Additional Notes

ANDERSON SERANGOON JUNIOR COLLEGE
PHYSICS 9749

Name: ________________________________ **Class: 24 /** ____

Topic 20: Nuclear Physics

Content:

A The nucleus, Isotopes
B Mass defect and nuclear binding energy
C Nuclear processes
D Radioactive decay
E Uses of radioactivity and biological effects of radiation

Lect	Concepts	Qns	Duration
1	A.1 – B.2	CYU 1–3, Ex 1	31 min
2	B.3 – C.4	CYU 4, Ex 2	32 min
3	C.5 – C.6	CYU 5, Ex 3–5	21 min
4	D.1 – D.4	Ex 6–7	32 min
5	D.5 – D.7	–	29 min
6	E.1 – E.2	Ex 8–10	28 min

Learning Outcomes:

Candidates should be able to:

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus
(b) distinguish between nucleon number (mass number) and proton number (atomic number)
(c) show an understanding that an element can exist in various isotopic forms each with a different number of neutrons in the nucleus
(d) use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$
(e) state and apply to problem solving the concept that nucleon number, charge and mass-energy are all conserved in nuclear processes
(f) show an understanding of the concept of mass defect
(g) recall and apply the equivalence between energy and mass as represented by $E = mc^2$ to solve problems
(h) show an understanding of the concept of nuclear binding energy and its relation to mass defect
(i) sketch the variation of binding energy per nucleon with nucleon number
(j) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission
(k) show an understanding of the spontaneous and random nature of nuclear decay
(l) infer the random nature of radioactive decay from the fluctuations in count rate
(m) show an understanding of the origin and significance of background radiation
(n) show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required.)
(o) show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required.)
(p) define the terms activity and decay constant and recall and solve problems using $A = \lambda N$
(q) infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0 \exp(-\lambda t)$ where x could represent activity, number of undecayed particles or received count rate
(r) define and use half-life as the time taken for a quantity x to reduce to half its initial value
(s) solve problems using the relation $\lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$
(t) discuss qualitatively the effects, both direct and indirect, of ionising radiation on living tissues and cells.

History of the Atom

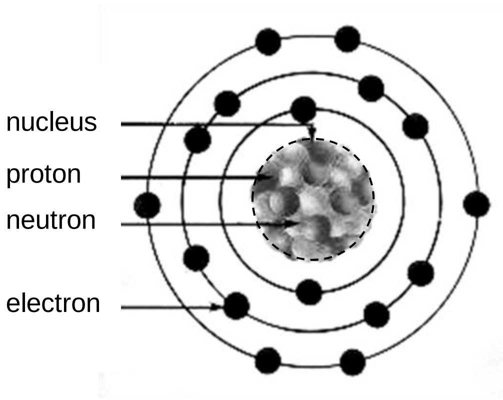
Discovery of Protons

- In 1919, using energetic α-particles to bombard nitrogen atoms, Rutherford found that protons were ejected after the collision. Protons were found to be positively charged and carried a charge of +e.
- It is concluded that
  - the positive charge in the nucleus is due to the presence of protons in the nucleus.
  - the number of protons is equal to the number of electrons in a neutral atom.
  - the mass of a proton is approximately 1840 times the mass of an electron.

Discovery of Neutrons

- In 1932, James Chadwick discovered neutrons inside the nucleus. Using α-particles to bombard beryllium, Chadwick found that neutrons were ejected in the collision.
- It is also discovered that:
  - neutrons carry no charge.
  - the mass of a neutron is nearly equal to the mass of a proton.

Simple Model of an Atom



Structure of Nucleus

An atom consists of a positively charged nucleus which makes up most of the atom's mass. The nucleus consists of protons which are positively charged and neutrons which carry no charge. The mass of a proton is almost equal to the mass of a neutron.

Strong Nuclear Force

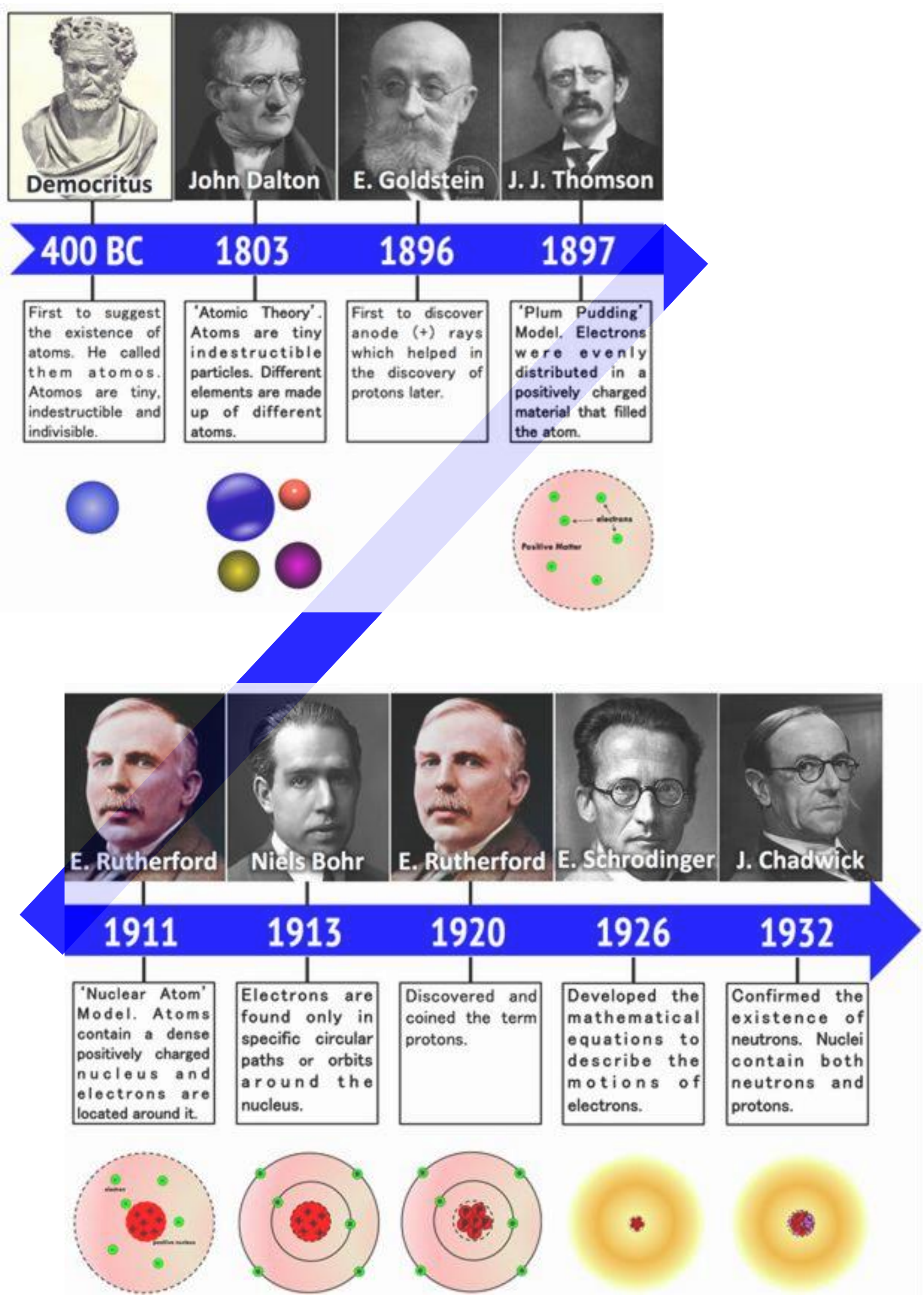
Inside the nucleus, the protons repel each other due to electrostatic repulsion of like charges. For the nucleus to be stable, there must exist other forces between the nucleons. These are called the strong nuclear forces which has a very short range. They provide a net attractive force greater than any repulsive electric forces when the nucleons are very close together.

Additional Notes

Electrons around the Nucleus

Electrons are considered to be orbiting in circular motion around the nucleus at relatively large distances away. The electrostatic forces of attraction between opposite charges (protons and electrons) provides the required centripetal force for the circular motion.

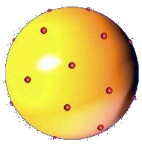
In the history of the atom, the concept of the atom was revisited and elaborated upon by many scientists and philosophers.



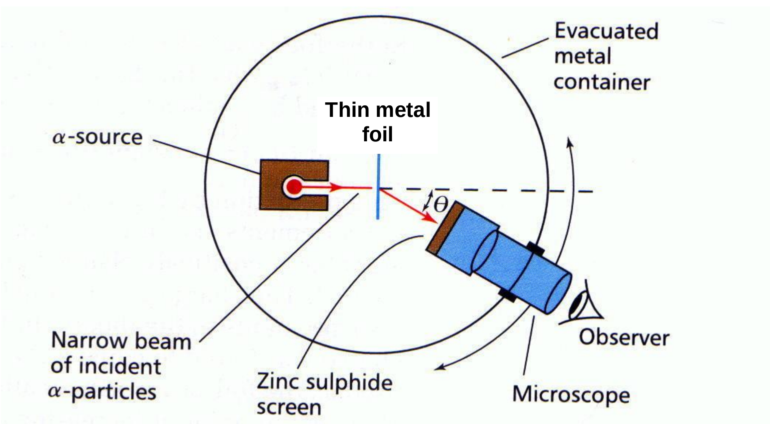
A The Nucleus, Isotopes

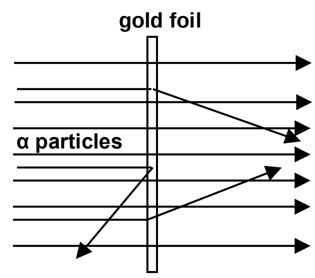
A.1 Alpha-particle Scattering Experiment

The 'plum pudding' model:



- In the "plum-pudding" model, first proposed by J. J. Thomson in 1904, the atom is composed of electrons surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "plums" surrounded by positively charged "pudding".
- In 1909, to test the "plum-pudding" model of the atom, Rutherford suggested the alpha-scattering experiment. Two of his students, Geiger & Marsden, carried out the experiment.
- Experimental Setup:





Alpha particle deflections by the atoms in a gold foil

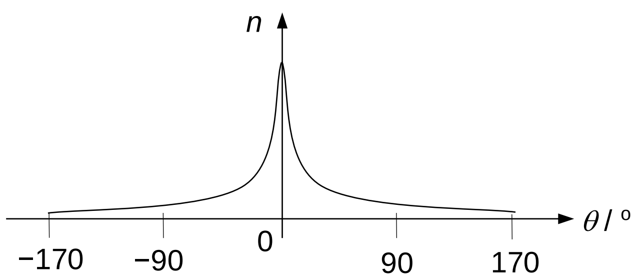
A narrow beam of α-particles is to ensure there is little divergence in the deflected beam to increase precision of measurement.

A thin gold foil is used so that the α-particles are scattered only once. It also ensures that the α-particles are not absorbed by the foil.

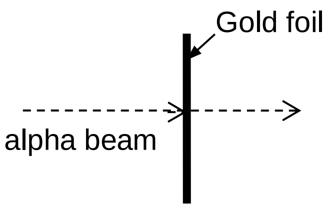
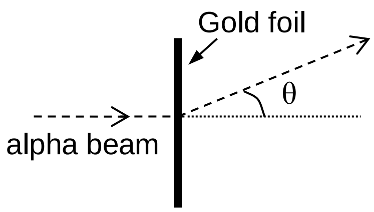
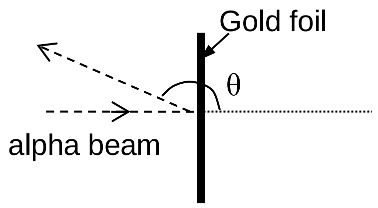
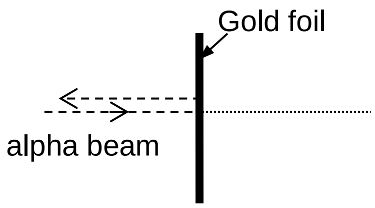
Scintillation is the process by which ionisation produced by charged particles excites a material and light is emitted by the de-excitation.

  - ➢ a narrow beam of positively charged alpha (α) particles (helium nuclei) were directed from a radioactive source onto a thin gold foil (~ 1 μm thick) in vacuum.
  - ➢ a zinc sulphide screen (fluorescent material) is mounted such that it could be set at different angles to the gold foil.
  - ➢ a scintillation is observed on the zinc sulphide screen when an α-particle strikes the screen. It could be deduced that an α–particle had been deflected from the gold foil to hit the zinc sulphide screen at that angle.
  - ➢ the entire apparatus was set up in a vacuum chamber to ensure that deflection of α-particles is entirely due to its interaction with gold atoms and not with the air molecules in the chamber.

- Results of alpha-scattering experiment:
  - ➢ The number of scintillations (n) observed at each scattering angle (θ) is summarized in the following graph:



Additional Notes

Observations	Interpretation
(a) Majority of the scintillations were observed at a scattering angle of around 0˚ to the direct path.  This indicated that a vast majority of the alpha particles were able to pass through the gold foil with little or no deflection.	Much of the atom is made up of empty space, hence the nucleus has a very small volume compared to the atom.
(b) A small proportion of scintillations were observed at an angle of more than about 10 ° to the direct path.  	The centre (or nucleus) of an atom is charged. Alpha particles, which are also charged, passing close to the nucleus will experience a repulsive force causing them to deviate.
(c) An extremely small proportion of scintillations were observed at a scattering angle of more than 90˚. Some of these particles were even detected to reflect back on the path they came from (back scattered). 	The alpha particles must have collided with comparatively massive particles to make an about-turn. The fact that so few particles did so confirms that the nucleus is very small, and that most of the atom is empty space.
From the results above, it can be concluded that 1. there is a very dense positively-charged centre core in the atom (nucleus). 2. the nucleus must be very small compared to the atom.	

Only alpha particles that pass very close to the nucleus, almost striking it head-on, will experience large enough repulsive forces to cause them to deviate through angles greater than 90 °.

Note that massive in this case means most of the mass is concentrated into a small region.

You are expected to explain how the Rutherford's model came about based on the observations of Alpha scattering experiment.

A.2 Rutherford's Model of the Atom

- Every atom has a central core, called the nucleus, which carries all the positive charge and almost all the mass of the atom. The electrons surround the nucleus.
- The atom is made up of mostly empty space; the nucleus occupies only a small fraction of the atom's total volume.

Typical values:
diameter of nucleus ≈ 10^{-15} m

diameter of atom ≈ 10^{-10} m

diameter of electron < 10^{-18} m

- Further experiments show that the diameter of the atom is about 10^{-10} m whereas the nucleus is about 10^{-15} m

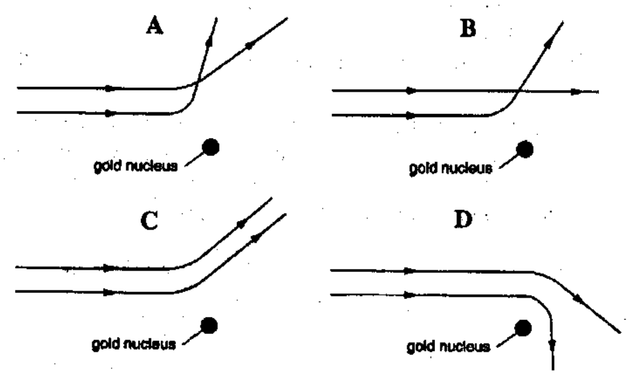
Check Your Understanding 1 (N97/I/29)

In an experiment to investigate the nature of the atom, a very thin gold film was bombarded with α-particles. What pattern of deflection of the α-particles was observed?

A. A few α-particles were deflected through angles greater than a right angle.
B. All α-particles were deflected from their original path.
C. Most α-particles were deflected through angles greater than a right angle.
D. No α-particles was deflected through an angle greater than a right angle. **Ans:** ____

Check Your Understanding 2 (N00/I/29)

Two alpha particles with equal energies are fired towards the nucleus of a gold atom. Which diagram best represents their paths (in the plane of the paper)?



Ans: ____

A.3 Terms and Notations for Representation of Nuclides

- The number of protons in the nucleus of an atom is called the proton number or the atomic number. It is denoted by the symbol *Z*.

- Nuclear particles, both protons and neutrons in a nucleus, are called **nucleons**.

Need to differentiate between nuclide, nucleus, nuclei and nucleons.

- The number of nucleons (protons and neutrons) in the nucleus of an atom is called the nucleon number or the mass number of the atom.
  It is denoted by the symbol *A*.

- Nuclide is a particular species (type) of nucleus that is specified by its proton number and neutron number.
  In symbol form, a nuclide can be represented by

$${}^{A}_{Z}\mathbf{X}$$

A: nucleon number or mass number
Z: proton number or atomic number
X: element

  - neutron can be written as: ${}^{1}_{0}n$ since it has *A* = 1 and zero charge
  - proton can be written as: ${}^{1}_{1}p$ or ${}^{1}_{1}H$ since the nucleus of hydrogen has only 1 proton
  - electron can be written as: ${}^{0}_{-1}e$ since there are no nucleons and has a negative charge

Check Your Understanding 3

$^{65}_{29}\text{Cu}$

Which statement is correct?

A. It is a nucleus with 65 nuclides, 29 protons, 36 neutrons
B. It is a nucleus with 65 nucleons, 36 protons, 29 neutrons
C. It is a nuclide with 65 nucleons, 36 protons, 29 neutrons
D. It is a nuclide with 65 nucleons, 29 protons, 36 neutrons **Ans:** ____

A.4 Isotopes

- Isotopes are atoms of the same element, having the same number of protons but different numbers of neutrons in their nuclei. They have the same atomic number but different mass number.

- Isotopes exhibit similar chemical properties as they have the same number of electrons but exhibit different physical properties as their masses are different.

- Examples of isotopes:

  - chlorine gas with 2 isotopes: $^{37}_{17}\text{Cl}$ and $^{35}_{17}\text{Cl}$

  - Hydrogen with 3 isotopes: $^{1}_{1}\text{H}$ with 1 proton
    $^{2}_{1}\text{H}$ (deuterium) with 1 proton and 1 neutron
    $^{3}_{1}\text{H}$ (tritium) with 1 proton and 2 neutrons

A.5 Atomic Mass Unit

- Masses at an atomic level are usually expressed in the unified atomic mass units, u.

- The unified atomic mass unit, u, is unit of mass, the size of which is equal to one-twelfth of the mass of a carbon−12 atom.

$$1\ u = \frac{1}{12} \text{ of the mass of a carbon−12 atom} = 1.66 \times 10^{-27}\ \text{kg}$$

- The relative atomic mass, u_r, of an atom is defined as the ratio of the mass of the atom to the unified atomic mass unit.

$$u_r = \frac{\text{mass of atom}}{u}$$

- Table 1 shows the mass and charge of proton, neutron and electron

Table 1: mass and charge of proton, neutron and electron

	mass	charge
proton	1.0073 u	+ e
neutron	1.0087 u	0
electron	0.00055 u	− e

($e = 1.60 \times 10^{-19}$ C)

Additional Notes

B Mass Defect and Nuclear Binding Energy

B.1 Einstein's Mass-Energy Equivalence

This means that mass can be converted to an equivalent amount of energy and vice versa.

- Einstein's mass−energy relation shows that mass and energy are interchangeable or equivalent, given by $E = mc^2$

  where E : energy (J)
  m : mass (kg)
  c : speed of light in vacuum (3×10^8 m s^{-1})

- A consequence of this mass−energy relation is that a small change in mass will bring about a large change in energy (due to the factor of c^2).

  $\Delta E = \Delta mc^2$ where ΔE : energy absorbed / released (J)
  Δm : mass difference (kg)

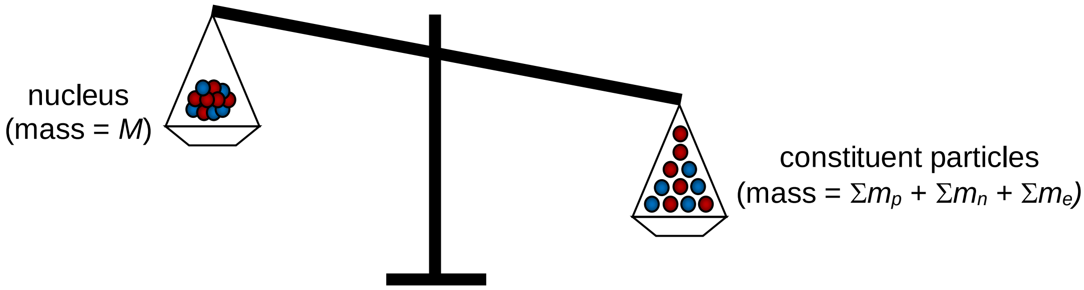
- Instead of conservation of energy, in nuclear processes, we have **conservation of mass−energy** where the total amount of energy, including energy that is equivalent to mass ($E = mc^2$), remains constant.

- Sometimes when expressing the amount of energy converted, scientist would make use of the units MeV instead of J.

  $1.0 \text{ MeV} = 1.0 \times 10^6 \times 1.6 \times 10^{-19}$
  $= 1.6 \times 10^{-13}$ J

B.2 Mass Defect

- The total mass of a stable atom or nucleus is always less than the sum of the masses of its constituent particles.



Hence, the difference in mass between the mass of the constituent particles of an atom and the (smaller) mass of the whole atom is known as **mass defect**.

Mass of electrons is usually neglected because $m_e \ll m_p$ and m_n.

mass defect, $\Delta m = \Sigma m_p + \Sigma m_n + \Sigma m_e - M$

Additional Notes

Example 1

Helium atom ${}^{4}_{2}He$ has 2 protons, 2 neutrons and 2 electrons. The mass of a ${}^{4}_{2}He$ atom is found to be 4.00260 *u*. What is the mass defect of ${}^{4}_{2}He$?
(refer to Table 1 on page 7 for the mass of a proton, a neutron and an electron)

If the question gives information on mass of electrons, it is an indication that the electron mass should be considered in computation.

Solution:

B.3 Binding Energy

- The mass of the nucleus is slightly less than the total mass of its individual nucleons. By mass-energy relation, energy is released / produced when the nucleus is formed from its individual nucleons.

The mass loss when individual constituent nucleons combine to form nucleus is transformed into other forms of energy such as KE or radiation.

- Similarly, by mass-energy relation, energy is required / supplied to split a nucleus into its individual nucleons. Hence,

> The **Nuclear Binding Energy** is defined as the energy **released** when the nucleus is formed from its constituent particles (protons and neutrons). It is the energy equivalent of the mass defect of a nucleus. **OR**
>
> The **Nuclear Binding Energy** can be defined as the energy **required** to separate the nucleus into individual protons and neutrons. It is the energy equivalent of the mass defect of a nucleus.

- Binding energy can be calculated as follows:

$$\text{Binding energy} = \Delta m \times c^2$$

- Binding energy is expressed in eV (electron−volt) where 1 eV = 1.6×10^{-19} J

Check Your Understanding 4 (N96/I/30)

A nucleus has a nucleon number *A*, a proton number Z, and a binding energy *B.* The masses of the neutron and proton are m_n and m_P respectively, and *c* is the speed of light. The mass of the nucleus is given by the expression

A $(A - Z)\,m_n + Zm_p - B/c^2$

B $(A + Z)\,m_n + Zm_p + B/c^2$

C $Am_n + Zm_p - B/c^2$

D $Am_n + Zm_p + B/c^2$

Ans: ____

Example 2

What is the binding energy of the nuclide $^{4}_{2}$He? (use mass-defect from Example 1 to solve)

Solution:

B.4 Binding Energy Per Nucleon Number

Nuclear Binding Energy per nucleon is the average energy released per nucleon when a nucleus is formed from its constituent particles (protons and neutrons). It is obtained by dividing the binding energy by the number of nucleons in the nucleus.

OR

Nuclear Binding Energy per nucleon is the average energy required per nucleon to break down a nucleus into its constituent particles (protons and neutrons). It is obtained by dividing the binding energy by the number of nucleons in the nucleus.

- The binding energy per nucleon of a nucleus is a measure of the nucleus stability. The higher the binding energy per nucleon, the more stable is the nucleus.

Worked Example

What is the binding energy per nucleon of $^{4}_{2}$He? (use your answer to example 2 to solve)

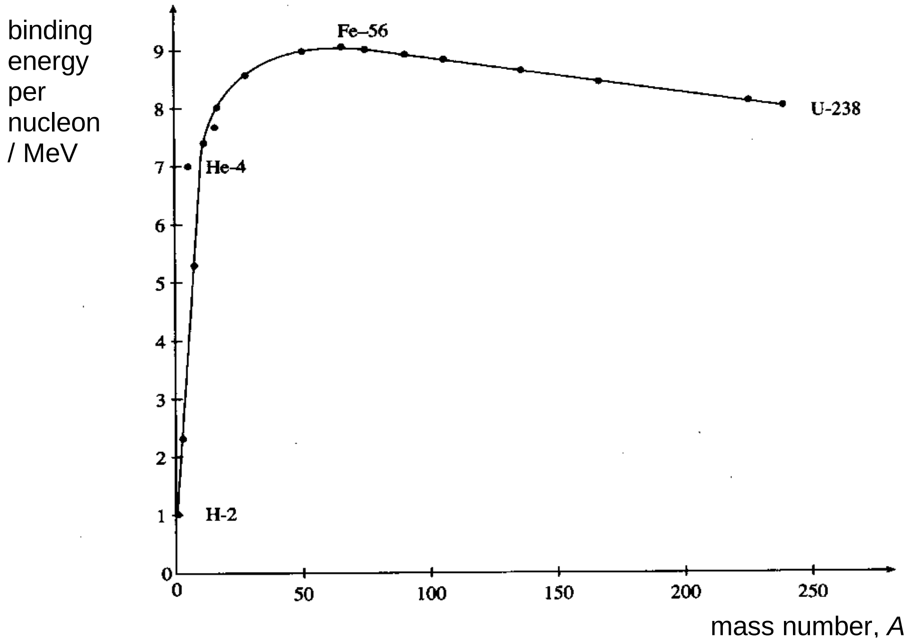
Solution:

The binding energy per nucleon of $^{4}_{2}$He = $\frac{28.479}{4}$ = 7.12 MeV

Additional Notes

B.5 Binding Energy Curve

- The binding energy curve is a plot of binding energy per nucleon against mass number.



- The most stable nuclides are those with the highest binding energy per nucleon which occurs at approximately **8.8 MeV**. Iron-56 ^{56}Fe is one of these stable nuclides. Nuclei on either side of the maximum are less stable.

C Nuclear Processes

C.1 Introduction

- A nuclear process (nuclear reaction) is one which causes nuclei to undergo changes.
- Examples of nuclear processes include:

  1. Nuclear fission: a heavy nucleus splits into two lighter nuclei of similar sizes. This reaction does not occur in nature naturally, the process is usually man-made.

  2. Nuclear fusion: light nuclei fuse to form a heavier nucleus with release of energy. This reaction occurs at very high temperature e.g. in the sun.

  3. Radioactive decay: a big unstable nucleus emits an alpha, beta or gamma photons to form daughter nucleus which is more stable. This reaction occurs in nature.

C.2 Equations to represent nuclear processes

'Reactants' refer to terms on the left hand side.

'Products' refer to terms on the right hand side.

- A nuclear process can be represented by a nuclear equation as shown below:

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{139}_{57}\text{La} + {}^{94}_{35}\text{Br} + 3\,{}^{1}_{0}\text{n}$$

indicating clearly the nucleon number, proton number, chemical symbols, products, reactants and the number of each nuclide/particles involved in each process.

- Below are examples of nuclear equations to represent nuclear processes (reactions).

For more info on the discovery of proton:

➢ Nuclear reaction leads to the discovery of proton
Bombardment of nitrogen with α−particles
(first nuclear transformation to be observed in Rutherford's Lab in 1919)

$$^{14}_{7}\text{N} + {}^{4}_{2}\text{He} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

For more info on the discovery of neutron:

➢ Nuclear reaction leads to the discovery of neutron
Bombardment of beryllium with α−particles
(carried out by Chadwick in 1932 which led to the discovery of neutrons)

$$^{9}_{4}\text{Be} + {}^{4}_{2}\text{He} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$$

C.3 Nuclear Fission and Fusion

Nuclear Fission is the splitting of a nucleus of high nucleon number into two smaller nuclei of <u>approximately equal</u> mass with the release of energy and <u>neutrons</u>.

For example:

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{236}_{92}\text{U}$$

$$^{236}_{92}\text{U} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n}$$

$^{235}_{92}\text{U}$ is induced by a neutron to form the highly unstable U−236 nucleus. Then it undergoes nuclear fission to form $^{92}_{36}\text{Kr}$ and $^{141}_{56}\text{Ba}$. Both $^{92}_{36}\text{Kr}$ and $^{141}_{56}\text{Ba}$ have <u>higher</u> binding energy per nucleon than $^{235}_{92}\text{U}$ (as seen in the binding energy curve on pg 11).

- A particular nuclide may split into many different pairs of fission products.
For example, ^{235}U can also undergo fission to form ^{140}Xe and ^{94}Sr

$$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{236}_{92}\text{U}$$

$$^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2\,{}^{1}_{0}\text{n}$$

Nuclear Fusion is the formation of a larger nucleus from two nuclei of low nucleon number, with release of energy.

For example: $^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$

One $^{2}_{1}\text{H}$ and one $^{3}_{1}\text{H}$ can join together to form $^{4}_{2}\text{He}$. $^{4}_{2}\text{He}$ has a higher binding energy per nucleon than $^{2}_{1}\text{H}$ and $^{3}_{1}\text{H}$.

- However, for a fusion process to occur, a large amount of energy is <u>first required to overcome the mutual electric repulsion of two nuclei</u> in order to fuse them. Hence fusion can only occur under extreme conditions, i.e. at very high temperature.

Additional Notes

C.4 Fusion and Fission on the Binding Energy Curve

- Since Fe-56 has one of the highest binding energy per nucleon, it is the element with the one of the greatest nuclear stability. Nuclides of other elements can take part in fission or fusion processes to form more stable nuclei which have higher binding energy per nucleon.
- The rising part of the binding energy curve shows that elements with low mass number can produce energy by fusion by forming nuclei with larger mass number and higher binding energy per nucleon.
- In contrast, the falling part of the curve shows that very heavy elements with high mass number such as uranium can produce energy by fission of their nuclei to lighter nuclei with higher binding energy per nucleon.
- To understand how energy is released in fission and fusion, refer to the next section.

C.5 Method of Calculation of Energy Released

- Method 1: Using differences in binding energy

Energy released = total binding energies of final nuclei – total binding energies of initial nuclei

- Method 2: Using differences in mass

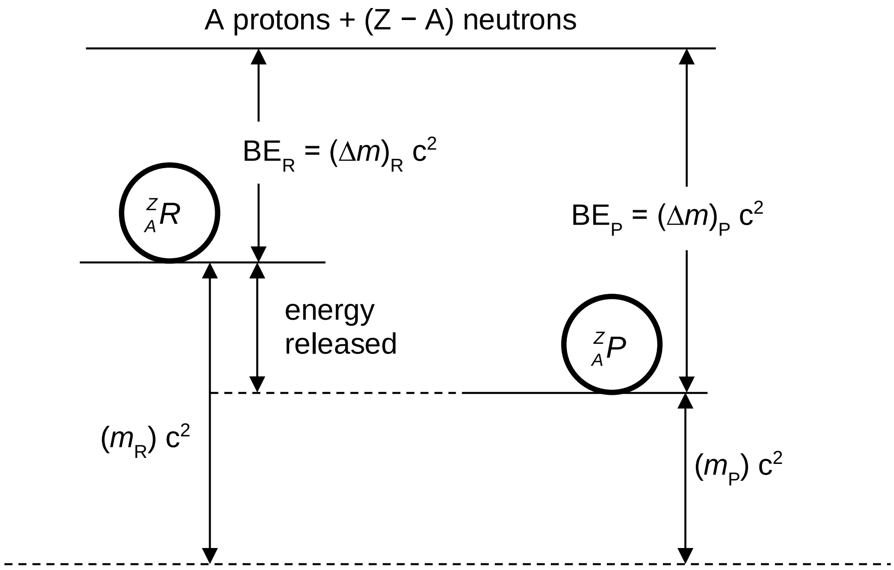
Energy released = (total initial mass – total final mass) × c^2

The term "mass defect" applies only to the situation where constituent nucleons forms the nucleus. In situations where a nuclear reaction results in a different nuclei, the term "mass difference" is used.

If the calculated energy released is negative, it implies that energy has to be supplied for the reaction to occur.

Consider $m_R > m_P$

R – reactants
P – products



Check Your Understanding 5 (J93/I/29)

Helium nuclei may result from the bombardment of Lithium nuclei with protons. The reaction can be represented by the following nuclear equation:

$$^{7}_{3}\text{Li} + ^{1}_{1}\text{p} \rightarrow 2\,^{4}_{2}\text{He} + \textit{energy}$$

The speed of light is c, and the masses of the particles are lithium m_L, helium m_H, proton m_P.

What is the energy released during such a reaction?

A $[2m_H - (m_L + m_P)]\,c^2$

B $[(m_L + m_P) - 2m_H]\,c^2$

C $(2m_H + m_L + m_P)\,c^2$

D $\frac{(m_L + m_P) - 2m_H}{c^2}$

E $\frac{2m_H - (m_L + m_P)}{c^2}$

Ans: ____

Example 3

Calculate the energy released when a uranium−235 nucleus undergoes fission according to

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$$

Given: Binding energy per nucleon of U = 7.591 MeV, Ba = 8.326 MeV, Kr = 8.513 MeV

Solution:

Example 4

Calculate the energy released when a uranium−235 nucleus undergoes fission according to

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,^{1}_{0}\text{n}$$

Given: masses of U = 235.0439 u, Ba = 140.9144 u, Kr = 91.9262 u

Solution:

C.6 Conservation Laws in Nuclear Reactions

The **nucleon number, charge, momentum and mass−energy** are all **conserved** in nuclear processes.

	Quantity Conserved	Example
1	Nucleon (mass) number The sum of the mass numbers of nuclides before and after the reaction is the same.	${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$ nucleon number of reactants = 2 + 3 = 5 nucleon number of products = 4 + 1 = 5
2	Charge The sum of the atomic numbers of nuclides before and after the reaction is the same.	${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$ atomic number of reactants = 1 + 1 = 2 atomic number of products = 2 + 0 = 2
3	Mass-energy Since mass and energy are equivalent and can be converted from one to the other (from the relation $E = mc^2$), the total amount of mass (convert to mc^2) and energy before the nuclear reaction must be the same as that after the reaction.	In the nuclear process ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$, mass of ${}^{2}H$ and ${}^{3}H$ = 2.014102 *u* + 3.016050 *u* = 5.030152 *u* mass of He and n = 4.002603 *u* + 1.008665 *u* = 5.011268 *u* i.e. the reactants have more mass-energy than the products. In order for mass-energy to be conserved, energy must be released when products are formed: Energy released = mass difference × c^2 = (5.03052 – 5.011268) $u \times c^2$ = 2.88×10^{-12} J This energy appears as the kinetic energy of the products and/or gamma radiation.
4	Linear momentum If there is no resultant external force acting on the system, the total momentum of the system must remain constant.	${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{0}n$ Total initial momentum of ${}^{2}_{1}H$ and ${}^{3}_{1}H$ = total final momentum of ${}^{4}_{2}He$ and neutron.

Example 5

The nuclear equation represents the bombardment of nitrogen with α-particles

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

Check whether the following conservation laws are applicable for this nuclear reaction.

Solution:

(i) atomic number
Total atomic number of the nuclides before reaction =
Total atomic number of the nuclides after reaction =

(ii) mass number
Total mass number of nuclides before reaction =
Total mass number of nuclides after reaction =

(iii) Conservation of mass-energy:

mass of N	=	14.00307 u	mass of O	=	16.99913 u
mass of He	=	4.00260 u	mass of H	=	1.00783 u
Total mass before	=	________ u	Total mass after	=	_______ u

(iv) Conservation of linear momentum:

Not enough information is given by the question to determine.

In general, nuclear reactions induced by α−particles cause the emission of either protons or neutrons depending on the energy of the α−particles and the nucleus under attack.

Additional Notes

Example 6

A stationary radium nucleus emits an α−particle spontaneously. Find the energy released and velocities of the α−particle and the recoiled Radon (Rn) nucleus.

$$^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He}$$

$$226.0254\ u \qquad 222.0176\ u \qquad 4.0026\ u$$

Solution:

Mass−energy, as well as momentum, is conserved in nuclear reactions. We can use this to determine whether a given reaction will occur or not.

If the total mass of the products is less than the total mass of the initial particles, then the energy will be released by the reaction – it will appear as kinetic energy of the outgoing particles.

But if the total mass of the products is greater than the total mass of the initial reactants, the reaction requires energy. The reaction will then not occur unless the bombarding particles have sufficient kinetic energy.

Example 7

Two deuterium ($^{2}_{1}\text{H}$) nuclei fuse to produce one $^{3}_{2}\text{He}$ nucleus and a neutron.
(a) Write down the nuclear equation for this reaction.
(b) Calculate the total mass defects before and after the fusion.
(c) Find the energy released in MeV.

mass of $^{2}_{1}\text{H}$ = 2.0141 u mass of $^{3}_{2}\text{He}$ = 3.0160 u
mass of proton = 1.0073 u mass of neutron = 1.0087 u

Solution:

D Radioactive Decay

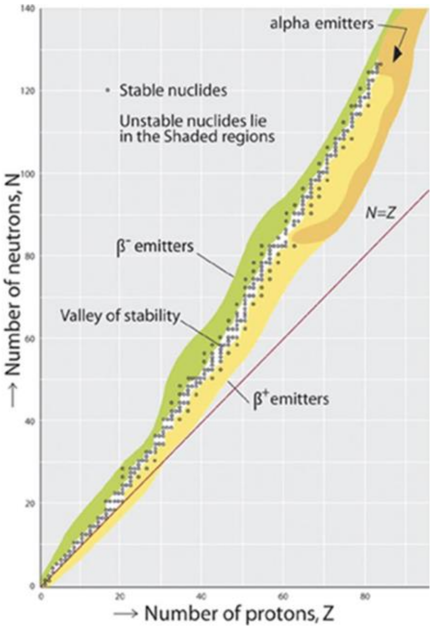
D.1 Introduction

- The positively charged and closely spaced protons in a nucleus have huge electrical forces of repulsion between them. At the same time, the neutrons and protons within the nucleus are bounded to one another by strong nuclear forces of attraction.

- One important aspect of the strong nuclear force is that it is a short-range force: acting over a very short distance. For electric forces, it can act over great distances and are therefore long-range forces. A delicate balance between these two opposing forces keeps most nuclei stable.

Radioactive decay is not considered a fission process because the products of the transformation does not require a trigger such as a third particle (usually neutron) to split up.

- If a nuclide contains too many or too few neutrons relative to the number of protons, the nucleus becomes unstable as the binding between the nucleons is reduced. As seen in the figure on the right, stable nuclides (dots on the graph) are located in an area known as the valley of stability and unstable nuclides lie in the shaded regions.



- Stable nuclei tend to have same number of protons as neutrons, up to about Z ≈ 20. Beyond this, stable nuclei contain more neutrons than protons as electrical repulsion increases, so a greater number of neutrons (which exert only the attractive nuclear force) are required to maintain stability. For Z above 82, there are no completely stable nuclides as no number of neutrons can overcome the greatly increased electric repulsion. A stable nucleus is one that stays together indefinitely and an unstable nucleus is one that comes apart, resulting in radioactive decay.

- **Radioactive decay** or **radioactivity** is <u>the spontaneous and random decay of a nucleus</u> with the emission of an alpha particle or a beta particle, and usually accompanied by the emission of a gamma ray photon, to obtain a more stable nucleus.

Additional Notes

- Natural radioactivity is observed in unstable isotopes that exist in nature. Artificial radioactivity can be acquired by isotopes as a result of nuclear reactions.

D.2 Spontaneous and random nature of radioactive decay

- Radioactive decay processes are spontaneous.

Spontaneous decay means the decay occurs on its own and is unaffected by environmental or external factors such as temperature and pressure.

Other external factors include:
- physical parameters (e.g. electric or magnetic fields in the environment)
- chemical composition (e.g. decay rate of Uranium remains the same in any sample containing Uranium like UCl_3, UO_2 or UF_3)

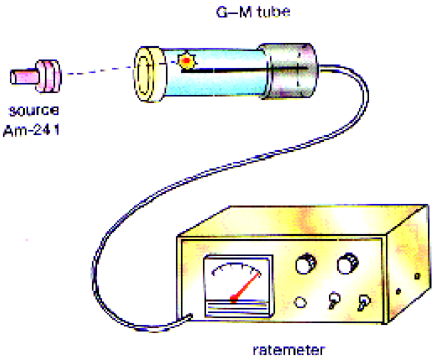
- Radioactive decay is also a random process.

Random decay means the nucleus has constant probability of decay per unit time. There is no way to predict when any individual nucleus will decay.

- Random decay means particles are not emitted at equal time interval, and it is also impossible to know the direction of emission of particles.

- This irregularity of decay may be demonstrated by the Geiger-Müller tube (GM tube). GM tube is used to detect radioactive particles, making use of their ionising property. When connected to a loud speaker, you can hear a 'click' for each radioactive particle entering the tube. The 'clicks' heard will be irregular.

Reference:
http://www.darvill.clara.net/nucrad/detect.htm



D.3 Background Radiation

- Not only does radiation come from elements in the form of radioactivity, some come from our natural environment, others from human activities and devices. The various sources include:

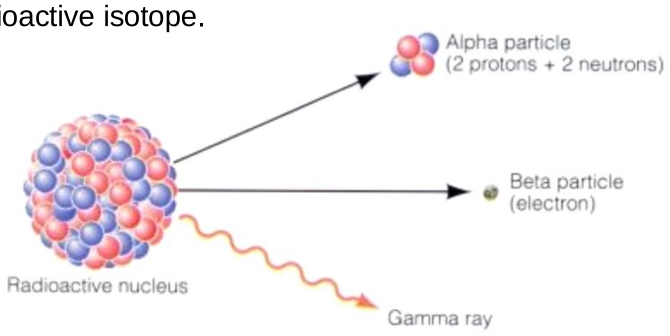
  - cosmic rays entering the Earth's atmosphere from outer space
  - radioactivity emissions from contaminated apparatus, rocks, soil and buildings

Because of these, any radiation detector placed in a location with no radioactive sources nearby will usually register a count of 20 to 50 per minute. This is known as background radiation.

- In experiments determining count rates from a specific radioactive source, and the emission from the radioactive source is high (e.g. 5000 counts per minute), the background count can be neglected without much error **BUT** the rule in conducting a radioactivity experiment is for the background count to be recorded first and its values deducted from the count rates obtained in subsequent readings.

D.4 Nature of α particles, β particles and γ rays

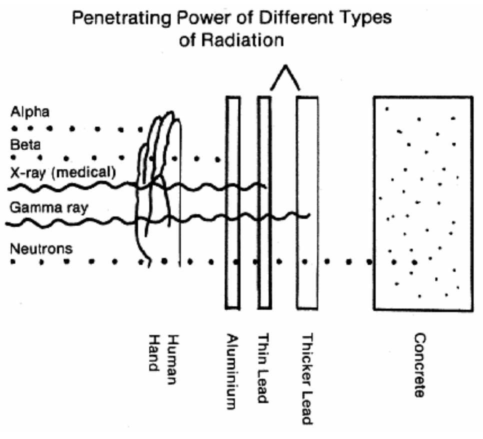
- In radioactive decay, alpha particles, beta particles or gamma rays are emitted from the nucleus of a radioactive isotope.



- **Alpha particles**
  - Some radiation that is emitted by radioactive isotopes travels as particles. One example is alpha particles, α-particles, which are ${}^{4}_{2}$He helium nuclei (2 protons and 2 neutrons for each ${}^{4}_{2}$He nucleus)
  - α−particles lose energy quickly. Because of their relatively large mass, charge and relatively low velocity, they are very likely to interact with other atoms and lose their energy, so they are effectively absorbed within a few centimeters of air. A hand, thin piece of paper or a few cm of air stops it. Typical alpha particle energies are in the range from 3 to 10 MeV.

- **Beta particles**
  - β−particles are fast moving electrons emitted from the nucleus. It was discovered experimentally that β particles are emitted from the same nucleus with a range of energies. Typical beta particle energies range from a few thousand electron volts to several MeV. They are absorbed by a few mm of aluminium; they can penetrate skin of a few cm but not concrete.

- **Gamma rays**
  - γ rays are electromagnetic radiation of very short wavelength (5×10^{-13} m to 40×10^{-10} m).
  - Gamma rays are distinguished from x−rays only by the fact that it comes from the nucleus of the atom of a radioactive isotope. Most gamma rays are somewhat higher in energy than x−rays and therefore are very penetrating.
  - Gamma rays are the most penetrating of all radiation. Typical range of energies is from an estimate of 20 000 eV to 10 MeV. They are absorbed by several cm of lead or concrete.



- Comparing ionising properties of α, β and γ (How is radiation absorbed):
  - Ionisation means to 'kick' electrons out of a neutral atom/molecules to produce an ion.

Additional Notes

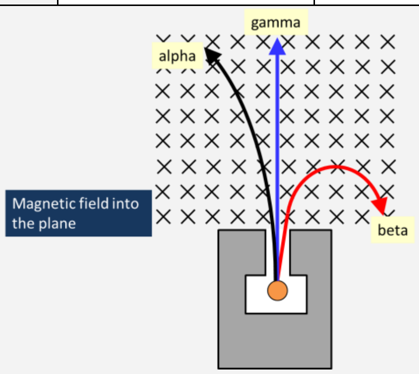
- In travelling the same distance in a given material, a γ-ray produces 1 ionised atom, a β-particle will on average produce about 1000 ionised atoms and an α-particle will produce 1,000,000 ionised atoms.

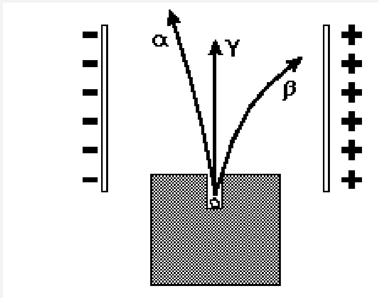
- α and β particles are better ionising agents as they are more massive and possess charge.

- A radiation's ionising power affects its penetration. The stronger the ionising power, the weaker the penetration since the radiation travels a shorter distance before all of its energy gets absorbed.

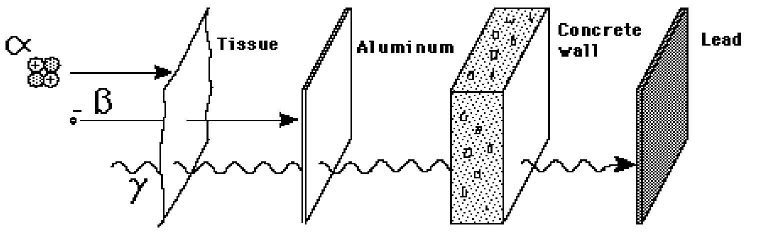
Table 2: Properties of α, β and γ

	α particles	β particles	γ rays
Charge **(where e = 1.60 x 10^{-19} C)**	+2e	− e	No charge
Rest mass	4 *u*	$\frac{1}{1840}u$	-
Speed	0.06 c (approx.)	up to 0.98 c	c
Energy	≈ 6 MeV	≈ 1 MeV	≈ 0.1 MeV
Electric Field Deflection	Yes - towards negative plate	Yes - towards positive plate	No
Magnetic Field Deflection	Yes	Yes	No





	α particles	β particles	γ rays
Ionising Properties	most ←		least
Penetrative Properties	least →		most
Range in air	A few cm	A few m	A few hundred m
Penetrating Properties	A few sheets of paper, few cm of air	A few mm of A*l*	Several cm of lead ≈10 cm

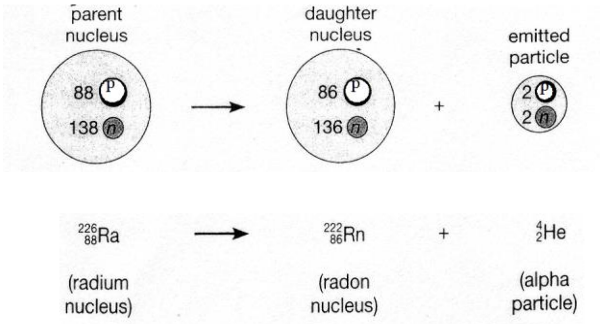


Additional Notes

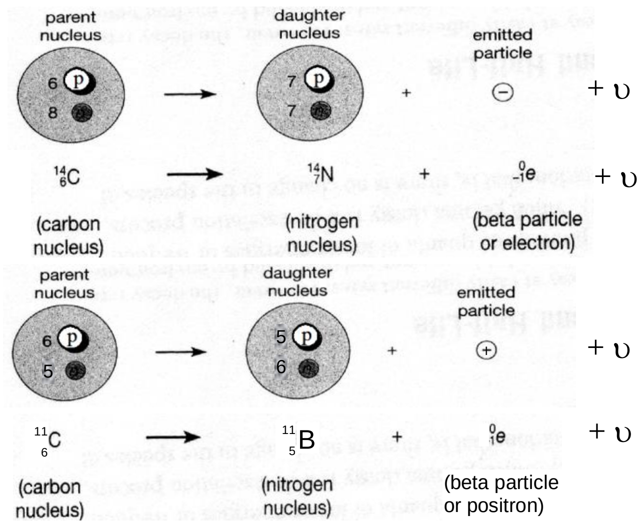
D.5 Changes in the Nucleus during Radioactive Decay

- When radioactivity involves the emission of either an electron or a helium nucleus, the unstable parent nucleus becomes a different element by either of these processes - α, β and γ emission. The new nucleus is called the daughter nucleus.

- α–emission:
  - An α–emission results in a new atom with a mass number 4 less and an atomic number 2 less than the original atom.
  - Examples of radioactive isotopes that emit α particles: polonium-210, americium-241

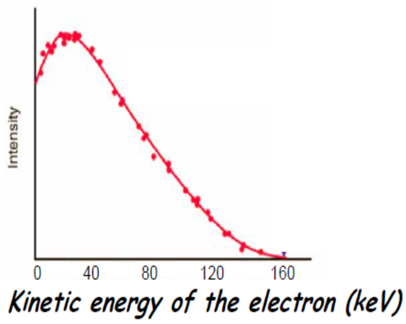


- β−emission
  - In a beta decay, the electron emitted is not an orbital electron of the atom. Instead it originates from the nucleus through a nuclear transformation in which a neutron changes into a proton: $^{1}_{0}n \rightarrow ^{1}_{1}p + ^{0}_{-1}e$.
  - Positive β–emission (positron decay) can also occur. This originates through a nuclear transformation in which a proton converts into a neutron: $^{1}_{1}p \rightarrow ^{1}_{0}n + ^{0}_{1}e$



  - Examples of radioactive isotopes that emit β particles: Caesium-137, Sodium-22
  - A β–emission results in a new nucleus with a mass number unchanged and an atomic number 1 more or 1 less than the original nucleus. During the process of a beta decay, a third particle called neutrino (υ) is released.
  - Existence of neutrino:

    1. In a beta decay from a single type of nucleus, the total energy released in the reaction is always a fixed value. If the daughter nucleus and beta are the only products, by conservation of momentum and energy, the daughter nucleus will gain very little kinetic energy as its mass is very much bigger than β-particles. Hence beta particle should gain almost all the energy released.



    2. However, experimentally, it is found that beta particles from a single type of nucleus are emitted over a range of continuous energies as opposed to alpha decay, in which alpha particles are emitted with

Additional Notes

discrete energies. This suggests a violation of conservation of energy and momentum.

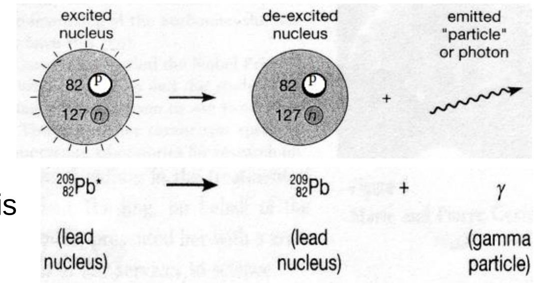
3. It was thus hypothesized there should be a particle which has a very small mass and no charge and acts almost like a photon, emitted during the decay. Hence, the energy released must be split randomly between beta and the other particle.

4. This other particle was included to account for conservation of linear as well as angular momentum. It is called the neutrino.

Neutrino was detected experimentally in 1956 and concluded to have the following properties:

- it has zero electric charge
- its mass is zero or very small (~ 10^{-36} kg or less)
- it interacts very weakly with matter and is therefore very hard to detect

- γ−emission:
  - γ−emission often follows α or β−emission which usually leaves the daughter nucleus in an excited nuclear state. The daughter nuclide would then return to a lower energy state by emitting high frequency electromagnetic wave called γ-rays. This process is called de-excitation.



* The asterisk sign indicates that the nucleus is in an excited state

  - Since γ−emission is caused by the change in nuclear energy states in the daughter nuclide, therefore, if nuclear state transition takes place in a few steps, γ−rays of several frequencies would be emitted.

$$^{24}_{12}Na \rightarrow \ ^{24}_{13}Mg^* + \ ^{0}_{-1}e$$
$$^{24}_{13}Mg^* \rightarrow \ ^{24}_{13}Mg + \gamma$$

  - A γ–emission results in both the atomic and mass number unchanged. This process is also known as isomeric transition.

D.6 Decay Curve

- Like all statistical random processes, the rate of decay of a radioactive sample is directly proportional to N, the number of radioactive atom present in the sample, i.e. $\frac{dN}{dt} \propto N$.
- During decay, the number of undecayed nuclei will decrease, hence $\frac{dN}{dt}$ is negative. The rate of decay is also known as the activity.
- **Activity**, A, is defined as the number of nuclear disintegrations per unit time.
- Thus $A = -\frac{dN}{dt} \propto N \Rightarrow A = -\frac{dN}{dt} = \lambda N$

where λ is a constant, known as the decay constant, of the radioactive nuclide. Different radioactive nuclides have different values of λ.

Additional Notes

- From $-\frac{dN}{dt} = A = \lambda N$ ➔ $\lambda = -\frac{dN}{dt} / N$

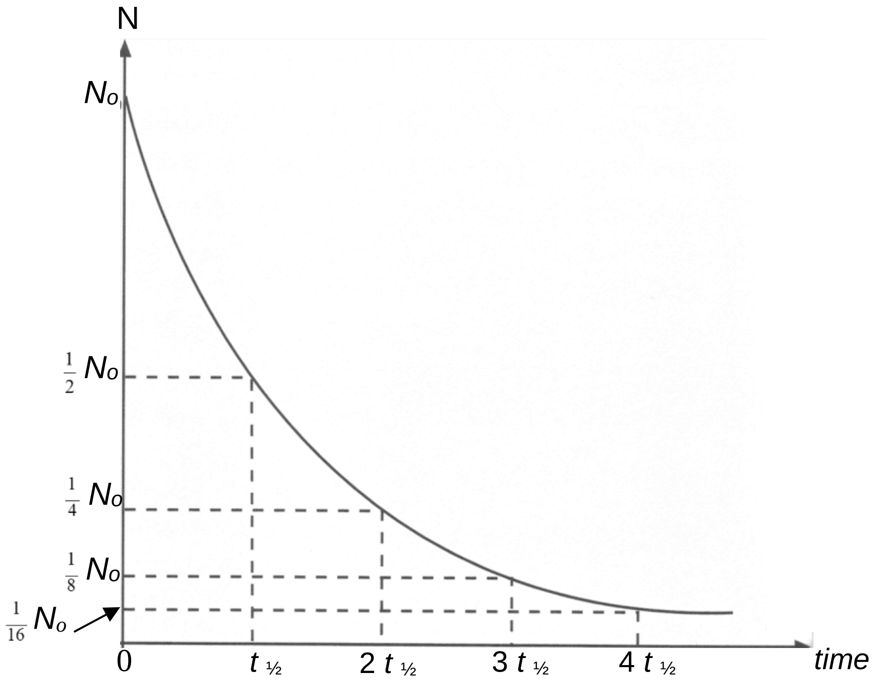
Decay constant is a property of a particular nuclide and is the probability of decay per unit time of a nucleus.

- Activity can be measured in terms of Becquerel (1 Bq = 1 disintegration per second).

- Suppose N_0 is the number of radioactive nuclei at time t = 0, then the Rutherford Law of radioactive decay gives at any time t,

$$N = N_0 e^{-\lambda t}$$

where N is the number of undecayed radioactive nuclei at time t. (The derivation of this equation is not required for the syllabus.) The decay curve therefore follows an exponential equation, and the graph is as shown below.



- Since activity and mass of undecayed nuclei are directly proportional to N, they also follow the same exponential decay curve and they can be represented by the following equations:

$$A = A_o e^{-\lambda t}$$
$$m = m_o e^{-\lambda t}$$

where A_o and m_o are the activity and mass of undecayed nuclei at time t = 0 respectively.

For derivation of decay equation:

- **Count rate** is the number of emissions received by a detector per unit time from a radioactive source, which may include background radiation.

- **Background count** is the number of counts recorded by a radiation detector from background radiation in the absence of a radioactive source.

Since count rate C received by a radioactive detector (e.g. the Geiger-Müller tube) is directly proportional to the activity, C also follows the same decay curve:

$$C = C_o e^{-\lambda t}$$

Additional Notes

- Hence the vertical axis of the decay curve above can also be activity, mass of undecayed nuclei or count rate.

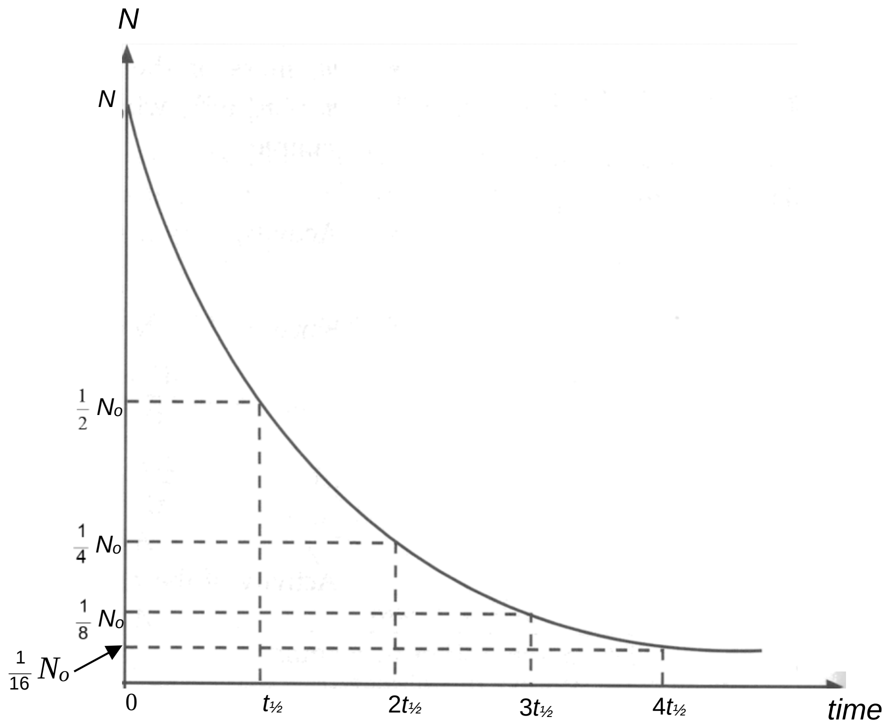
D.7 Half-life

- The **half-life** $t_{1/2}$ is the average time taken for the activity of a particular radioactive nuclide to fall to half of its initial value.

Memorise

- Thus if N_o is the original amount of radioactive nuclide present in the sample, at a time equals to half-life later, the number of radioactive nuclide is ½ N_o.

$$N_o \xrightarrow{t_{1/2}} \tfrac{1}{2} N_o \xrightarrow{t_{1/2}} \tfrac{1}{4} N_o \xrightarrow{t_{1/2}} \tfrac{1}{8} N_o \xrightarrow{t_{1/2}} \tfrac{1}{16} N_o \xrightarrow{t_{1/2}} \text{ and so on}$$



- Relation between λ and $t_{1/2}$:

If at $t = 0$, $N = N_0$, then $N = N_0 e^{-\lambda t}$
When $t = t_{1/2}$, $N = ½ N_0$

Hence, $\quad ½ N_0 = N_0 e^{-\lambda t_{½}}$

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

* If question gives the background radiation count rate, do remember to remove it.

n need not be an integer.

- In summary, the decay law: $x = x_0 e^{-\lambda t}$

where x can be: (i) activity (A)
(ii) count rate (C)*
(iii) number of radioactive nuclei (N)
(iv) mass (m)
(v) no. of mole (n)
x_0 : the initial value of x at time $t = 0$

- From the decay law, it can also be derived:

$$x = x_0 \left(\frac{1}{2}\right)^n$$ where n: number of half-life that has lapsed

Example 8
The half-life of radium is 1590 years. In how many years will one gram of the pure element
(a) lose one centigram
(b) to be reduced to one centigram
(1 centigram = 1 × 10^{-2} g = 0.01 g)

Solution:

Example 9
A radioactive source contains 10^{-6} g of plutonium (mass no. 239). The source is estimated to emit a total of 2300 α-particle per second in all directions. Calculate the half-life of plutonium in years. (N_A = 6.02 × 10^{23} mol^{-1})

Solution:

Additional Notes

Example 10

A unit for radioactivity is the curie (Ci). One curie is the rate of decay of 3.7×10^{10} nuclei per second. The initial activity of a sample of thorium-234 which has a half-life of 24 days is 10 µCi.

(a) What is the activity of the sample after 72 days?

(b) What is the time taken for the activity of the sample to become 2.5 µCi?

Solution:

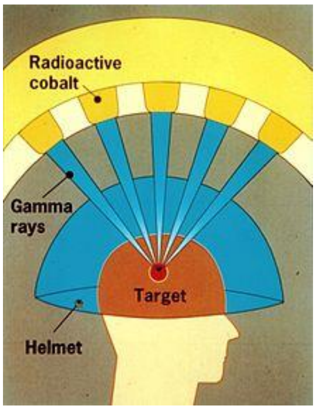
E Uses of Radioactivity and Biological Effects of Radiation

E.1 Uses of Radioactivity

- Cobalt–60 emits emit penetrating rays which can be used to penetrate deep into welding to reveal faults and to check for consistency of thickness of sheets of paper or plastic.

- Radioactive nuclide can also be used as tracer. Nitrogen-15 is used as tracer in plants and Technetium-99 is used to trace blood flow in the heart and lungs.

- Gamma radiation like Cobalt-60 can also be used to kill cancerous cells by focusing a high dose of it at the defect cells. Gamma Knife therapy uses doses of radiation to kill cancer cells and shrink tumors, delivered precisely to avoid damaging healthy brain tissue. Gamma Knife radiosurgery is able to accurately focus many beams of gamma radiation to converge on one or more tumors. Each individual beam is of relatively low intensity, so the radiation has little effect on intervening brain tissue and is concentrated only at the tumor itself. The non-invasive nature of gamma knife makes it very suitable for treating tumors in the brain region.



- Archaeological Dating with Carbon-14
  Carbon dating is a variety of radioactive dating which is applicable only to matter which was once living and presumed to be in equilibrium with the atmosphere, taking in carbon dioxide from the air for photosynthesis. Presuming the rate of production of carbon-14 to be constant, the activity of a sample can be directly compared to the equilibrium activity of living matter and the age calculated.

  http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/cardat.html

E.2 Biological Effects of Radiation

- Immediate effects of over exposure to radiation includes nausea, vomiting, bloody diarrhoea, loss of hair, coma and even death. Delayed effect includes leukemia, weaken immune system and will even affect your offspring.

- Precautions against radiation hazards
  - Radioactive sources should be kept in well protected, lead lined rooms and boxes.
  - Signs should be put up to inform people of the present of radioactive source.
  - No direct contact should be made when handling the radioactive sources
  - Special protective clothing should be worn during experiment.
  - No food and drinks should be taken when doing radioactive experiment.

- Effects of radiation on health
  Radioactive materials that decay spontaneously produce ionizing radiation, which has sufficient energy to strip away electrons from atoms (creating two charged ions) or to break some chemical bonds. Any living tissue in the human body can be damaged by ionizing radiation in a unique manner. The body attempts to repair the damage, but sometimes the damage is of a nature that cannot be repaired or it is too severe or widespread to be repaired. Also mistakes made in the natural repair process can lead to cancerous cells.

  - **Alpha particles**

    As any heavy charged particle, alpha particles lose their energy within a very short distance in dense media, causing significant damage to surrounding biomolecules. The health effects of alpha particles depend heavily upon how exposure takes place. Generally, external alpha irradiation (exposure) is not harmful because alpha particles are completely absorbed by the thin layer of dead skin cells in the outermost layer of the skin as well as by a few centimeters of air.

    However, if alpha emitters have been inhaled, ingested (swallowed), or absorbed into the blood stream, sensitive living tissue can be exposed to alpha radiation. There is resulting biological damage to vital organs and increase in the risk of cancer; in particular, alpha radiation is known to cause lung cancer in humans when alpha emitters are inhaled.

    The death of Marie Curie at age 66 from leukemia, was likely caused from radiation exposure, such as alpha particles from radium, during her years of important research on the subject.

    The 2006 assassination of Russian dissident Alexander Litvinenko is thought to have been caused by poisoning with polonium-210, an active alpha emitter.

  - **Beta particles**

    Direct exposure to beta particles is a hazard, because emissions from strong sources can redden or even burn the skin. However, emissions from inhaled or ingested beta particle emitters are the greatest concern. Beta particles released directly to living tissue can cause damage at the molecular level, which can disrupt cell function. Because they are much smaller and have less charge than alpha particles, beta particles generally travel further into tissues. As a result, the cellular damage is more dispersed.

Additional Notes

- **Gamma rays**

Both direct (external) and internal exposures to gamma rays or X-rays are of concern. Gamma rays can travel much farther than alpha or beta particles and have enough energy to pass entirely through the body, potentially exposing all organs. A large portion of gamma radiation largely passes through the body without interacting with tissue - the body is mostly empty space at the atomic level and gamma rays are vanishingly small in size.

Gamma rays can ionize atoms in tissue directly or cause what are known as "secondary ionizations." Ionizations are caused when energy is transferred from gamma rays to atomic particles such as electrons (which are essentially the same as beta particles). These energized particles then interact with tissue to form ions through secondary ionizations. Because gamma rays are photons and thus interact less frequently with matter than alpha and beta particles, they are more penetrating and the damage they cause can occur much farther into tissue (that is, farther from the source of radiation).