Additional Notes

# Topic 16: Electromagnetism

Electromagnetism Lecture Plan

| Lect | Concept | Est. duration of videos | Qns |
|---|---|---|---|
| 1 | A, B, C,D | 43 min | Eg1-3; CYU1 |
| 2 | E Eg2-5 | 43 min | Eg4,5; CYU2 |
| 3 | F | 48 min | Eg6,7; CYU3,4 |

**Content:**

A Concept of a Magnetic Field
B Magnetic fields due to currents
C Effects of ferrous core on magnetic field of a solenoid
D Force on a current-carrying conductor
E Force between current-carrying conductors
F Force on a moving charge

**Learning Outcomes:**

Candidates should be able to:

(a) show an understanding that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets.

(b) sketch flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid.

(c) use $B = \mu_o I / 2\pi d$, $B = \mu_o NI / 2r$, and $B = \mu_o nI$ for the flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively.

(d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of a ferrous core.

(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force.

(f) recall and solve problems using the equation $F = BIl \sin\theta$, with directions as interpreted by Fleming's left hand rule.

(g) define magnetic flux density.

(h) show an understanding on how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance.

(i) explain the forces between current-carrying conductors and predict the direction of the forces.

(j) predict the direction of the force on a charge moving in a magnetic field.

$\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$

(k) recall and solve problems using $F = BQv \sin\theta$.

(l) describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields.

(m) explain how electric and magnetic fields can be used in velocity selection for charged particles.

Additional Notes

## A Concept of a Magnetic Field

- Recall:
  - A gravitational field is a region in space where a mass would experience gravitational force acting on it.
  - An electric field is a region in space where a force acts on a stationary charge.

- Similarly,
  - A magnetic field will be a region where a magnetic pole will experience a force.
    - However, it is not possible to have an isolated magnetic monopole as they always exist in pairs (i.e. North and South poles existing within same object), unlike charges which can exist as isolated positive or negative

- Hence, a magnetic field is defined as:

| |
|---|
| A region of space where a magnetic pole, a current-carrying conductor or a <u>moving</u> charged particle will experience a force. |

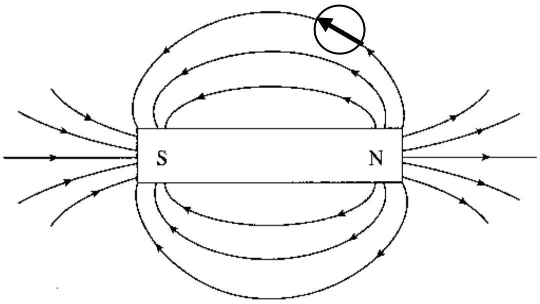
A magnetic field may be distinguished from an electric field in that <u>a charged particle experiences a force in a magnetic field only when it is moving (not parallel) to the field.</u> In an electric field, electric forces acts on charges whether it is moving or not.

- The magnetic field may exist at a point as a result of the presence of:
  (a) either a permanent magnet or
  (b) a conductor carrying an electric current
  in the vicinity of that point.

### Magnetic Lines of Force

- Just like gravitational and electric fields, a magnetic field can be represented by lines of force or field lines.

- Magnetic field lines run from the north-pole to the south-pole <u>outside the magnet</u>.

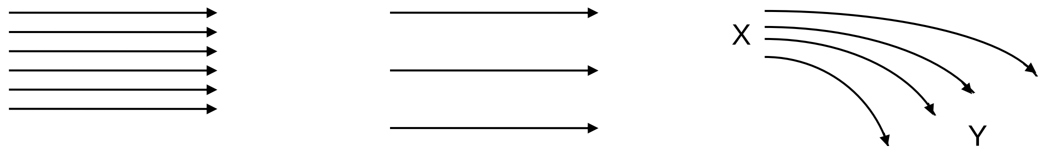



- The direction of a magnetic field at a point is taken as the direction where the <u>north-seeking pole of a compass</u> would point when the compass is placed there (only if the field is much stronger than Earth's magnetic field).

- If the field lines are parallel and equally spaced, the associated field is uniform. Otherwise the field will vary in its magnitude from one point to another.

Additional Notes

- The field is stronger if the lines of force are relatively closer together and weaker if they are relatively widely separated from one another.




strong uniform field | weak uniform field | non-uniform field – stronger at X than at Y

**Seeing links**: Recall that electric field lines are also drawn such that a) the electric field direction at any point is tangent to the electric field line at that point. b) Electric field lines never touch or cross. c) The closeness of the lines indicates the strength of the field.

- Field lines do not cross one another.

- If two or more magnets are present, their field strength at a point in the field is the vector sum of their individual field strengths.

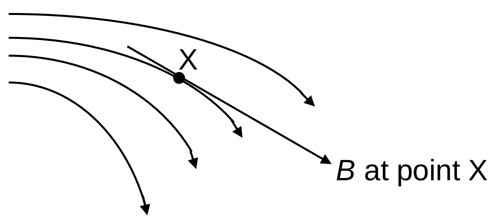
### Magnetic Flux Density

- The magnetic flux density, $B$, is a measure of the strength of the magnetic field in a particular region of space, i.e. $B$ is high if many lines passing normally through a unit area.

Refer to pg 13 for the definition of magnetic flux density.

- Unit of the magnetic flux density is tesla (T) or weber per metre squared (Wb m$^{-2}$).

- The direction of magnetic flux density at a point in space is along a tangent to the magnetic field at that point.



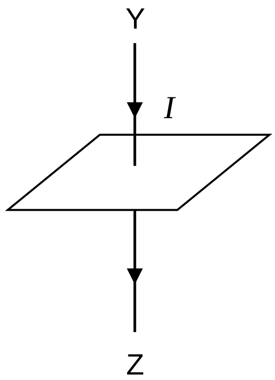
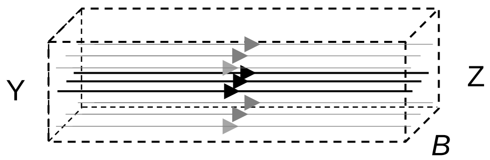

- Some typical values of magnetic flux density:
  - Earth's magnetic field (at Earth's surface): 24 - 66 μT
  - A large permanent magnet: 0.1 T
  - Electromagnet: approximately 1 T
  - INUMAC, world's most powerful MRI: 11.75 T

## B Magnetic Fields due to Currents

- A conductor carrying an electric current produces a magnetic field. Different configurations of current carrying conductors give rise to different field patterns and associated magnetic flux densities.

- Convention for representing current / field directions:

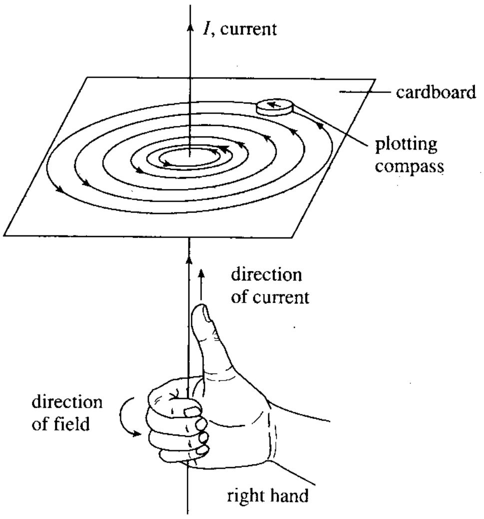
| | |
|---|---|
| $\otimes$ / $\times$ | represents a current / field directed <u>into</u> the plane of paper |
| $\odot$ / ● | represents a current / field directed <u>out of</u> the plane of the paper |

| | | View from Point Y | View from Point Z |
|---|---|---|---|
| Representing a <u>current</u> | <u>Current flowing from Y to Z in a conductor</u><br> | $\otimes$ $I$ | $\odot$ $I$ |
| Representing a <u>field</u> | <u>Magnetic field lines in the direction from Y to Z</u><br> | X X X<br>X X X<br>X X X<br>*B* | ● ● ●<br>● ● ●<br>● ● ●<br>*B* |

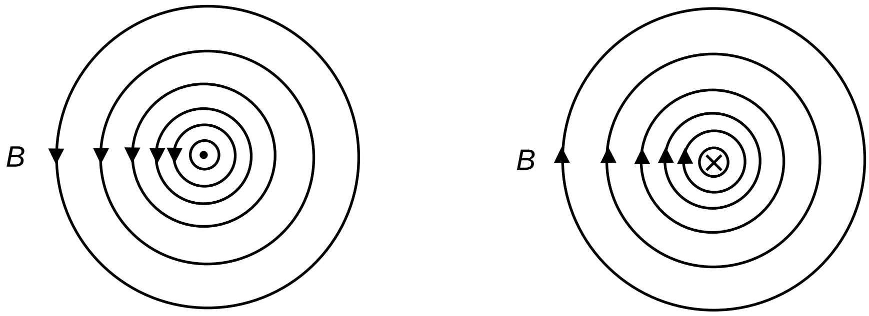
Additional Notes

**(a) Long straight wire**

- Magnetic flux pattern
  The field lines in the vicinity of a long, current-carrying wire take the form of concentric circles. The direction of the field lines, indicated by arrows, can be predicted using the Right Hand Grip Rule.




**Right Hand Grip Rule**
- Grip the wire using the right hand, with the thumb pointing in the direction of the current.
- The fingers will point in the direction of the *B*-field.




Current out of paper (viewed from top of cardboard)

Current into paper (viewed from bottom of cardboard)

- Drawing Guidelines:
  - The circles are concentric.
  - The spacing between the circles should increase as distance from wire increases.
  - You should draw at least 3 field lines to show the difference in spacing.

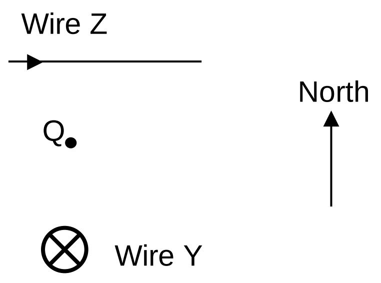
Spacing between the circles should increase due to

$$B = \frac{\mu_0 I}{2\pi r}$$

Additional Notes

For wire Y, recall that direction of magnetic flux density at a point in space is along a tangent to the magnetic field at that point. (pg 16-3)

## Check your Understanding 1




What is the direction of the magnetic flux density due to each wire at point Q?

| | Direction of magnetic flux density due to wire Z | Direction of magnetic flux density due to wire Y |
|---|---|---|
| **A** | into the page | eastwards |
| **B** | out of the page | eastwards |
| **C** | out of the page | westwards |
| **D** | into the page | westwards |

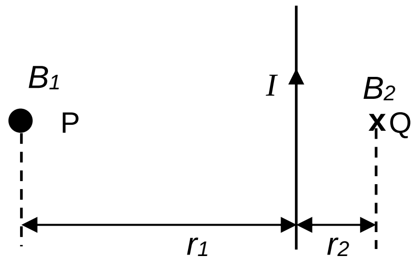
- Expression for magnetic flux density

  The magnetic flux density $B$, at a point distance $r$ away from a long straight wire carrying a current $I$, can be found using:

Do NOT memorize this formula – it is provided in formula list

However it is necessary to remember that
(1) $B \propto I$
(2) $B \propto \frac{1}{r}$.

$$B = \frac{\mu_0 I}{2\pi r}$$ where $\mu_0$ = permeability of free space ($4\pi \times 10^{-7}$ H m$^{-1}$)




In the diagram above, using Right Hand Grip Rule, the magnetic field at point P is directed out of the plane of the paper with a magnetic flux density of $B_1 = \frac{\mu_o I}{2\pi r_1}$. At point Q, the field is directed into the plane of the paper with a magnetic flux density of $B_2 = \frac{\mu_o I}{2\pi r_2}$.

Additional Notes

**Example 1**

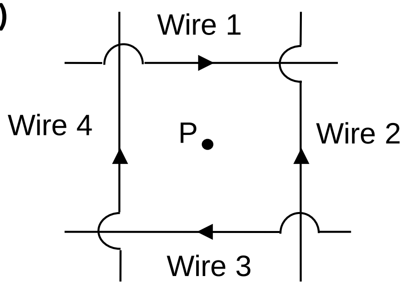
Four wires, each carrying a current $I$, are arranged in 2 different configurations.

If P is the centre of the squares, and given that the magnetic flux density at P due to one wire alone is $B$, determine, for each configuration,

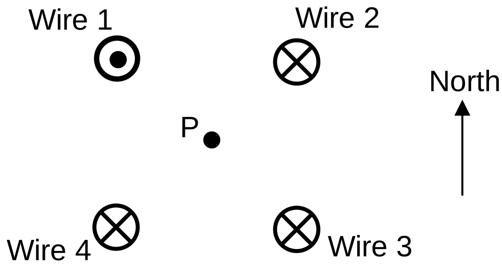
**(i)** the expression for the net magnetic flux density at P, and

**(ii)** state its corresponding direction.

**(a)**

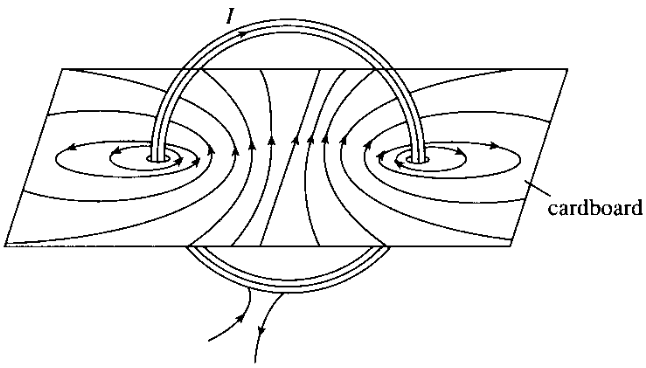



**(b)**




## (b) Flat circular coil

- <u>Magnetic flux pattern</u>




Take note of the reversal of the roles of the thumb and fingers as compared to applying it to a long straight wire.

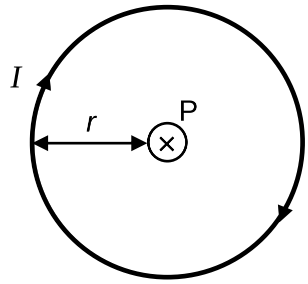
**Right Hand Grip Rule**
- Grip the wire using the right hand with the fingers pointing in the direction of the current.
- The thumb will point in the direction of the field <u>inside the coil</u>.

Additional Notes

Do NOT memorize this formula – it is provided in formula list

However it is necessary to remember that
(1) $B \propto I$
(2) $B \propto \frac{1}{r}$.

- Expression for magnetic flux density




The magnetic flux density at the centre, P, of a current-carrying flat circular coil, in a vacuum, of $N$ turns is:

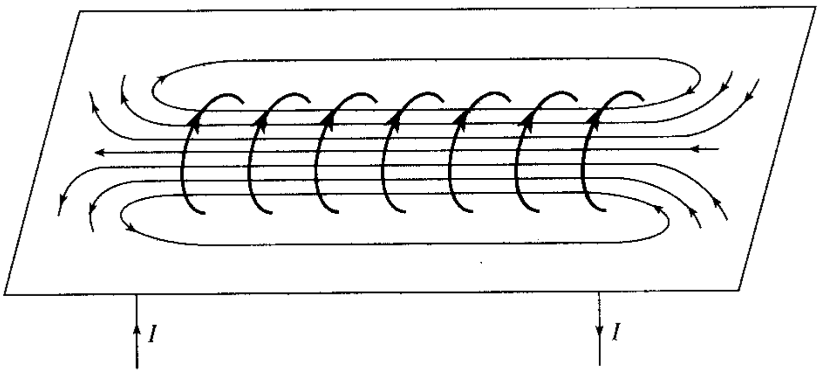
$B = \dfrac{\mu_0 NI}{2r}$ where $r$ = radius of the coil (m)

**(c) Solenoid**

Take note of the reversal of the roles of the thumb and fingers as compared to applying it to a long straight wire.

- Magnetic flux pattern
When a current flows in a solenoid, the magnetic field pattern produced is identical to that of a bar magnet. Hence the solenoid has poles just like a bar magnet.

To identify the pole at each end of the solenoid, recall that the field lines of a magnet exits from the north pole and enters into the south pole. Use the Right Hand Grip Rule to determine which of the 2 ends is a north pole.

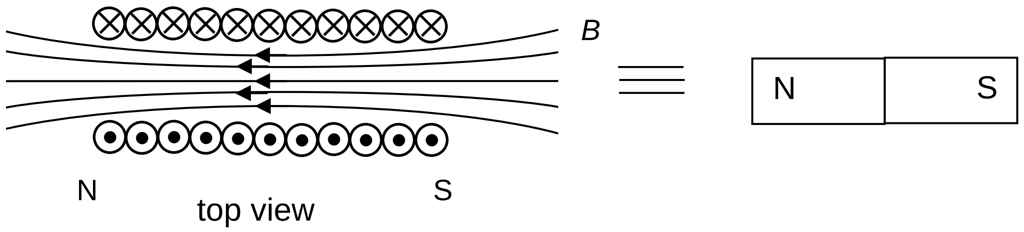



Within the solenoid, magnetic field lines are directed from South to North.

Outside the solenoid, magnetic field lines points from the out of the North pole and into the South pole.

**Right Hand Grip Rule**
- Grip the wire using the right hand with the fingers pointing in the direction of the current.
- The thumb point in the direction of the field inside the solenoid.

Additional Notes




- Expression for magnetic flux density

  At the centre of an infinitely long solenoid,

  $B = \mu_0 nI$ where $n$ is the number of turns per unit length

- Important note:

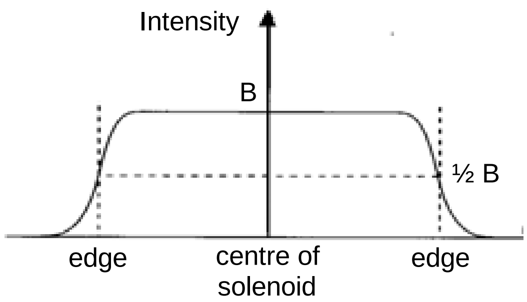
  - At the ends of the solenoid, the magnetic flux density is half of that at the centre of the solenoid.

  - The magnetic flux density is constant throughout the whole inner region, and over the whole cross-section (i.e. uniform flux density.)

$B_{inside} = \mu_o nI$

$B_{end} = ½\ B_{inside} = ½\ \mu_o nI$

n = no. of turns per unit length

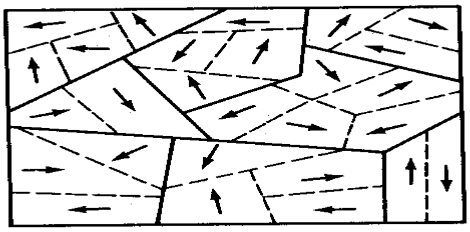



  - The magnitude of flux density can be greatly increased by the insertion of a ferromagnetic material such as iron, cobalt or nickel into its core.

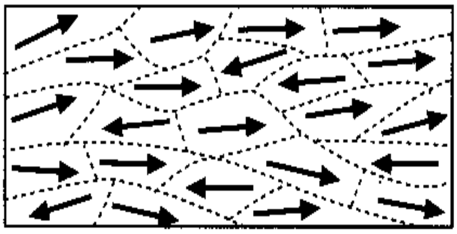
Do NOT memorize, but once again, it is useful to remember that
(1) $B \propto I$,
(2) $B \propto \frac{N}{l}$ or $n$

## C Effects of Ferrous Core on Magnetic Field of a Solenoid

- <u>Ferromagnetic material</u>:
  - characterized by numerous small magnetized regions called domains. Tiny atomic (elementary) magnets reside in each domain.
  - In the unmagnetised state, all the elementary magnets are arranged randomly, as shown in fig 16.13.
  - Material displays little or no magnetism.

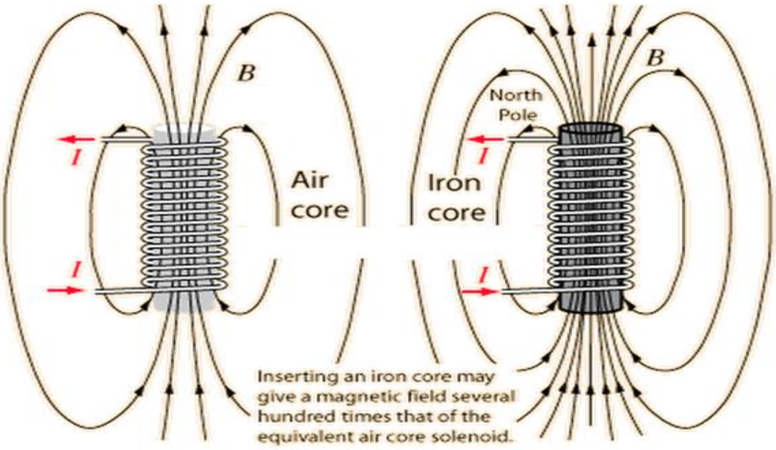


  - When subjected to an external magnetic field e.g. when placed within a solenoid, elementary magnets line up in the direction of the external field as shown in fig. 16.14.
  - Now that the elementary magnets are all parallel, they give rise to a stronger field i.e. the permeability of matter $\mu$ increases. Material is magnetized (i.e. displays magnetism).




  - Examples of ferromagnetic materials are iron, cobalt, nickel and alnico (an alloy of aluminium, nickel and cobalt).

- <u>Effects of Ferrous Core in Solenoid</u>:
  - By putting a ferrous (iron) core inside a solenoid, it has ability to <u>strengthen the magnetic flux density</u>. There are more field lines and the field lines are closer together as shown below.




For air core, $B_0 = \mu_0 n I$

For iron core, $B = \mu n I$

Since permeability of the iron $\mu$ is larger than $\mu_0$,

$B > B_0$

Additional Notes

- Large increase in magnetic flux density is due to the high degree of alignment of the magnetic domains in the ferromagnetic material. Once the solenoid is switched on, the previously unaligned domains “line up” and “contribute” to the field, hence magnetic flux density increases.

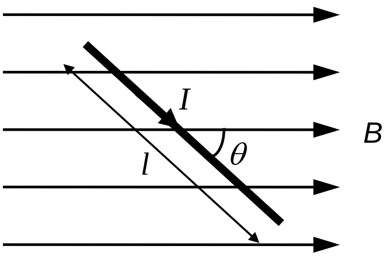
- Iron is a particularly good core for solenoids as it is easily magnetised and demagnetised. This means that when the current is switched off the iron does not stay magnetic. Iron is said to be magnetically soft.

- Applications:
  - In transformers where more will be covered under topic of *Alternating Current*.
  - In electromagnets where more will be covered under topic of *Electromagnetic Induction.*

Additional Notes

## D Force on a current-carrying conductor

- A metallic conductor carrying a current $I$ placed in a uniform magnetic field $B$ will experience a force.
- Only the component of the $B$-field perpendicular to the current $I$ will result in a force on the conductor




- Magnitude of the force
  The magnitude of this force was proven experimentally to be

(Note: $B$ is external magnetic flux density acting on current carrying conductor)

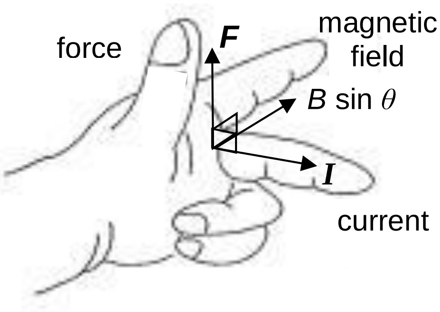
$$F = B_{\perp} I\, l$$
$$= (B \sin\theta)\, I\, l$$
$$= B\, I\, l \sin\theta$$

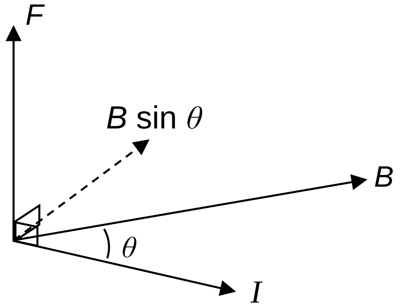
where $F$ = electromagnetic force (N);
$B_{\perp}$ = component of magnetic flux density (T) perpendicular to current;
$I$ = current in conductor (A);
$l$ = length of conductor (m); and
$\theta$ = the angle between the conductor and magnetic field.

If the conductor is parallel to the field lines, then $\boldsymbol{\theta}$ = 0°, and $\boldsymbol{F}$ = 0. (No magnetic force acts on the conductor).

If the conductor is at right angles to the field lines, then $\theta$ = 90°, and the magnetic force acting on the conductor would be maximum ($\boldsymbol{F} = \boldsymbol{B\, I}\, l$).

- Direction of the force
  The direction of the force on the conductor can be predicted by using Fleming's Left Hand Rule.



Additional Notes

- From $B = \dfrac{F}{IL\ sin\theta}$, the definition of magnetic flux density (B) and the Tesla (T) can be derived:

Magnetic flux density
The force acting per unit current per unit length on a conductor placed perpendicular to the magnetic field.

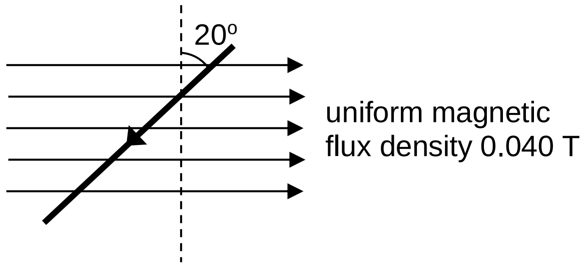
memorize

Tesla
The **tesla** (symbol: **T**) is the SI unit of magnetic flux density.
$T = N\ m^{-1}\ A^{-1} = Wb\ m^{-2}$

Wb is the symbol for the unit weber. You will learn more about it in the next topic: Electromagnetic Induction (for H2 only).

**Worked Example**

The given diagram shows a wire of length 3.0 cm, and carrying a current of 5.0 A, situated at an angle 20° to the vertical, in a uniform magnetic field of flux density 0.040 T.




What is the magnitude and direction of the force on the wire due to the field?

**Solution:**

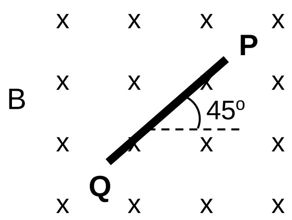
Force = $BIl \sin\theta$ *(where θ is the angle between the field and the wire)*
= (0.040)(5.0)(0.03)sin 70° = 0.0056 N

By Fleming's Left Hand Rule, the force is acting out of the plane of the paper.

Additional Notes

**Example 2**

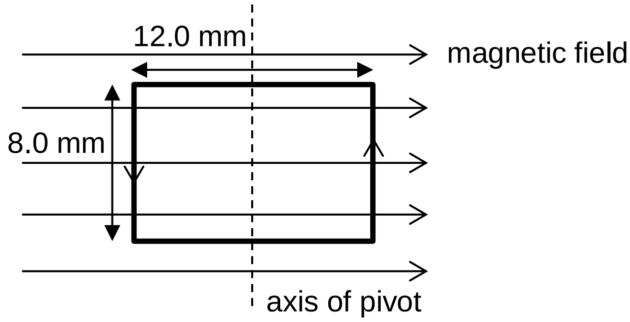
A conductor PQ of length 5.0 m carries a current of 2.0 A flowing from P to Q. The conductor is placed in a uniform magnetic field with flux density 0.05 T.




Determine the magnitude and direction of the force acting on the wire.

**Example 3**

A 20-turn rectangular coil of breadth 8.0 mm and length 12.0 mm is placed pivoted at the centre and placed in a uniform magnetic field of flux density 0.010 T such that the 2 longer sides of the coils are parallel to the field and the 2 shorter sides are perpendicular to the field as shown. A current of 5.0 mA is passed through the coil in an anticlockwise direction.

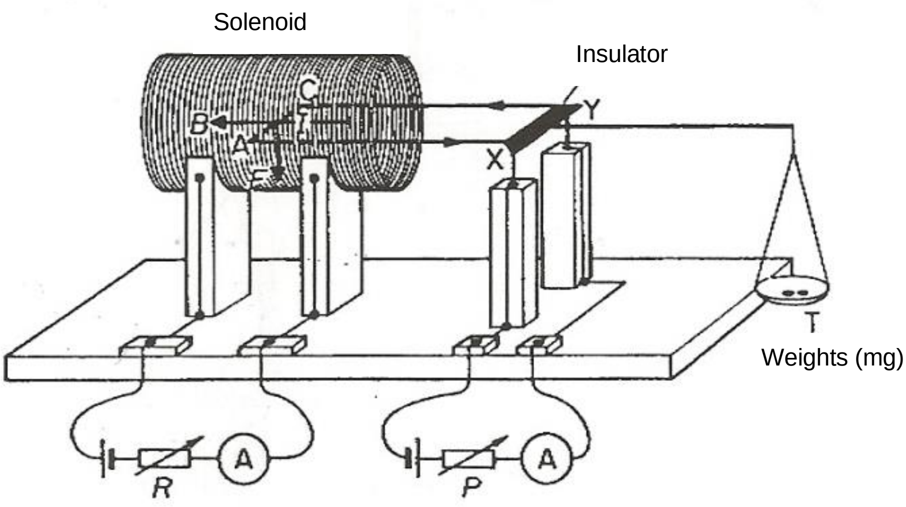



Determine the magnitude and direction of forces acting on each side of the coil.

Additional Notes

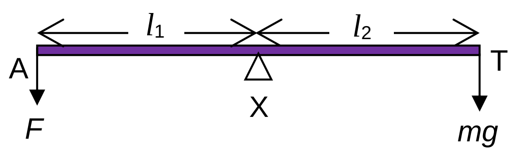
## Measurement of magnetic flux density using a current balance

- The current balance is used to measure magnetic flux density.




- The components:
  - Solenoid – to generate magnetic field;
  - Wire frame which has one side inserted in the middle of the magnetic field generated by the solenoid;
  - Probe AC – placed at the centre of the solenoid where the field is to be measured;
  - Weights – placed at T so as to balance the electromagnetic force *F* exerted on AC.

- When a current runs through the rectangular wire frame YCAX in the anticlockwise direction, a downward electromagnetic force *F* is exerted on the portion of the wire AC.




(side view)

- To balance this force, we can either vary the
  (a) length $l_2$; or
  (b) mass of the weight *m*.

- When the system is in rotational equilibrium, the Principle of Moments is applied about axis XY,

$$Fl_1 = mgl_2$$

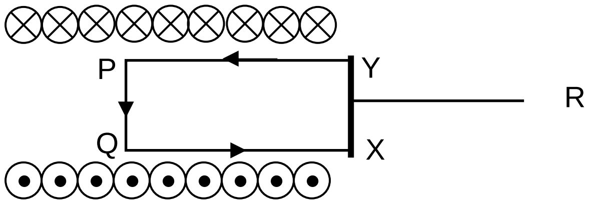
Since $F = BIl_{AC}$,

$$BIl_{AC}\,l_1 = mgl_2$$

$$B = \frac{mgl_2}{Il_{AC}l_1}$$

**Example 4**

A wire frame PQR is supported on two knife-edges X and Y so that the section PQXY of the frame lies within a solenoid as shown.



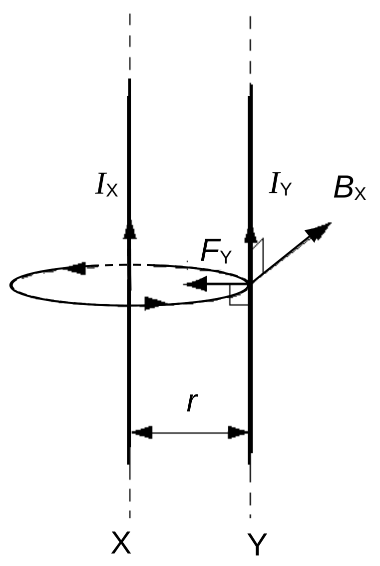
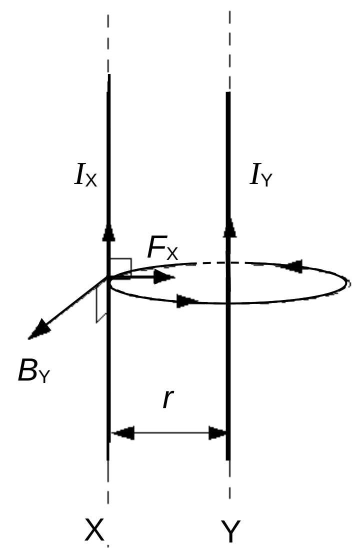

The magnetic flux density within the solenoid is 3.1 mT. The length of PQ is 5.0 cm and that of QX is 15.0 cm. When the current flowing in the frame is 3.5 A, a rider of mass 0.10 g is placed between the pivot XY and R to keep the frame horizontal.

Determine the position of the rider from the pivot XY.

Additional Notes

## E Force between current-carrying conductors

- Consider two wires X and Y carrying currents $I_X$ and $I_Y$ flowing upwards.

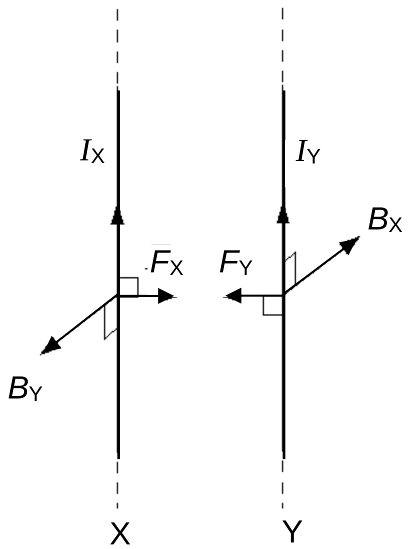
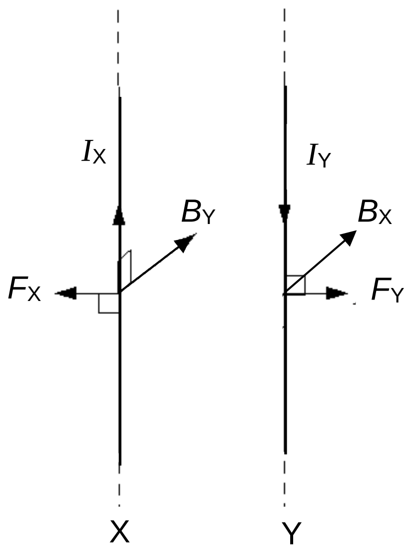
| Electromagnetic force on Y | Electromagnetic force on X |
|---|---|
|  |  |
| • Using the Right Hand Grip Rule for wire X, the magnetic field due to current $I_X$ at the position of wire Y is $B_X$, directed perpendicularly into the plane of the paper. | • Using the Right Hand Grip Rule for wire Y, the magnetic field due to current $I_Y$ at the position of wire X is $B_Y$, directed perpendicularly out of the plane of the paper. |
| • <u>Direction of force</u>: By Fleming's Left Hand Rule, the force acting on wire Y is $F_Y$, towards the left. | • <u>Direction of force</u>: By Fleming's Left Hand Rule, the force acting on wire X is $F_X$, towards the right. |
| • <u>Magnitude of force</u>: $F_Y = B_X I_Y l$<br>Since $B_X = \dfrac{\mu_0 I_X}{2\pi r}$<br>Hence $\dfrac{F_Y}{l} = \dfrac{\mu_0 I_X I_Y}{2\pi r}$ | • <u>Magnitude of force</u>: $F_X = B_Y I_X l$<br>Since $B_Y = \dfrac{\mu_0 I_Y}{2\pi r}$,<br>Hence $\dfrac{F_X}{l} = \dfrac{\mu_0 I_Y I_X}{2\pi r}$ |

Take note that the magnetic field created by the current in any wire <u>does not affect itself</u>.

When using Fleming's Left Hand Rule, the index finger indicates the direction of an <u>external magnetic field</u>.

Additional Notes

- Important note:
  - $\frac{F_X}{l}$ and $\frac{F_Y}{l}$ are of the same magnitude, but act in opposite directions, on different wires. They constitute an action-reaction pair (Newton's Third Law)
  - Despite the difference in the magnitudes of the currents in both wires, they experience the same magnitude of force per unit length.
  - For wires carrying currents in the same direction, they will be attracted to each other; for wires carrying currents in the opposite directions, they will be repelled from each other.

| Conductors carrying current in the same direction | Conductors carrying current in opposite direction |
| --- | --- |
| **Attraction**  | **Repulsion**  |





***Nature of Science***

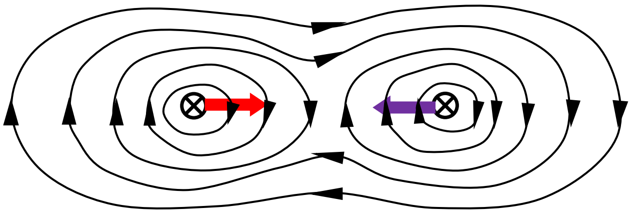
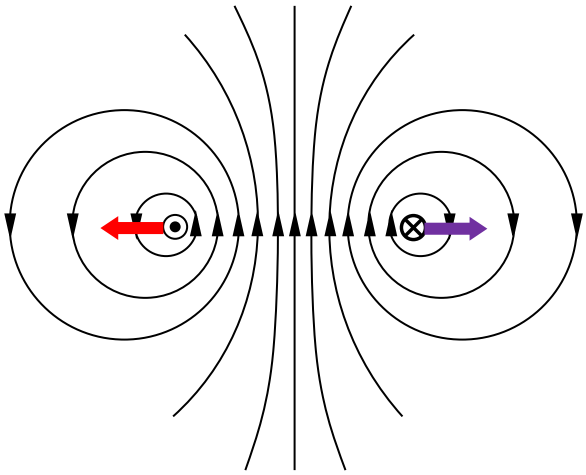
*The first person to discover evidence that electricity and magnetism are related phenomena was Danish scientist Hans Christian Ørsted. In the midst of a lecture on electricity, Ørsted noticed that a wire carrying current was able to deflect a compass needle. Furthering Ørsted's experimental work, French scientist André-Marie Ampère* ***showed that two parallel wires carrying electric currents attract or repel each other****, depending on whether the currents flow in the same or opposite directions, respectively. He* ***generalised the experimental results into physical laws****, one of which is in the same form you see at the bottom of page 17 – also known as the Ampere's force law.*

*This is an example how* ***science is an evidence-based, model-building enterprise concerned with the natural world.*** *From the evidence which are observations and experimental findings, Ampère was able to build a model in the form of physical law to describe the relationship or patterns in nature – how the force depends on the current through both the wires.*

*This example also shows that* ***creativity is found in all aspects of scientific research,*** *from coming up with a question, creating a research design, interpreting and making sense of findings.*

*In recognition of his contributions, the ampere was established as a standard unit of electrical measurement for current.*

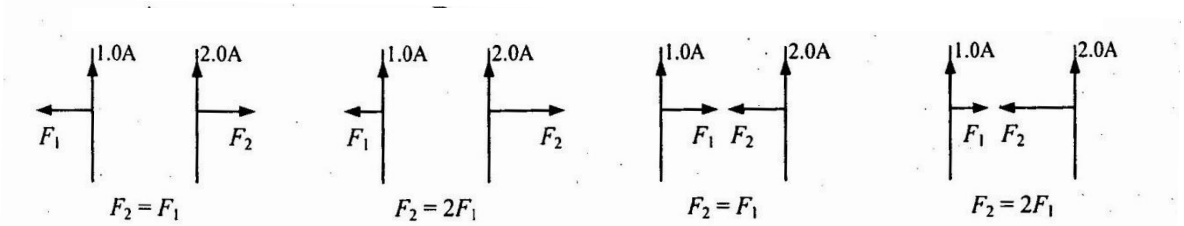
Additional Notes

| Configuration of wires | Resultant magnetic field | Conclusion |
|---|---|---|
| Currents in same direction (eg:into page) |  | Wires move towards each other |
| Currents in opposite directions |  | Wires move away from each other |

**Check your Understanding 2**

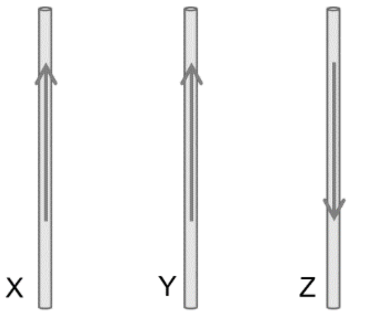
The diagram shows two long, straight, parallel wires carrying currents of 1.0 A and 2.0 A. The forces per unit length experienced by the wires are $F_1$ and $F_2$ respectively. Which diagram correctly shows the directions and relative magnitudes of $F_1$ and $F_2$?

A B C D

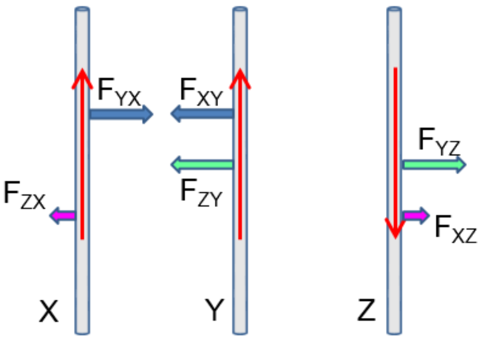


**<u>Worked Example</u>**

3 long, straight, parallel, equally spaced wires carry identical currents. Rank the magnitude of the force acting on each wire in descending order of magnitude.




**Solution:**



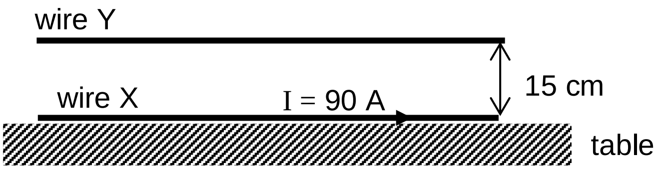

Based on the direction of the current in each wire, each wire will experience the electromagnetic forces as shown above.
Note: the further distance between wires X and Z accounts for a smaller electromagnetic force

Ranking: Y Z X (in descending order)

**Example 5**

Wire X, with a current of 90 A flowing from left to right, is placed on a table. Wire Y, with length 1.5 m and mass 1.0 g, is parallel to the wire X but lies 15 cm above it. Wire Y is held in suspension magnetically.

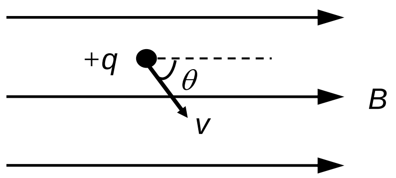
Determine the magnitude and direction of the current in wire Y.
(Given for a straight wire, $B = \mu_o I/2\pi r$)



Additional Notes

## F Force on a moving charge

- Consider a positive charge of magnitude $+q$ moving with speed $v$ at an angle $\theta$ in a uniform magnetic field $B$.




- Magnitude of force
  The force experienced by a current carrying conductor in a magnetic field is the resultant of the forces acting on the moving charges which constitute the current.

  Recall that $F = B_{\perp} I l$
  $F = BIl \sin\theta$

  Since $I = \dfrac{q}{t}$ , $F = \dfrac{Bql}{t} \sin\theta$

  Note that $v = \dfrac{l}{t}$, where $v$ is the velocity of the charge $q$,
  hence the force on charge $q$ can be simplified to

$$\boxed{F = Bqv \sin\theta}$$

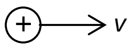
Recall:
For a charged particle to experience a magnetic force, it has to be moving (not parallel) to the field (pg 2)

- Direction of force
  The direction of the force on the charge can be predicted by using Fleming's Left Hand Rule.

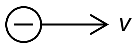
  The finger representing current will now be directed in the same direction as $v$ of the positive charge and opposite direction as $v$ of a negative charge.

  Hence, the direction of the force on the moving positive charge is out of the page.

Recall:




current to the right




current to the left

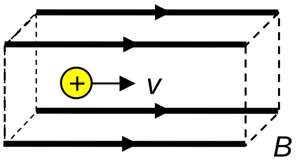
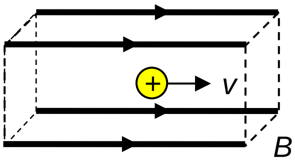
- Important note:

  - The electromagnetic force $F$ which the charged particle experiences in a magnetic field is a deflecting force. In contrast to electric and gravitational forces, this electromagnetic force does not act in the direction of the magnetic field.

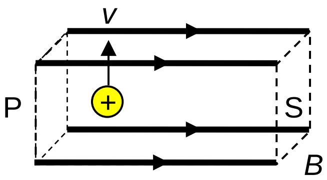
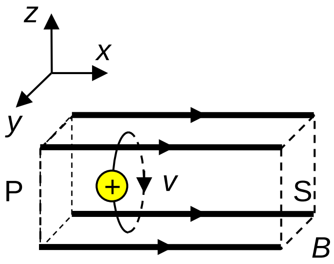
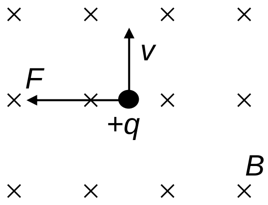
  - The electromagnetic force $F$ acts perpendicularly to the velocity $v$ of the moving charge $q$. Hence, it does no work on the charged particles (since WD = F × dist moved in direction of F) which implies that $F$ will only change the direction of $v$ and not its magnitude.

Additional Notes

**Possible paths taken by a moving charged particle (of mass *m* and carrying charge *Q*) in a magnetic field**

| | Initial Path | Subsequent Path | Explanation |
|---|---|---|---|
| ***Case 1: v* parallel to *B*** |  |  | • Since magnetic field is parallel to motion of particle, there is no electromagnetic force acting on the particle.<br>• Particle will continue to move with speed *v* in a straight line parallel to the magnetic field. |

Additional Notes

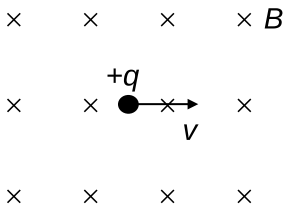
| | Initial Path | Subsequent Path | Explanation |
|---|---|---|---|
| ***Case2: v* perpendicular to *B* (ie: *v* is in the z plane)** |  |  | *viewed from side P*<br><br>• By Fleming's Left Hand Rule, the particle will experience an electromagnetic force that is perpendicular to the field and its movement.<br>• Since the force is always perpendicular to the direction of the motion of the particle, it will not change its speed but only the direction of its velocity. Hence, the particle is moving in a circular motion.<br>• Its acceleration and the force will be constantly pointing towards the centre of a circular path.<br>• Electromagentic force provides the centripetal force for circular motion.<br><br>$\boldsymbol{B} Q \boldsymbol{v} = \frac{m\boldsymbol{v}^2}{r}$<br><br>$m$ – mass of charge<br>$r$ – radius of circular motion<br><br>• Particle will move in a circular path in the *y-z* plane. |

Additional Notes

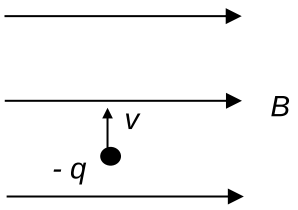
## Check Your Understanding 3

Indicate on each of the figures below, the subsequent paths taken by the charged particles moving with speed *v*.

**(a)**




**(b)**




**Worked Example**

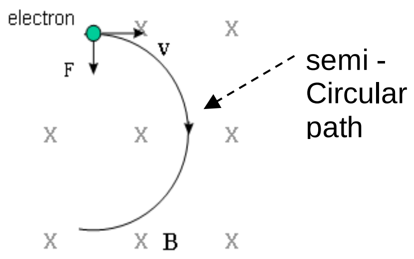
A uniform magnetic field with magnitude 1.20 mT, is directed vertically downward throughout a laboratory chamber. An electron with kinetic energy $8.48 \times 10^{-16}$ J enters the chamber, moving horizontally from west to east.

a) Draw a diagram to indicate the direction of the force and the motion of the electron in the field.

b) Determine the magnitude of the magnetic deflecting force that acts on the electron as it enters the chamber.

Note that once the electron exited the region of magnetic field, it should travel in a straight line at constant speed.

a)




b)

$$KE = \tfrac{1}{2} mv^2$$

$$8.48 \times 10^{-16} = \tfrac{1}{2} (9.11 \times 10^{-31})\, v^2$$

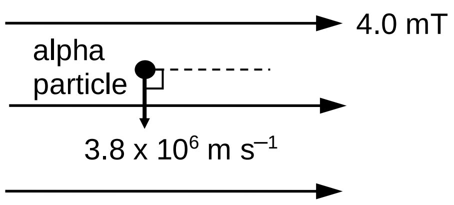
$$v = 4.315 \times 10^{7} \text{ m s}^{-1}$$

$$F = Bqv \sin 90^{\circ} = Bqv = 8.28 \times 10^{-15} \text{ N}$$

Additional Notes

**Example 6**

An alpha particle with a charge of 2*e* and a mass of $6.64 \times 10^{-27}$ kg is moving at a speed of $3.8 \times 10^{6}$ m s$^{-1}$, in a uniform magnetic field, perpendicular to the direction of the field. The flux density is 4.0 mT.




a) Determine the radius of the path of the alpha particle.

b) State and explain any differences in the period and angular velocity if the same alpha particle enters the same field with twice the speed.

Additional Notes

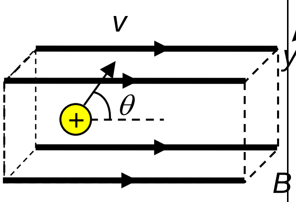
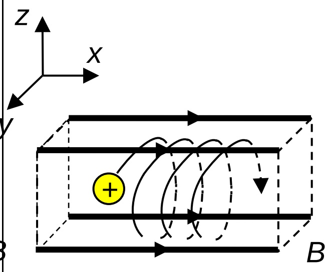
**Seeing links:** This is not the first time you had analysed motion by resolving the components of velocity.

*Recall:* Projectile Motion

x-direction: constant speed

y-direction: uniform accelerated motion towards the ground

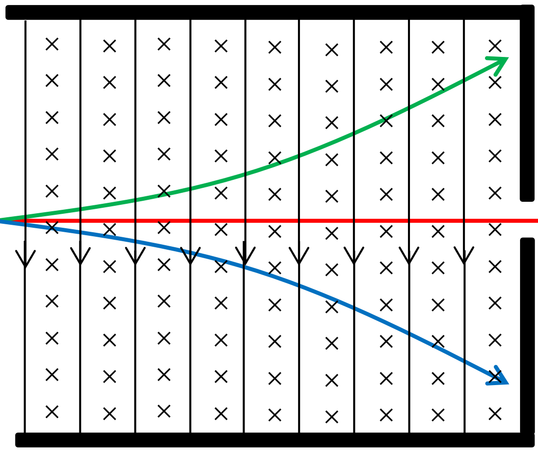
A helical path is **not** the same as a spiral path.

| | Initial Path | Subsequent Path | Explanation |
|---|---|---|---|
| **Case 3: *v* at an angle $\theta$ to *B*** *(ie: v is the x-z plane)* |  |  | • *v* can be resolved into its perpendicular components along:<br>➢ *x*-axis ($v\cos\theta$)<br>➢ *z*-axis ($v\sin\theta$)<br>• $v\cos\theta$ results in uniform velocity along the *x*-direction. (ie: case 1)<br>• $v\sin\theta$ results in circular motion in the *y*-*z* plane. (ie:case 2)<br>• The combined effect gives a <u>helical path</u>. |

Additional Notes

**Crossed (perpendicular) electric and magnetic fields**

- For **electrons** entering the crossed fields,




| Path | Direction of deflection | Forces experienced by charged particle | Speed of charged particle |
|---|---|---|---|
| 1 | undeflected | $F_B = F_E$ | Bqv = qE<br>**v = E/B** |
| 2 | downwards | $F_B > F_E$ | Bqv > qE<br>**v > E/B** |
| 3 | upwards | $F_B < F_E$ | Bqv < qE<br>**v < E/B** |

*E*: electric field strength
*B*: magnetic flux density

- The above principle applies to positive charged particles; the only difference being that the forces are in the opposite direction compared to that for negative charges.

- Application: By suitable selection of the ratio of $\frac{E}{B}$, charged particles of certain velocity may be collected from their undeflected paths. This is the basis on which a velocity selector works.

Additional Notes

## Check your Understanding 4

A beam of electrons enters an electric field at right angles to the field lines while a similar beam enters a magnetic field at right angles to the flux. The electrons in both beams will experience a force due to the fields.

Are the respective forces dependent on the speed of the electrons?

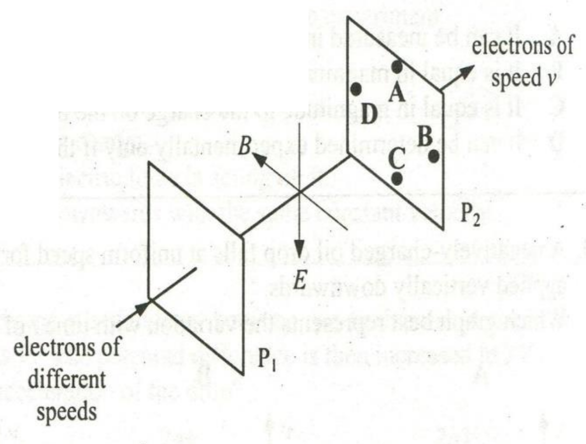
| | electric force | magnetic force |
|---|---|---|
| **A** | No | No |
| **B** | No | Yes |
| **C** | Yes | No |
| **D** | Yes | Yes |

## Example 7

A method of velocity selection of charged particles is illustrated in the given diagram. Plate $P_1$ has a hole through which a beam containing electrons of different speeds passes. Parallel to $P_1$ is plate $P_2$, also with a hole in it. A uniform magnetic field of flux density $B$ and a uniform electric field of intensity $E$ are applied at right angles to each other in the region between the plates as shown.

By adjusting the magnitudes of $B$ and $E$, only electrons with a particular speed $v$ will pass undeflected through the holes in $P_1$ and $P_2$.

State and explain at which of the points could the electrons of a speed greater than $v$ strike plate $P_2$.



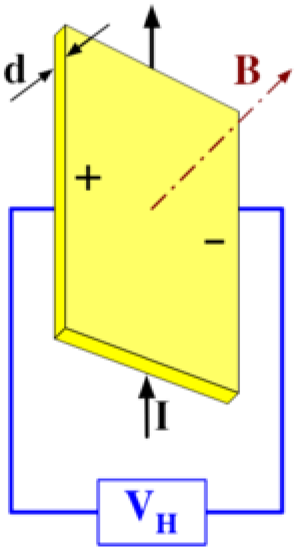
Additional Notes

**Appendix 1**

## Hall Effect

The Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current.

he Hall effect comes about due to the nature of the current in a conductor. Current consists of the movement of many small charge carriers, typically electrons, holes, ions or all three. Moving charges experience a force, called the Lorentz force, when a magnetic field is present that is perpendicular to their motion.[2] When such a magnetic field is absent, the charges follow approximately straight, 'line of sight' paths between collisions with impurities, phonons, etc. However, when a perpendicular magnetic field is applied, their paths between collisions are curved so that moving charges accumulate on one face of the material. This leaves equal and opposite charges exposed on the other face, where there is a scarcity of mobile charges. The result is an asymmetric distribution of charge density across the Hall element that is perpendicular to both the 'line of sight' path and the applied magnetic field. The separation of charge establishes an electric field that opposes the migration of further charge, so a steady electrical potential builds up for as long as the charge is flowing.




As a result, the Hall effect is very useful as a means to measure either the carrier density or the magnetic field.

When a current-carrying semiconductor is kept in a magnetic field, the charge carriers of the semiconductor experience a force in a direction perpendicular to both the magnetic field and the current. At equilibrium, a voltage appears at the semiconductor edges.

One very important feature of the Hall effect is that it differentiates between positive charges moving in one direction and negative charges moving in the opposite. The Hall effect offered the first real proof that electric currents in metals are carried by moving electrons, not by protons. The Hall effect also showed that in some substances (especially p-type semiconductors), it is more appropriate to think of the current as positive "holes" moving rather than negative electrons. A common source of confusion with the Hall Effect is that holes moving to the left are really electrons moving to the right.

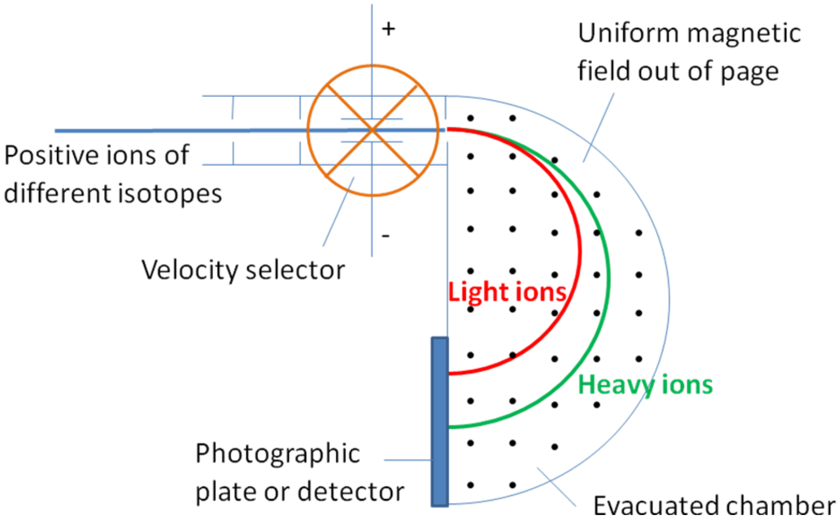
**Appendix 2**

**Mass Spectrometer**

*Joseph John Thomson* investigated the deflection of beams of positive ions using parallel electric and magnetic fields to deflect positive ions from a gaseous element. His results demonstrated the existence of isotopes, atoms of an element with different atomic masses.

*Thomson's* research on positive ions was taken further and developed into the mass spectrometer which is used to measure atomic masses to 8 significant figures!

The Bainbridge mass spectrometer uses a velocity selector to restrict the speed of the ions in the beam to a narrow range. The ion beam is then deflected by a uniform magnetic field on a circular path.




**Bainbridge mass spectrometer**

The deflection of each type of ion can then be calculated by equating the electromagnetic force to the centripetal force acting on the ions.

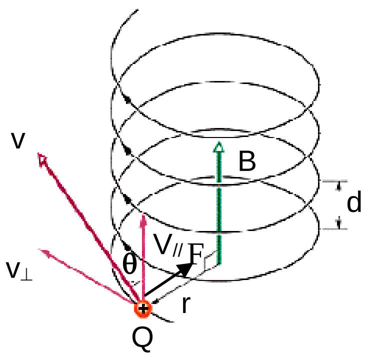
$$Bqv = mv^2 / r$$
$$m = Bqr / v$$

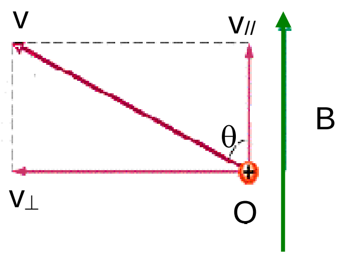
Thus ions of different isotopes carrying equal charge at the same speed are deflected on slightly different paths. They are therefore detected at slightly different positions. A modern mass spectrometer uses an electronic detector linked to a computer.

Additional Notes

**Appendix 3**

**Electron injected at some other angle (i.e. not 90°) into a magnetic field – Helical paths**




Suppose an electron is projected at an angle (other than 90°) into a magnetic field with a velocity *v*, as shown in the above diagram. The velocity *v* can be resolved into 2 components, one parallel to the magnetic field $v_{//}$ and one perpendicular to the field $v_\perp$.

$$v_\perp = v \sin\theta \qquad v_{//} = v\cos\theta$$

The perpendicular component of velocity $v_\perp$, being perpendicular to the magnetic field, will result in a force *F* acting on the electron. The direction of this force *F* is always perpendicular to $v_\perp$ (from Fleming's Left Hand Rule) and it will cause the electron to move in a circular motion.

$$\text{Using } F = B\,q\,v_\perp = \frac{mv_\perp^{\ 2}}{r}$$

$$\therefore\ r = \frac{mv_\perp}{Be} = \frac{mv\sin\theta}{Be}$$

If *T* is the period of making a complete revolution, then

$$T = 2\pi\frac{m}{Be} \quad \text{------------} \quad (1)$$

At the same time as $v_\perp$ causes the circular motion of the electron, $v_{//}$ causes the forward motion of the electron (in the particular case shown in the diagram, the forward motion is upwards).

$$d = v_{//}\,T = (v\cos\theta)\,(T) \quad \text{where T can be found from eqn (1)}$$

The combined effects of both paths results in the helical path as shown in the above diagram.

**Key Formulae**

| Term | Equation | Where |
|---|---|---|
| Force on a current-carrying conductor, $F$ | $F = B_\perp I\, l$ | $B_\perp$ = component of magnetic flux density (T) perpendicular to current;<br>$I$ = current flowing through the conductor<br>$l$ = length of conductor |
| Force on a charge moving in a magnetic field, $F$ | $F = B_\perp q\, v$ | $B_\perp$ = component of magnetic flux density (T) perpendicular to motion of charged particle;<br>$q$ = amount of charge on the charge carrier<br>$v$ = velocity of charge |
| Force on a charged particle moving in circular motion in a magnetic field, $F$ | $F = B_\perp q\, v$<br>$= \dfrac{mv^2}{r} = mr\omega^2$ | $B_\perp$ = component of magnetic flux density (T) perpendicular to motion of charged particle<br>$q$ = amount of charge on the charged carrier<br>$v$ = velocity of charge<br>$m$ = mass of charged particle<br>$r$ = radius of circular path<br>$\omega$ = angular velocity of charged particle |
| Force on a charged particle passing through <u>undeflected</u> in a velocity selector, $F$ | $F = Bqv = qE$ | $B$ = magnetic flux density<br>$q$ = amount of charge on the charged carrier<br>$v$ = velocity of charge<br>$E$ = electric field strength |

Additional Notes

## Glossary

| | |
|---|---|
| **Magnetic field** | A region of space where a magnetic pole, a current-carrying conductor or a <u>moving</u> charged particle will experience a force. |
| **Magnetic flux density (*B*)** | The force acting <u>per unit current</u> <u>per unit length</u> on a conductor placed <u>perpendicular</u> to the magnetic field. |

## Electromagnetism

- **Force on a current-carrying conductor**
  - magnitude: $F = B_{\perp}IL$
  - direction: Flemings Left-hand Rule
  - Current Balance Experiment
- **Force on a moving charge**
  - magnitude: $F = B_{\perp}Qv$
  - direction: Flemings Left-hand Rule
  - Analyse the deflection of a moving charge by uniform Electric Field and uniform Magnetic Field
- **Magnetic Fields due to currents**
  - Right-hand Grip Rule
    - Long straight wire
    - Flat circular coil
    - Long solenoid
- **Force between current-carrying conductors**
  - Analyse the forces due to magnetic fields of current-carrying conductors