$	Coulomb's law: $F = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Qq}{r^2}$
4.	Field (strength or intensity)	Def: Force per unit mass $g = \dfrac{F_G}{m}$ (N kg^{-1})	Def: Force per unit <u>+ve</u> charge $E = \dfrac{F_E}{q}$ (N C^{-1})
5.	Field set up by isolated mass / charge	M g r $g = G\dfrac{M}{r^2}$	Q E r $E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r^2}$
6.	Potential	$\phi = \dfrac{\text{work done}}{\text{mass}}$ (J kg^{-1})	$V = \dfrac{\text{work done}}{\text{charge}}$ (J C^{-1} or V)
7.	Pot. due to isolated mass M / charge Q	M ϕ r $\phi = -G\dfrac{M}{r}$	Q V r $V = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Q}{r}$

Additional Notes

		Gravitational	Electrostatic
8.	Pot due to multiple mass / charge system	 $\phi_p = -G\dfrac{M_1}{r_1} - G\dfrac{M_2}{r_2} - G\dfrac{M_3}{r_3}$	 $V = \dfrac{1}{4\pi\varepsilon_o}\left(\dfrac{Q_1}{r_1} + \dfrac{Q_2}{r_2} + \dfrac{Q_3}{r_3}\right)$
9.	Potential energy	Def: Work done to assemble the system of masses	Def: Work done to assemble the system of charges
10.	Potential energy of 2-mass / 2-charge system	 $U_G = -G\dfrac{Mm}{r}$	 $U_E = \dfrac{1}{4\pi\varepsilon_o}\dfrac{Qq}{r}$
11.	Relation bet. field and potential	$g = -\dfrac{d\phi}{dr}$	$E = -\dfrac{dV}{dr}$
12.	Relation bet. force and potential energy	$F_g = -\dfrac{dU_G}{dr}$	$F_E = -\dfrac{dU_E}{dr}$

Appendix 1

Electric potential energy:



Force T: An external system exerts on q
Force F: Q exerts on q

Consider a small test charge q being brought from ∞ to a point P a distance r from a point charge Q.

To move the charge from ∞ requires the force T to be at least equal to force F.

Work done δW on q by some external system to move it a distance δx towards P is:

$$\begin{aligned}\delta W &= -T\,\delta x \\ &= -q\,E\,\delta x \\ &= -\frac{Q\,q}{4\,\pi\,\varepsilon_o\,x^2}\,\delta x\end{aligned}$$

The total work done W in bringing q from ∞ to P is:

$$W = -\frac{Q\,q}{4\,\pi\,\varepsilon_o}\int_{\infty}^{r}\frac{dx}{x^2}$$

$$= -\frac{Q\,q}{4\,\pi\,\varepsilon_o}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= -\frac{Q\,q}{4\,\pi\,\varepsilon_o}\left[-\frac{1}{r}-(-\frac{1}{\infty})\right]$$

$$= \frac{Q\,q}{4\,\pi\,\varepsilon_o\,r}$$

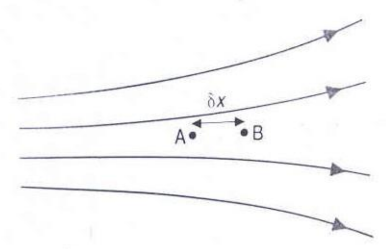
The total work done on the charge q = **potential energy U** of charge q.

and Potential energy, $U = \dfrac{Q\,q}{4\,\pi\,\varepsilon_o\,r}$

Appendix 2

Relation between E and V:

The work done by some external system on the +ve distance δx from A to B must be -ve.



Non-uniform field

i.e. δW = - E q δx

Potential difference = work done per unit charge.

$$\delta V = \frac{-\,E\,q\,\delta x}{q}$$

$$E = -\frac{\delta V}{\delta x}$$

In the limit, as δx → 0

$$\boxed{E = -\frac{dV}{dx}}$$

$\dfrac{dV}{dx}$ is called the *potential gradient*.