Additional Notes

Topic 19: Quantum Physics

Content:

A The energy of a photon and the photoelectric effect
B Wave-particle duality
C Energy levels in atoms
D Line spectra
E X-ray spectra
F The uncertainty principle

Lecture SOW			
Lect	**Concepts**	**Questions**	**Est. Video Time**
1	A.1 – A.4	CYU1	38 min
2	A.5 – A.8	CYU2, CYU3 and Example 1	41 min
3	B.1, B.2 and C	Example 2, Example 3, Example 4, CYU 4	40 min
4	C.1 - D.3	CYU5 and CYU6	37 min
5	E.1 – E.3	Example 5, Example 6	37 min
6	F	Example 7, Example 8 and Example 9	14 min

Learning Outcomes:

Candidates should be able to:

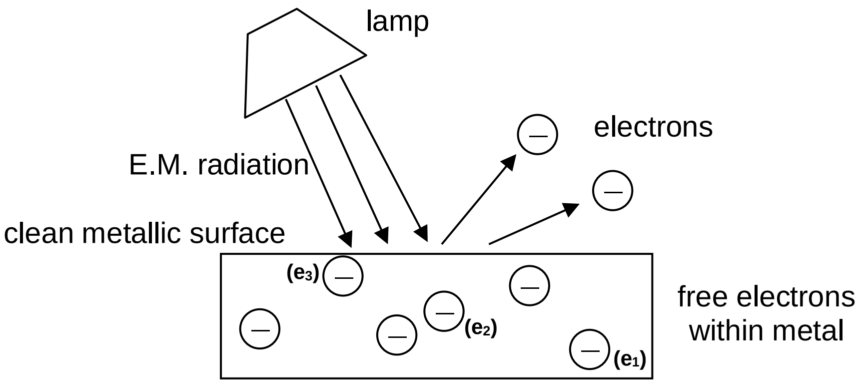
(a) show an appreciation of the particulate nature of electromagnetic radiation
(b) recall and use $E = hf$ for the energy of a photon
(c) show an understanding that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for the wave nature
(d) recall the significance of threshold frequency
(e) recall and use the equation $\frac{1}{2} mv_{max}^2 = eV_s$, where V_s is the stopping potential.
(f) explain photoelectric phenomena in terms of photon energy and work function energy
(g) explain why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency
(h) recall, use and explain the significance of the equation $hf = \phi + \frac{1}{2}mv_{max}^2$
(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles
(j) recall and use the relation for the de Broglie wavelength $\lambda = \frac{h}{p}$
(k) show an understanding of the existence of discrete electron energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to the observation of spectral lines
(l) distinguish between emission and absorption line spectra.
(m) recall and solve problems by using the relation $hf = E_2 - E_1$
(n) explain the origins of the features of a typical X-ray spectrum
(o) show an understanding of and apply $\Delta p \Delta x \gtrsim h$ as a form for the Heisenberg position-momentum uncertainty principle to new situations or to solve related problems.

A The Photoelectric Effect

Gold leaf oscilloscope showing photoelectric effect

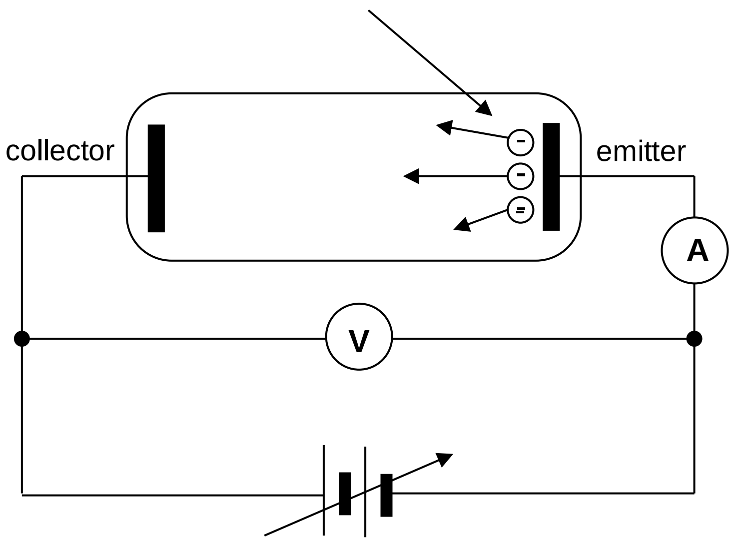
- In earlier topics we learnt that phenomena such as interference and diffraction are exhibited by light as it is passed through slits in Young's double slits experiment.
- These observations provide evidence for a wave nature of electromagnetic radiation.
- However, when we look at observations associated with the photoelectric effect, we discover a completely different aspect of light.
- We find that the energy of electromagnetic radiation is quantized; it is emitted and absorbed in particle-like packages of definite energy.

Photoelectric effect is the ejection of an electron from a metal surface when the surface is irradiated with electromagnetic radiation of a high enough frequency.



Video on Photoelectric Experiment:

A.1 The Experimental Setup



Additional Notes

- **The components** include:
  - emitter and collector plates
  - vacuum tube (to ensure that emitted photoelectrons are not obstructed by air molecules)
  - variable emf (to produce a potential difference between emitter and collector)
  - voltmeter measures the potential difference V between emitter and collector, taken with respect to the emitter
  - ammeter measures the magnitude of the photocurrent (i)

The photoelectric effect

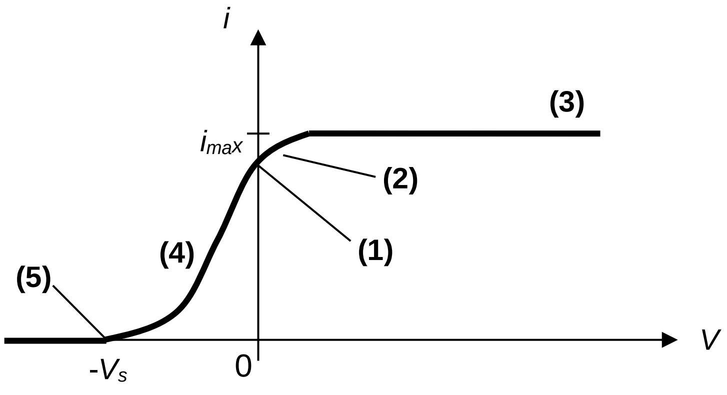
The emitter plate (a metal plate) is irradiated with electromagnetic radiation, and electrons are emitted from the emitter plate. These photoelectrons leave the plate with a range of kinetic energies – some move faster than the others.

The stream of photoelectrons which manage to reach the collector plate complete the circuit and constitute a photocurrent (i).

Photoelectrons are electrons emitted via the photoelectric effect.

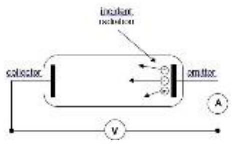
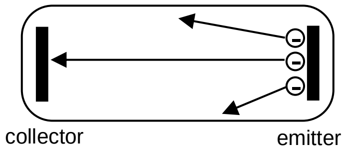
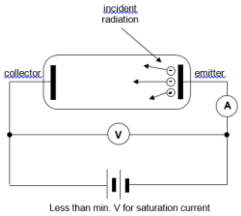
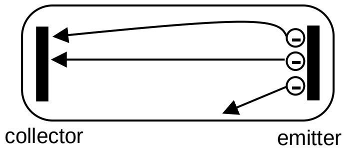
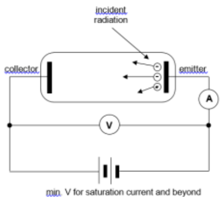
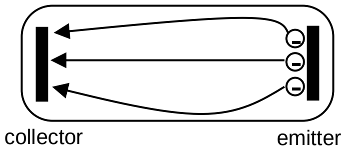
A.2 Measuring the maximum KE of photoelectrons

When radiation of sufficiently high frequency is irradiated on the emitter, by varying the potential difference V across the collector and emitter, the photocurrent i detected may change, as shown in the graph below.



Additional Notes

The following scenarios explain the various parts labelled in the graph.

Scenario	Explanation
1 	 Collector is at the same potential as emitter (*V* = 0). No electric field is exists between the plates. Electrons emitted in random directions. Only photoelectrons travelling towards the collector reached it (i.e. not all electrons reached collector) Rate of electrons leaving emitter > rate of electrons arriving at collector
2 	 Collector is at a slightly higher potential than emitter. More electrons reach the collector (i.e. electrons that may have been directed away from collector is now attracted towards it), although the electric field is not strong enough to draw all of them towards the collector. As compared to Scenario 1, now the rate of electrons arriving at collector increases, thus photocurrent increases.
3 	 Collector is at <u>much</u> higher potential than emitter. Electric field between plates is <u>strong enough</u> such that <u>all</u> photoelectrons reach the collector. Rate of electrons leaving emitter = rate of electrons arriving at collector Any further increase in potential difference between the plates does not lead to increase in photocurrent. Photocurrent is maximum (saturation current).

Saturation Current: Even though potential difference increases and there is a greater force on each electron and they accelerate more, the total number of electrons emitted per second did not increase. E.g. in 1 second, 10 electrons are emitted. The current is determined by $I = dQ/dt$, which is the total charge reaching collector plate per unit time. Even if these 10 electrons move with higher speed between emitter and collector, total charge reaching per second remains the same. Drift velocity cannot be applied directly as the space between collector and emitter is a vacuum.

Additional Notes

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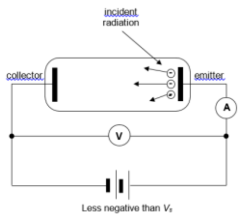

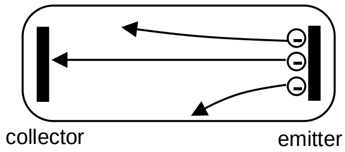

The polarity of the e.m.f. source is reversed. Collector is at a lower potential than emitter.

Electric field causes photoelectrons with lower KE to be repelled from collector. Only those with greater KE reach the collector. Hence, less electrons reach the collector.

As compared to Scenario 1, now the rate of electrons arriving at collector decreases, thus photocurrent decreases.

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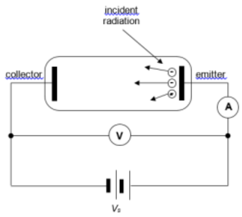
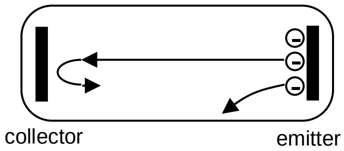




From Scenario 4, any increase in the potential difference (*V* became more negative) lead to decrease in photocurrent.

At the value of *V* when photocurrent just reached zero, the electric field is strong enough such that even photoelectrons with the greatest KE do not have sufficient energy to reach the collector. They are just turned back.

This potential difference is known as the **stopping potential** (V_s).

When this happens, a photoelectron with KE_{max} would have used up all its energy to do work against the electric field, i.e.

$$KE_{max} = eV_s$$

The stopping potential V_s is the minimum potential difference to reduce the photoelectric current to zero. All the kinetic energy of the most energetic electron will be converted into electrical potential energy, eV_s.

Hence by measuring the stopping potential V_S, we can determine the maximum kinetic energy with which electrons leave the emitter plate.

Saturation Current:
When all photoelectrons emitted per unit time reach the collector and give the maximum photocurrent. Since the number of emitted photoelectrons per unit time depends on intensity, the photocurrent will remain constant no matter how large *V* is made.

Stopping potential:
Note that stopping potential is when current JUST reached zero, not when current is zero. There are multiple values for current being zero, but only one for which it JUST reached zero. Max KE also implies there is a range of KE

$KE_{max} = eV_s$ represents the conversion of energy, Loss in KE = Gain in EPE.

A.3 Predictions from classical wave theory

The photoelectric effect experiment investigates how the photocurrent depend on the voltage across plates and on the frequency and intensity of light. Based on the picture of electromagnetic radiation as a wave, we would make the following predictions:

Wave-model prediction 1:

- Electrons *should* absorb energy continuously from the electromagnetic waves. As the light intensity increased, energy is transferred into the metal at a higher rate and the electron *should* ejected with a greater maximum kinetic energy.

- Since $K_{max} = eV_S$, this *should* lead to a greater stopping potential. **The maximum kinetic energy and hence stopping potential *should* depend on intensity of incident radiation, and *should not* depend on its frequency.**

Wave-model prediction 2:

- The greater the intensity of the light, the greater will be the energy per unit time reaching the illuminated plate. When the intensity of light falling on the plates is sufficiently high, we expect electrons to be emitted from the plate.

- Since intensity of a wave depends on amplitude, not frequency, the emission of electrons *should not* depend on frequency – **electrons *should be* emitted for radiation of all frequencies if the incident light beam is intense enough.**

Wave-model prediction 3:

- Since energy is arriving in a continuous manner, at low light intensities, a certain time interval *should be* needed for the electron to absorb the incident radiation before it acquires enough energy to escape from the metal.
- **We *expect* a measurable time lag between the start of radiation of light and the start of emission of electrons.**

The experimental results proved very different from these predictions.

Additional Notes

A.4 Observations from the experiment:
Applet download: http://phet.colorado.edu/en/simulation/photoelectric

Observation 1:

Experiment repeated with different intensities I, keeping frequency unchanged

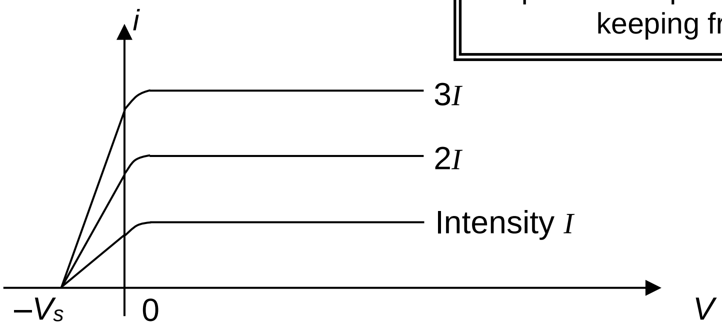


Fig. 19.1.4a

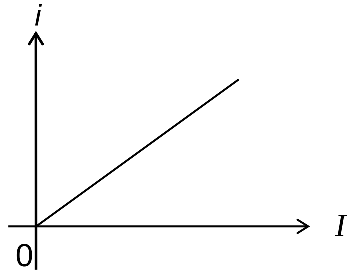


Fig. 19.1.4b

When the **intensity *I*** of the incident radiation is increased (i.e. I to $2I$ to $3I$),

- the stopping potential remains unchanged. Since $K_{max} = eV_S$, none of the electrons are emitted with higher KE. That is to say, *dim illumination can produce photoelectrons of the same energy as bright illumination!* (Fig. 19.1.4a)

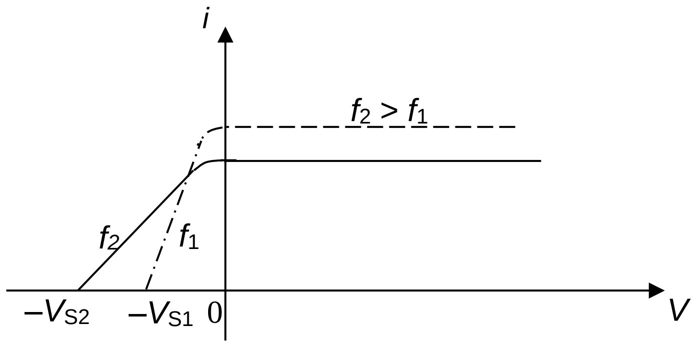
- the rate of electron emission, and hence the photoelectric current increases. When frequency f and p.d. V are kept constant, the photoelectric current i is proportional to intensity I of the incident light. (Fig. 19.1.4b)

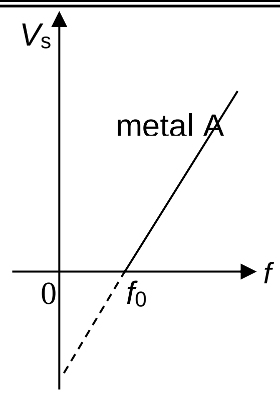
Hence, **increasing the intensity of the radiation has no effect on the maximum energy of the electrons.** This contradicts the first prediction we made using the wave-model.

This contradicts wave prediction 1 in A.3

Observation 2:

Experiment repeated with different frequencies, keeping intensity unchanged.





When incident light of ***different frequency*** but the same intensity is used,

- there was no emission below a certain threshold frequency even with high intensity of radiation.
- stopping potential V_S gets larger as the incident frequency increases,
- rate of emission of photoelectrons (i.e the photoelectric current) decreases as the incident frequency increases.

This contradicts wave prediction 2 in A.3

This contradicts wave prediction 1 in A.3

Threshold wavelength is the longest wavelength of radiation that ejects electrons from a particular surface, since *λ=c/f*

For a given metal, no electron is emitted if the frequency of the radiation is below a certain threshold frequency, even with high intensity of radiation. This contradicts the second prediction we made using the wave-model.
At frequencies above the threshold frequency, the maximum kinetic energy of the emitted electron increases linearly with the frequency of the radiation, even with low intensity of radiation. This contradicts the first prediction we made using the wave-model.

The **threshold frequency** is the lowest frequency of radiation that ejects electrons from a particular metal surface.

Observation 3:

This contradicts wave prediction 3 in A.3

Emission of photoelectrons starts with no observable time lag, even for very low intensity of incident light, assuming the frequency *f* exceeds the threshold frequency. This contradicts the third prediction we made using the wave-model.

No one could provide an account for these contradictions, until, in 1905, Albert Einstein came up with a solution. His proposal involved nothing less than casting a new light on the nature of light.

Check your understanding 1:

Which of the following observation(s) contradicts wave-model prediction of photoelectric effect?

I There is no observable time lag between irradiation of metal with EM waves and emission of photoelectrons.
II As intensity of light increases, photocurrent increases.
III When frequency of light drops below a certain value, there is no emission of photoelectrons regardless of intensity.
IV Increasing frequency of light increases kinetic energy of electrons.

Additional Notes

A.5 Light as a particle

- Einstein put forward the new quantum theory to explain the observations of the photoelectric effect.

- He postulated that light behaves as particles (photons) each carrying energy of $E_{photon} = hf$.

A photon is a quantum of electromagnetic energy which is dependent only on the frequency of the radiation.

The amount of energy E of a photon is given by

$$E = hf$$

where h = Planck constant (6.63×10^{-34} J s)
f = frequency of electromagnetic radiation (Hz)

These equations **only** apply for **photons**, NOT electrons or any other particles.

or, using the relationship $c = f\lambda$ for light

$$E = h\frac{c}{\lambda}$$

where c = speed of light in vacuum (3.00×10^{8} m s^{-1})
λ = wavelength of radiation (m)

These equations implies higher frequency more photon energy or lower wavelength more energy. E.g. red light photons will have less energy than blue light photons as red has a longer wavelength / lower frequency.

A.6 Einstein's photoelectric equation

- Each free electron in the surface of the metal is kept within the metal by the attractive force of the metal.

- A photon of the incident light gives *all* its energy *hf* to a *single* electron in the metal.

- Interaction between a photon and an electron is considered one-on-one i.e. each photon can only be absorbed by one electron.

- The transfer of energy from photon to electron is instantaneous (recall: no observable time lag between radiation and emission).

- It is also an all-or-nothing process. i.e. either the electron gets all the photon's energy, or none at all (photon not absorbed).

- The minimum amount of energy required to eject an electron from the metal surface is called the work function energy (ϕ) of the metal.

- Different metals have different work function energies.

Additional Notes

Work function is related to threshold wavelength and threshold frequency as:

$\Phi = hf_0$

$\Phi = hc/\lambda_0$

Work function energy ϕ is the minimum energy required to eject an electron from the metal surface.

- For an electron to escape from the surface of a metal, it must absorb a photon of energy that is equal to or greater than the work function energy of the metal.

When $E_{photon} = \phi = hf_0$,

the electron only has sufficient energy to escape from the metal.

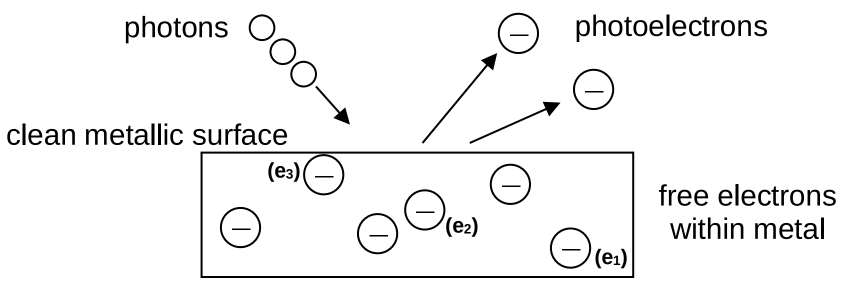
It may be reabsorbed back into the metal, unless it is pulled away by an electric force.

When $E_{photon} > \phi$,

the electron has more than enough energy to escape from the metal.

The remaining energy is converted into KE of the photoelectron.

- By Conservation of Energy, $E_{photon} = \phi + KE_{photoelectron} \left(+E_{loss}\right)$
When an incident photon with energy *hf* is absorbed by an electron, a portion of it, ϕ, is used to free the electron from the metal,and the remainder is retained by the electron as its kinetic energy, which has a maximum value.



- E_{loss} occurs when the electrons are deep in the metal (e.g. e_1 in figure above). As these electrons make their way to the surface of the material, they collide with the lattice ions and other free electrons, thereby losing energy.
- e_2 also loses energy in the metal, but less. e_2 therefore leaves the material with greater KE than e_1.
- e_3, which is at the surface of the metal, leaves the metal with the maximum KE (KE_{max}). This is the basis for **Einstein's Photoelectric Equation:**

$$E_{photon} = \phi + KE_{max}$$

Additional Notes

A.7 Equivalent terms in Einstein's Photoelectric Equation

Energy of Photons E_{photon}	**Work function energy** ϕ	**Max KE of photoelectrons** KE_{max}
hf_{photon}	hf_0	$\frac{1}{2}mv_{max}^2$
$h\frac{c}{\lambda_{photon}}$	$h\frac{c}{\lambda_0}$	eV_s

- Another unit for energy is the <u>electronvolt</u> (eV).

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

- 1 eV is the work done on an electron to move it through a potential difference of 1 V.

Recall: work done on a charge = charge × p.d. across which the charge is moved

$$W = qV$$

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C}) \times (1 \text{ V}) = 1.60 \times 10^{-19} \text{ J}$$

Check your understanding 2:

Which of the following statement(s) about photon energy is true?

I Electrons need not absorb all of the energy of a single photon.
II Energy of the photon is the KE of the photoelectron.
III Electronvolt is a unit for potential difference. 1 eV represents 1.60 x 10^{-19} V.
IV None of the above.

Check your understanding 3:

Which of the following statement(s) about work function energy of photoelectric effect is true?

I All electrons in the same metal requires the same amount of energy, which is the work function energy, to be emitted.
II Different types of metals have different work function energies.
III All photoelectrons will have $KE = E_{photon} - \phi$

Additional Notes

Based on $E = hf$, or $E= hc/\lambda$, longer wavelength or lower frequency implies lower photon energy.

Worked Example 1

The work function energy ϕ of potassium is 2.0 eV.

(a) Determine the value of ϕ in joules (J).

(b) Calculate the threshold frequency of potassium.

(c) Calculate the threshold wavelength of potassium.

(d) Explain whether incident radiation of wavelength 7.20 × 10^{-7} m can be used to emit electrons from potassium.

Solution

(a) work function $\phi = 2.0 \times 1.60 \times 10^{-19} = 3.20 \times 10^{-19}$ J

(b) $\phi = hf_0$

threshold frequency $f_0 = \dfrac{\phi}{h} = \dfrac{3.2 \times 10^{-19}}{6.63 \times 10^{-34}} = 4.83 \times 10^{14}$ Hz

(c) $c = f_0 \lambda_0$

threshold wavelength $\lambda_0 = \dfrac{c}{f_0} = \dfrac{3.00 \times 10^{8}}{4.83 \times 10^{14}} = 6.21 \times 10^{-7}$ m

Alternatively,

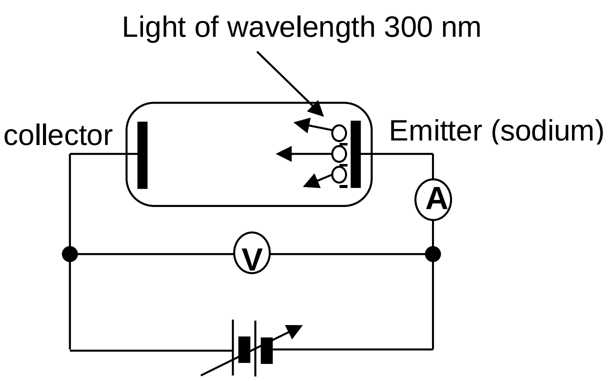
$$\phi = \frac{hc}{\lambda_o}$$

$$\lambda_o = \frac{hc}{\phi} = \frac{6.63 \times 10^{-34}(3.00 \times 10^{8})}{3.2 \times 10^{-19}} = 6.21 \times 10^{-7} \text{ m}$$

(d) At 7.20 × 10^{-7} m, wavelength of radiation is greater than the threshold wavelength. The incident photons have energy lower than the work function of potassium. Therefore, no electrons will be emitted.

Example 1

A freshly prepared sodium surface is illuminated with light of wavelength 300 nm as shown in the setup below. The work function energy for sodium metal is 3.94 × 10^{-19} J. Determine the stopping potential for the setup.



Additional Notes

Solution

A.8 Explanation of the photoelectric effect using Einstein's theory

(i) Observation 1:

- **Increasing the intensity of the radiation has no effect on the maximum energy of the electrons**

Experiment repeated with different intensities I, keeping frequency unchanged

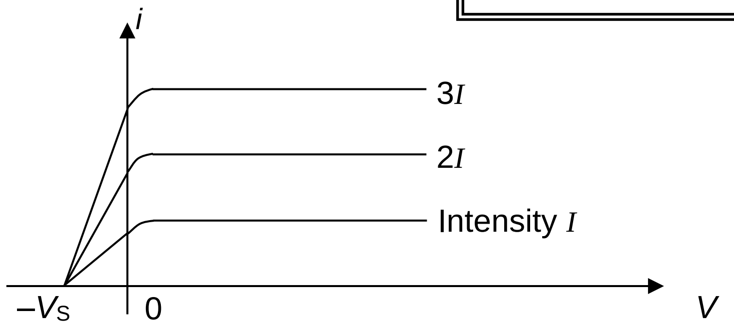


Fig. A.8a

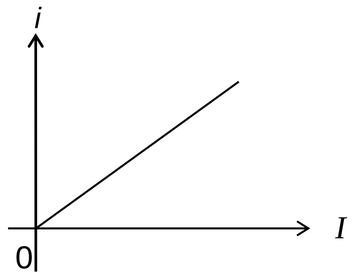


Fig. A.8b

Explanation of Observation 1 using Einstein's Theory:
Equation $KE_{max} = h f - \phi$ shows KE_{max} is independent of the light intensity. The maximum kinetic energy of any electron depends only on the light frequency and the work function energy.

Additional Notes

From the equation, it can be seen that intensity is proportional to rate of incidence of photons. Do note that rate of incidence of photons is not equal to rate of emission of photoelectrons, since not all photons will be absorbed and not all absorption will lead to emission.

- Photoelectric current is proportional to the intensity of EM radiation

$$\text{Intensity of radiation } I = \frac{\text{Power of radiation incident on surface}}{\text{Area illuminated}}$$

$$= \frac{\text{Total energy incident on surface}}{\text{Time taken x Area illuminated}}$$

$$= \frac{\text{number of photons x energy of 1 photon}}{\text{time taken x area illuminated}}$$

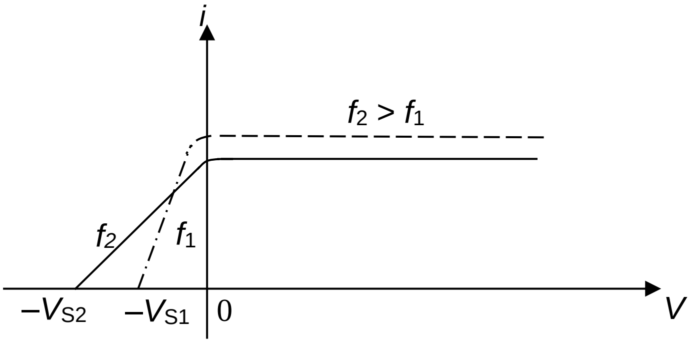
$$= \frac{nhf}{tA} = \frac{n}{t}\left(\frac{hf}{A}\right) \text{ where } \frac{n}{t} \text{ is the rate of photons arriving on the surface}$$

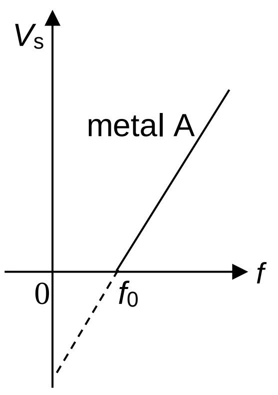
An increase in radiation intensity I, when frequency f is kept constant, results in an increase in the rate of photons arriving, but not in the energy of individual photon. Thus the number of photoelectrons emitted per second is increased but not its maximum kinetic energy. This increases the photocurrent *i*.

(ii) Observation 2:

- **For a given metal, no electron is emitted if the frequency of the radiation is below a certain threshold frequency, even with high intensity of radiation.**

- **Above the threshold frequency, the maximum kinetic energy of the emitted electron increases linearly with the frequency of the radiation, even with low intensity of radiation,**





Additional Notes

Explanation of Observation 2 using Einstein's Theory:
The increase in maximum KE with higher frequency radiation is due to the greater photon energy of such radiation.

Since $E = hf$
As f increases, energy of photon increases,
so from Einstein's photoelectric eqn: $hf - \phi = \frac{1}{2} mv^2_{max} = eV_s$
Thus when f increases, KE_{max} increases *(or* v_{max} increases *or* V_s increases*)*.

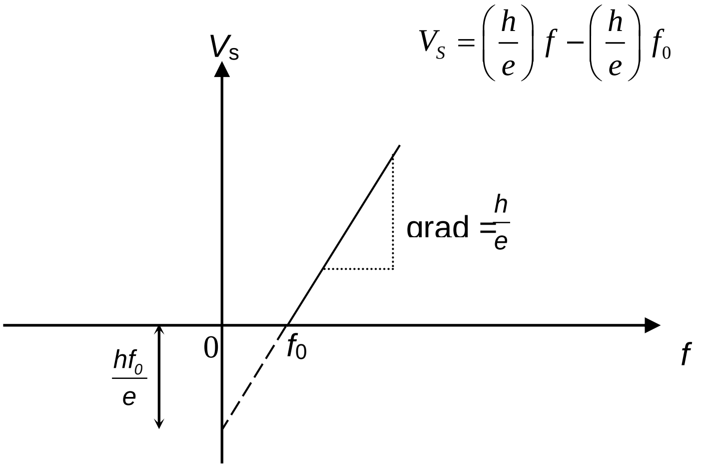
When $f < f_o$, no individual photon imparts enough energy to an individual electron to cause ejection, regardless of light intensity. Hence there is no photoelectric emission.

To explain the linear graph of V_s against f, we can use the photoelectric equation:

$$KE_{max} = h f - \phi$$

$$eV_s = h f - h f_0$$

$$V_s = \left(\frac{h}{e}\right) f - \left(\frac{h}{e}\right) f_0$$



From Einstein's photoelectric equation, we can predict a linear relationship between the maximum electron kinetic energy KE_{max} and the light frequency *f*.

Experimental observation of a linear relationship between KE_{max} and f confirmed the validity of Einstein's theory.

How would you sketch the graph if the experiment is repeated with metal surface having a larger work function?

The gradient of the graph is *h/e* and the intercept on the horizontal axis gives the threshold frequency below which no photoelectrons are emitted, regardless of the light intensity.

(iii) Observation 3:

- **Emission of photoelectrons starts with no observable time lag, even for very low intensity of incident light.**

  Explanation of Observation 3 using Einstein's Theory:
  Instantaneous emission of electrons with no observable time lag is consistent with the photon model of light. The incident energy appears in small packets, and there is a one-to-one interaction between photons and electrons. Even if the incident light has very low intensity, there may be very few photons arriving per unit time interval, but each one can have sufficient energy to eject an electron immediately.

Example 2

A 80 W lamp gives light of wavelength 550 nm. Calculate the number of photons emitted per unit time.

Thinking Process

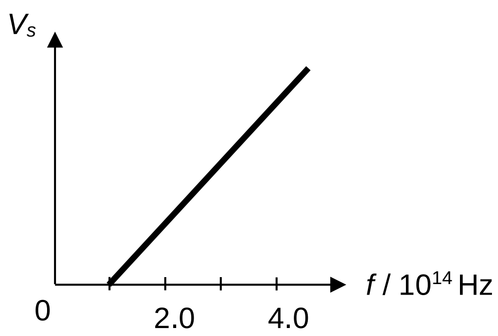
How is the number of photons emitted per unit time related to the information (power and wavelength) given in the question?

- Photons of a particular wavelength carry a certain amount of energy.
- Since power is energy per unit time, the number of photons per unit time gives rise to the energy per unit time of 80 W.

Solution

Example 3

The graph shows how the stopping potential of photoelectrons varies with the frequency of incident radiation for metal X.



(a) Determine the gradient of the graph.

(b) State the threshold frequency of metal X.

(c) Calculate the work function energy of metal X.

(d) If radiation of frequency 9.0×10^{14} Hz falls onto the plate, what is the maximum kinetic energy of the photoelectrons emitted?

Additional Notes

Thinking Process

- **What do we look out for when graphs are given in the question?**
  - When dealing with graphs, it is always important to look at the axes.
  - Gradient and area under graphs may have physical meanings, but not always.

(a) No values are given on the vertical axes – the convention way of calculating gradient using coordinates is not going to work.

How else can we determine the gradient of a line?

- By looking at the equation of the line: The only equation that links V_s with f is Einstein's photoelectric equation.

(b) Command word is 'state'. This suggests that the value required is easily obtained.

(c) Direct application of equation.

There are not many ways of calculating KE_{max} (recall previous example).

Which is a more appropriate method to find KE_{max}?

- Since neither values of V_s nor v_{max} are given, use Einstein's photoelectric equation.

Solution

A.9 Summary of the photoelectric effect:

The following are observations that contradict predictions from the wave-model of light.

(i) Increasing the intensity of the radiation has no effect on the maximum energy of the electrons

(ii) Above the threshold frequency, the maximum kinetic energy of the emitted electron increases linearly with the frequency of the radiation, even with low intensity of radiation.

(iii) For a given metal, no electron is emitted if the frequency of the radiation is below a certain threshold frequency, even with high intensity of radiation.

(iv) Emission of photoelectrons starts with no observable time lag, even for very low intensity of incident light.

By employing a particle-model of light, Einstein was able to successfully account for these observations. Hence, the photoelectric effect provides evidence for the particulate nature of electromagnetic radiation.

Einstein was award the Nobel Prize in 1921, "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect".

B Wave-Particle Duality

B.1 De Broglie's Theory

It is useful to note that

$$KE = \frac{1}{2}(\frac{p^2}{m})$$

So that you can fiind wavelength of particle without having to find speed of particle first.

If a particle acts like a wave, it should have a wavelength and a frequency. De Broglie suggested that a particle of mass *m* moving with speed *v* should have a wavelength λ related to its momentum *p* = *mv* in exactly the same way as for a photon. The **de Broglie wavelength** of a particle is then

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

de Broglie's equation

De Broglie's proposal was a bold one, made at a time when there was no direct experimental evidence that particles have wave characteristics. But within a few years, his ideas were resoundingly verified by a diffraction experiment with electrons. The Nobel Prize in Physics 1929 was awarded to Louis de Broglie "for his discovery of the wave nature of electrons".

i.e. You should use De Broglie's equation to find wavelength of particles. BUT, you cannot use De Broglie's equation to find wavelength of an electron, then use *E* = *hc/λ* to find energy of the electron. This is wrong and it will not match the KE of the electron.

Check Your Understanding 4:

The wavelength of an electron was calculated using De Broglie's equation as 400 nm. What is the kinetic energy of the electron?

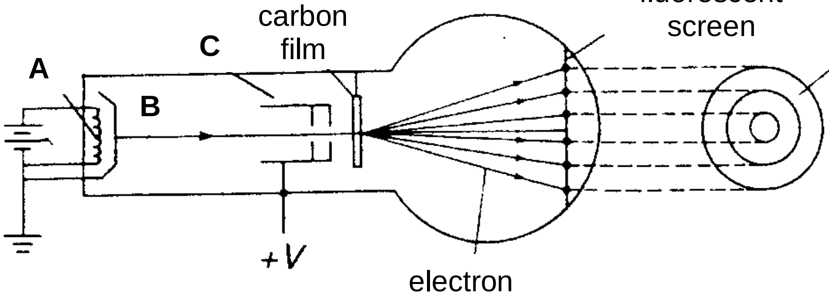
A 4.97×10^{-19} J

B 1.51×10^{-24} J

Additional Notes

B.2 Electron Diffraction Experiment

- The electron diffraction experiment provides evidence that electrons may exhibit wave-like behaviour.



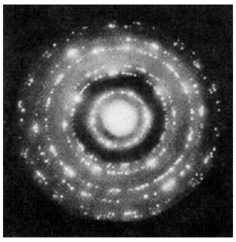
- Components of set-up:

  A: heated cathode which emits electrons (thermionic emission)
  B: electron beam
  C: anode at high positive potential (to accelerate the electrons)
  Carbon film: regular arrangement of the atoms diffract electrons (spaces between the carbon lattice act like the 'lines' in a grating)
  Fluorescent screen: to detect the locations of arriving electrons (when electrons strike the screen, it fluoresces)

❖ Process:

A potential difference is set up between the cathode and anode that accelerates the electrons. (i.e. during this process, the loss in electric potential energy is converted to a gain in kinetic energy). When the accelerated electrons reach the anode, they possess a momentum such that their de Broglie wavelength (see Section B.1) is in the same order of magnitude as the separation between atoms in the carbon film.

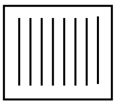
❖ Observation and Explanation:



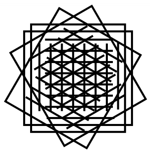
- Upon the electrons passing through the carbon film, the pattern produced on the screen is observed to be similar to the <u>interference pattern produced by electromagnetic radiation</u>. This is the electron diffraction phenomenon.
- It can be concluded that the electrons must have undergone diffraction with the carbon film acting as 3-D diffraction grating, hence exhibiting wave-like property.

<u>Note:</u>
- The regions of maxima are where the rate of incidence of electrons is high.
- The regions of minima are where the rate of incidence of electrons is practically zero.

Instead of having 'lines' only in one direction,



the carbon film gives 'lines in many directions because of its 3D structure.



∴ instead of getting the usual diffraction pattern consisting of straight fringes, a circular diffraction pattern is now observed.

Example 4

Observations at the microscopic level led to the idea of wave-particle duality.
Explain why sound waves are not normally considered as having particle-like properties, nor raindrops are not normally considered as having wave-like properties.

Thinking Process

We need to make reasonable estimate of sound's wavelength and velocity. When put these values into de Broglie's equation, we yield a ridiculous result for its mass, thus showing that its mass is undetectable.

We need to make reasonable estimate of raindrop's mass and velocity. When we put these values into de Broglie's equation, we yield a ridiculous result for its wavelength, thus showing that its wavelength is undetectable.

Solution

Additional Notes

C Energy Levels in Atoms

- To account for the existence of the bright and dark lines observed in the emission and absorption line spectra respectively, Niels Bohr (Nobel Prize, 1922) proposed that the atomic structure follows a 'planetary model'.

- In Bohr's model, the electrons in an atom move in fixed orbits around a central positive nucleus, just as how planets orbit around the Sun.

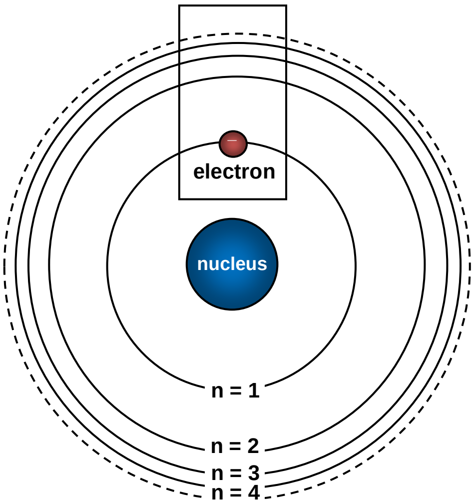
- The required centripetal forces to keep the electrons in their orbits are provided by electrostatic forces between the electrons and the nucleus, instead of gravitational forces in the case of the solar system.

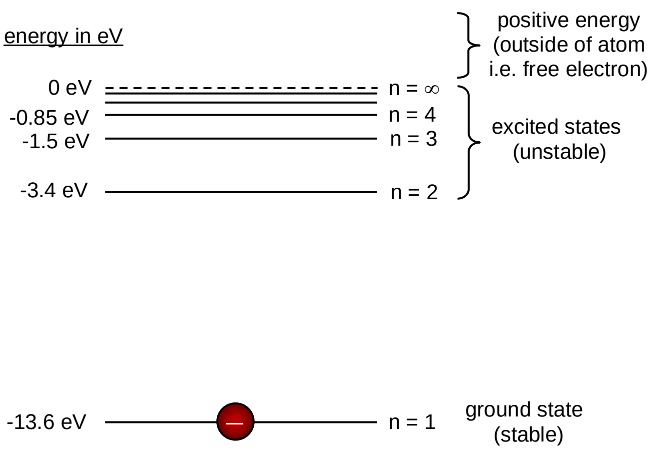
- However, this model could not be explained using classical physics and was in conflict with the electromagnetic theory which predicts that the accelerating electrons radiate (hence lose) energy and will eventually spiral into the nucleus.

Accelerating charges produce changing electric and magnetic fields. Changing electric fields produce magnetic fields and changing magnetic fields produce electric fields. This interplay between induced electric and magnetic fields leads to propagating electromagnetic waves.

http://labman.phys.utk.edu/phys222core/modules/m6/production_of_em_waves.html

- To overcome this flaw, Bohr further suggested the following important proposals:
  - Electrons are allowed to revolve around certain definite orbits, each associated with a definite energy level in such a way that they do not emit radiation;
  - An electron can jump from one energy level to another energy level by the absorption or emission of a quantum of radiation of frequency *f*; and
  - Energy atom of different elements has a characteristic set of energy levels.





The energy of the electrons are negative because forces are attractive between the nucleus and electrons. It is also an indication that the electrons lacks energy to escape from its current position.

- Important notes:
  - An (free) electron outside an atom has zero energy when it is at rest and positive energy when it is in motion (kinetic energy).
  - An electron within an atom has negative energy and can only take the value of the quantized energy level it is in.
  - An electron can pass from one energy level to another by absorbing or emitting energy – to jump to a higher energy level, it needs to absorb energy; to transit to a lower energy level, it needs to emit energy.
  - Within an isolated atom, the energy is absorbed or emitted as a single photon whose energy *hf corresponds* exactly to the difference in energy levels between the initial and final energy levels, $E_f - E_i$.

$$E_{photon} = hf = |E_f - E_i|$$

When atoms are isolated, they have discrete energy levels. When atoms are sufficiently close, they form energy bands instead. https://www.e-education.psu.edu/matse81/node/2227

  - When every electron in an atom is at the lowest available energy level, the atom is said to be at the ground state or most stable state. If an electron absorbs the right amount of energy, it will jump to a higher energy level and the atom is said to be excited. The energy that it needs to absorb is called excitation energy.
  - When electron is at a higher energy level, it is less stable and tends to de-excite and drop to lower energy level within a short period of time (if there are vacancies at lower energy states). The de-excitation process is random. This de-excitation process is accompanied by the release of a photon.

This random de-excitation explains why filament lamp is not a coherent light source.

  - Within an atom, electrons in the lower energy levels are held strongly by the nucleus. Electrons in the higher energy levels are more loosely held and require less energy to escape.
  - For the hydrogen atom, when its electron (the hydrogen atom has only one electron) jumps from the ground state to the highest energy level (at this state, the electron is no longer bound to the atom), the energy absorbed is called the ionization energy.

Ionization energy is NOT the work function.

  - If an electron absorbs more energy than the ionization energy, the excess energy will appear as kinetic energy of the (free) electron.

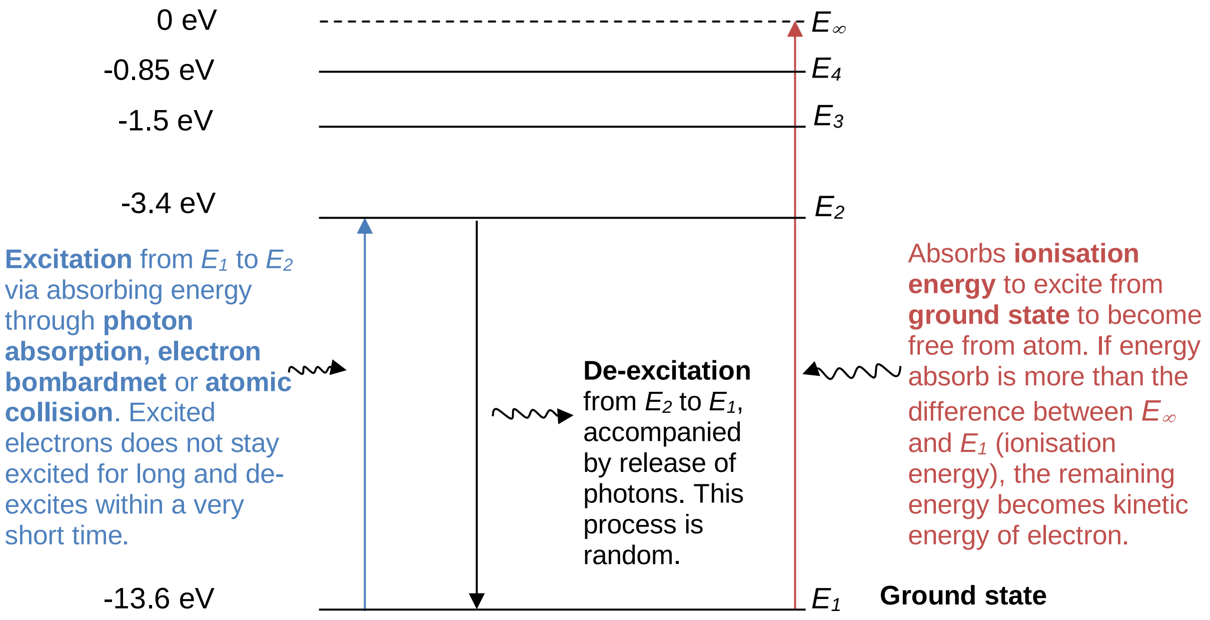
Additional Notes

C.1 Different Methods of Exciting an Atom (or Electron)

	Method of excitation	Necessary conditions	Remarks
1	Photon absorption	The photon will only be absorbed if its energy is exactly equal to the energy difference between the initial energy level of the electron and any other higher energy levels. A photon is a packet of energy. Photon absorption is hence an all-or-nothing process.	This is how an absorption line spectrum is obtained. If the photon is not absorbed, it will simply pass through the gas.
2	Electron bombardment	The bombarding electron must have KE greater than or equal to the energy difference between the initial energy level of the electron and any other higher energy levels. A free electron can lose any fraction of its KE during collision with an atom.	This is how a discharge tube works. The electrodes placed across the gas will pass an electric current through the gas. When the free electrons from the current collide with the gas atoms, the atoms will be excited. When they de-excite, they give off EM waves such as light.
3	Atomic collision	When two or more atoms collide inelastically, the loss of KE of the system is used to excite the colliding atoms. Amount of KE lost must be sufficient to result in excitation. An atom can also lose any fraction of its KE during the collision.	

Additional Notes

Summary



Energy Level Diagram of Isolated Hydrgen Atom

- For scenario where excitation is to higher energy levels such as E_3 from E_1, there are a few possibilities of how it may de-excite. For example, some electrons excited to E_3 may de-excite directly to E_1. Other electrons may de-excite to E_2 before de-exciting back to E_1.
- Electrons do not stay excited for sustained duration.
- Electrons at higher energy level tends to de-excite to lower energy levels (e.g. ground state) eventually.

Check your understanding 5:

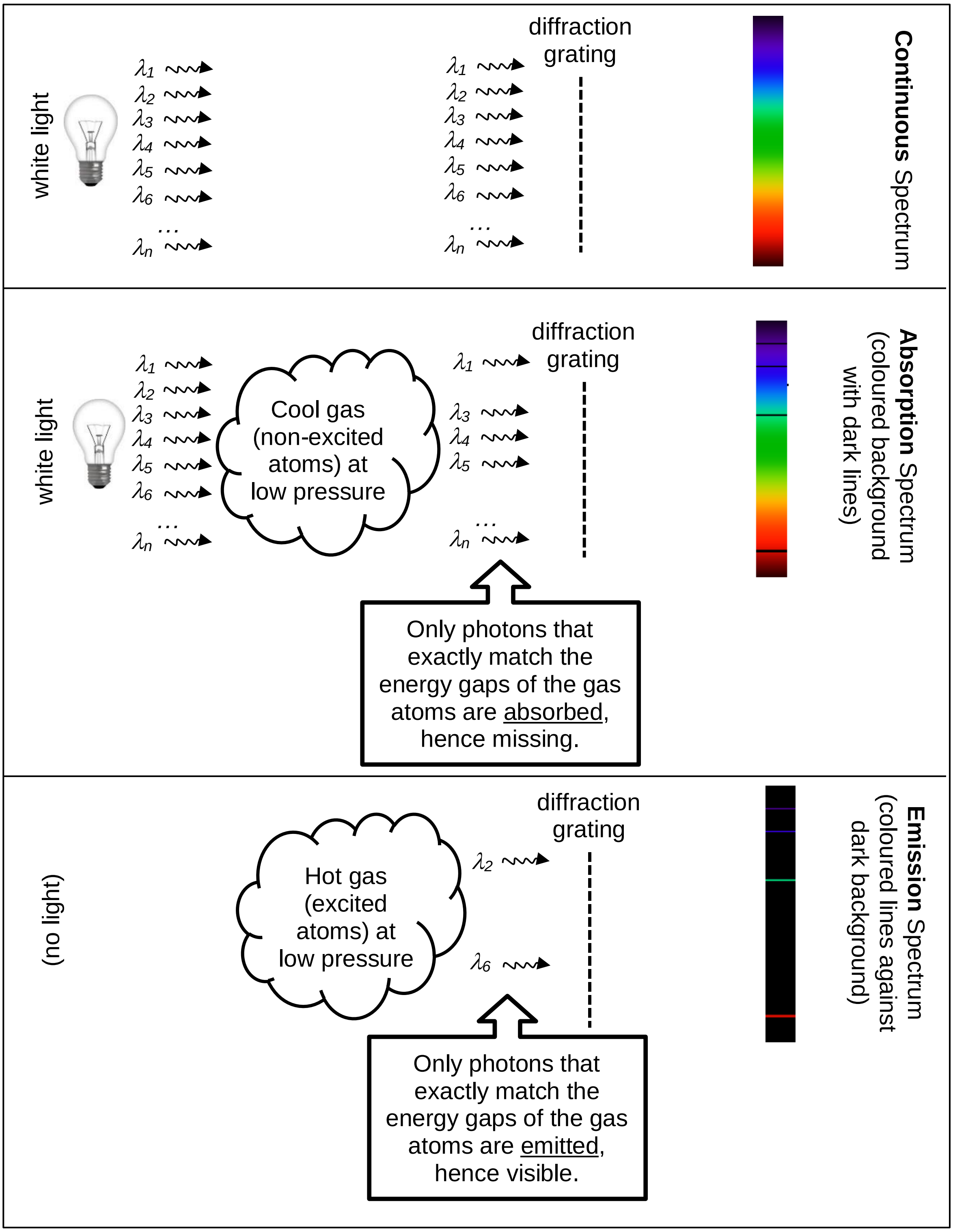
With reference to energy level diagram on pg 19-21, for an electron to excite from level n = 2 to n = 3, it needs to

A absorb photon energy of 1.9 eV or more
B absorb an exact amount of photon energy of -1.9 eV
C absorb an exact amount of photon energy of 1.9 eV
D emit photon energy of 1.9 eV

Additional Notes

D Line Spectra

- Absorption and emission line spectra provide evidence that there are **discrete energy levels in an <u>isolated atom</u>**.



In cases where atoms are not isolated, energy bands are formed instead.

The gases are kept at <u>low pressure</u> to ensure that the atoms are sufficiently far apart to be considered isolated.

To view the line spectra in colour

Note that it is NOT the electrons that are emitted towards the diffraction grating, but the photons generated by de-excitation of the electrons.

Additional Notes

D.1 Continuous Line Spectrum

A continuous rainbow spectrum is obtained when sunlight (or white light) is dispersed using a prism or diffraction grating (without passing through any gas).

D.2 Absorption Line Spectrum

- In an absorption line spectrum, a sample of cool gas at low pressure (e.g. hydrogen gas) is placed between the directed source of white light and the diffraction grating (or prism).
- When unexcited gas atoms absorbs photons from the white light, they get excited and transit to a more excited state, provided the photon energies exactly matches the energy difference between the ground state and any higher levels.
- The leads to the absorption of only the photons with specific energies (and hence , specific frequencies, since $E_{photon} = hf$.
- These photons are then re-emitted randomly in all directions.
- The well-defined and distinct dark lines in the spectrum correspond to the specific frequencies (or wavelengths) of light that are absorbed by atoms in the gas which the white light passed through.

Cool gas to ensure the gas atoms are not already excited, otherwise, it may not be absorbing photons.

Absorption line spectrum is a continuous spectrum crossed by dark lines due to some missing frequencies and is produced when white light passes through a cooler gas or vapour.

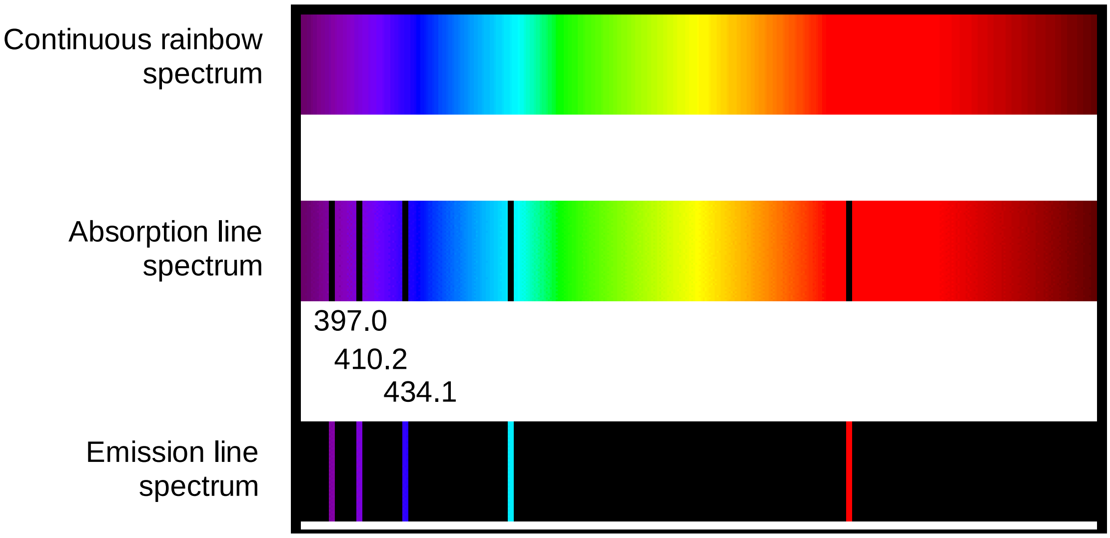
D.3 Emission Line Spectrum

An animation on emission line spectrum

- **How the gas is heated:** Gas at a low pressure is contained in a discharge tube. When the high voltage (p.d.) discharge tube is switched on, free electrons are accelerated and collide with the atoms of the gas thus causing the gas to become hot and the electrons within the gas atoms to be excited, that is, their electrons occupy higher energy levels.
- During subsequent de-excitation to lower energy levels, the gas atoms emit photons of energies corresponding to the difference between the initial and final energy levels.
- Since these energy levels are discrete, only photons of specific energies can be emitted. This result in a spectrum consisting of well-defined and distinct colored lines superimposed on a dark background which characterizes an emission line spectrum.

Emission line spectra consist of quite separate bright lines of definite wavelengths on a dark background and are given by luminous gases and vapours at low pressure.

Additional Notes



To view the line spectra in colour

The positions where the dark lines occur in the Fraunhofer series are used to determine the types of gases present in the sun's atmosphere.

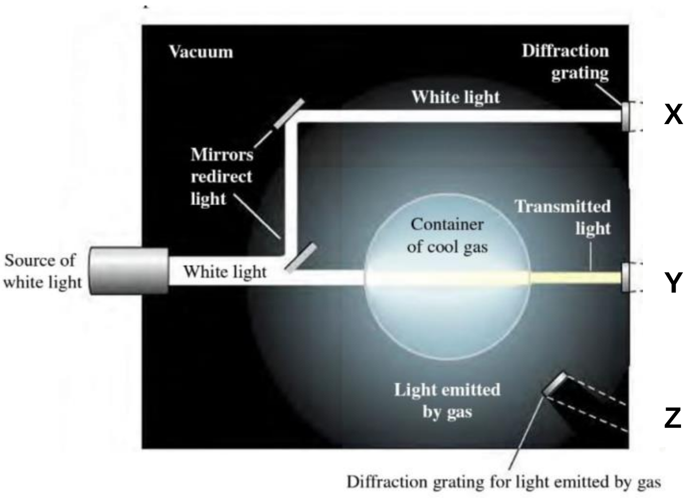
<u>Important notes</u>:

- For the <u>same element</u>, the positions where the bright lines occur on the emission spectrum <u>coincide</u> with the dark lines of the absorption spectrum.
- The positions where the bright and dark lines occur are different for different elements.
- Hence, by studying the positions of the lines, information on the type of element(s) present can be determined.
- For example, the line spectra from starlight can be used to identify the different types of elements in the atmosphere of these stars.

Check your understanding 6:

From the list below, identify the spectrum that can be observed at the following locations:

A. absorption line spectrum
B. continuous spectrum
C. emission line spectrum



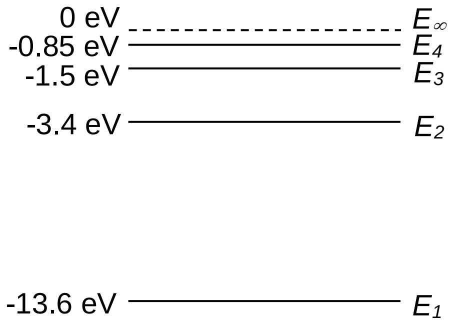
X:

Y:

Z:

Example 5

The first 4 energy levels of a hydrogen atom are shown below.



For hydrogen gas which is cool and at low pressure,

(a) Determine the ionization energy of the hydrogen atom.

(b) Determine the minimum frequency of the photon that is required to cause an electron in the ground state of the hydrogen atom to be ionized.

(c) Describe what would be likely to happen to photons of energies 11.9 eV, 12.1 eV and 15.0 eV if they pass through gaseous hydrogen atoms in the ground state.

(d) Bombarding electrons collide with gaseous hydrogen atoms in the ground state.

- **(i)** Determine the amount of energy absorbed by the gaseous hydrogen atom electrons in the ground state,
  - 1) when the bombarding electrons each carry energy of 5.0 eV, and
  - 2) when the bombarding electrons each carry energy of 11.0 eV.
- **(ii)** Determine the amount of energy retained by the bombarding electrons,
  - 1) when the bombarding electrons each carry energy of 5.0 eV, and
  - 2) when the bombarding electrons each carry energy of 11.0 eV.

(e) If a hydrogen atom absorbs a photon of energy 12.75 eV, show, on the energy level diagram, all transitions that can possibly be detected.

Solution

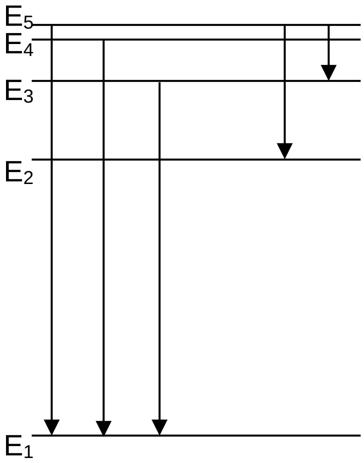
Additional Notes

The **Lyman** Series constitutes the transitions that end at n=1; the **Balmer** at n=2 and **Paschen** at n=3.

Visible light (wavelength: 400nm to 800nm) are part of the **Balmer** series.

Example 6

The diagram shows five energy levels of an atom, one being much lower than the other four. Five transitions between the levels are indicated, each of which produces a photon of definite energy and frequency.



Sketch the line spectrum that corresponds to the set of transitions indicated when it is being viewed through a spectroscope.

Solution

Additional Notes

E X-Ray Spectra

E.1 Discovery of X-Ray Spectra

- Discovered by Wilhelm Roentgen in 1895, he coined this form of radiation X-rays because it was unlike any other types of radiation seen at that time.

- X-rays are electromagnetic waves of short wavelengths in the range of 10^{-9} m to 10^{-12} m. They travel at the speed of light (c = 3.00 × 10^{8} m s^{-1}).

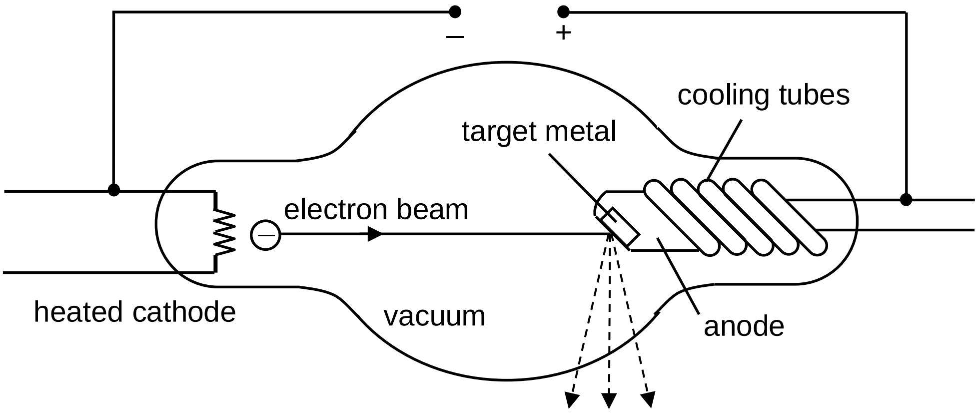
- Their highly-penetrative properties are useful in medical applications e.g. to image bones and in Computerised Tomography (CT) scanning.

- X-rays are also used to identify elements found in samples.

First X-ray scan – hand of Roentgen's wife

E.2 The X-ray Tube

- X-rays are produced when metal is bombarded with <u>high-energy electrons</u>.



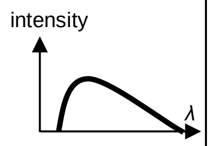
Conservation of Energy
From cathode to anode:
Loss in EPE = Gain in KE

Additional Notes

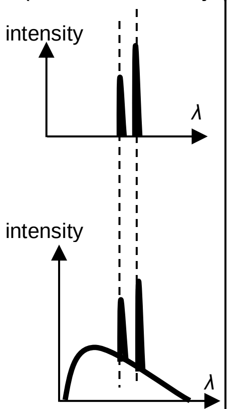
- The components of an X-ray tube include:

Component	Purpose / Details
Heated cathode	Emits electrons via thermionic emission
High voltage supply	Accelerates electrons across the large potential difference to very high speeds
Target metal (at potential of anode)	Usually metals with large atomic numbers to increase probability of x-ray production e.g. tungsten, copper or iron
Cooling tubes	Electrons strike the metal target at very high speeds and will collide with the lattice ions and transfer their kinetic energy to them. This causes the target metal to heat up. A very small percentage of the bombarding electrons will result in the emission of X-ray photons. A cooling system is required to remove the heat from the anode due to the numerous impacts of the electrons on the target metal. This prevents the target metal from melting.

(A) Bremsstrahlung radiation (continuous spectrum)



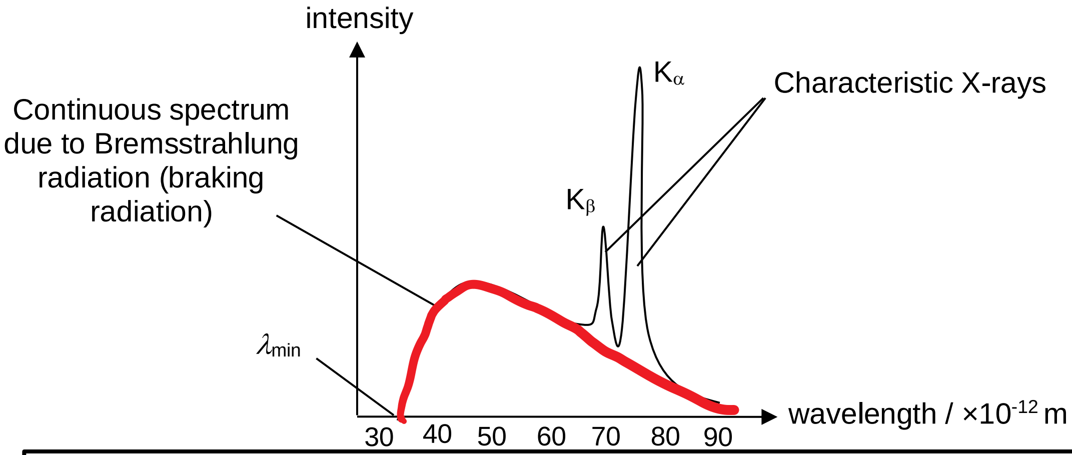
(B) Characteristic radiation (characteristic X-rays)



(A+B) Combined effect of radiation

E.3 The X-ray Spectra

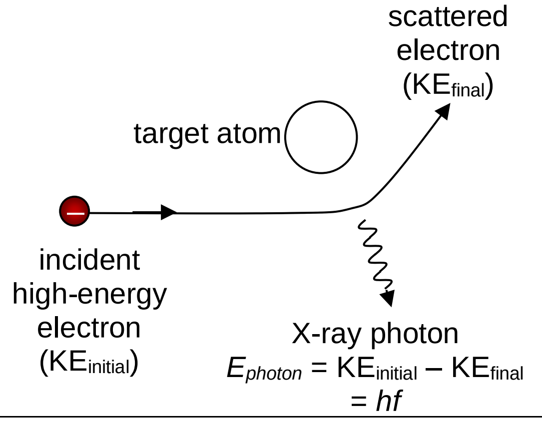
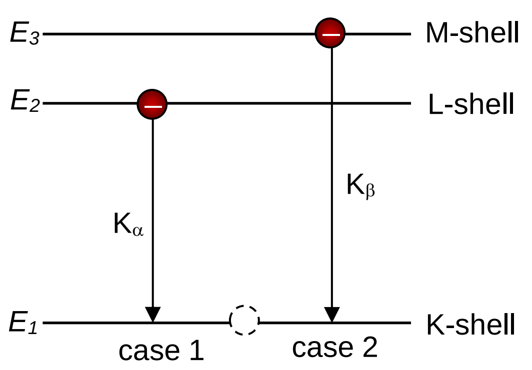
- A typical X-ray spectrum consists of a broad, continuous spectrum of radiation (due to braking radiation) with a cut-off wavelength (λ_{min}), superposed on peaks of sharply defined wavelengths (characteristic X-rays).



- **Braking radiation** (Bremsstrahlung radiation) describes the radiation which is emitted when fast-moving electrons are rapidly slowed down as they pass through the electric field around an atomic nucleus. This effect results in the **continuous spectrum**.

- **Characteristic X-rays** are emitted when an electron in an upper state (L- or M-shells) of an atom drops down to fill the vacated lower state (K-shell) that has its electron dislodged by the bombarding electrons. The pattern of the characteristic X-rays is unique to each element.

- The processes which give rise to the 2 different spectra are:

Continuous Spectrum (Bremsstrahlung/ Braking Radiation)	**Characteristic X-ray Spectrum**
	
When a high-energy electron passes by a target atom, it is scattered. It loses energy ($KE_{initial} - KE_{final}$) as it decelerates and the energy it loses is converted into the energy of one X-ray photon. The further the electron is passing by the atom, the lesser the energy it losses. The scattered electron goes on to be scattered by another atom, losing more energy in the process. The electron-scattering process continues until the electron has lost all its initial KE. All the X-ray photons generated by this process form part of the continuous X-ray spectrum. Important note: An X-ray photon with minimum wavelength (λ_{min}) is obtained if the incident electron loses all its KE during the first scattering process i.e. the value of λ_{min} is dependent on initial KE of the incident electron, hence the accelerating potential between the anode and cathode. $KE_{electron} = E_{most\ energetic\ photon} = \frac{hc}{\lambda_{min}}$ $so,\ \lambda_{min} = \frac{hc}{KE_{electron}} = \frac{hc}{eV}$ where V = accelerating potential (V) λ_{min} is independent of the target metal.	If a high-energy electron collides with an electron from the inner K-shell of a target atom with sufficient energy to remove this innermost electron from the atom, a vacancy is created in the K-shell. Case 1: An electron from the L-shell drops into the vacancy in the K-shell. This transition is accompanied by the emission of a X-ray photon with energy ($E_2 - E_1$). This is called the K_α line. Case 2: An electron from the M-shell drops into the vacancy in the K-shell. This transition is accompanied by the emission of a X-ray photon with energy ($E_3 - E_1$). This is called the K_β line. Important note: The wavelengths of the characteristic X-rays are dependent on the target metal. They are independent of the accelerating potential. $\lambda_{K_\alpha} = \frac{hc}{E_2 - E_1}, \quad \lambda_{K_\beta} = \frac{hc}{E_3 - E_1}$

Additional Notes

Other characteristic X-rays are formed when electrons drop from upper levels into vacancies other than those in the K-shell.

E.g. L lines are produced when vacancies in the L-shell are filled by electrons dropping from higher shells.

These lines occur at positions of longer wavelengths, and are not shown in the graph

Note that the energy changes involved are as below:

From cathode to anode:
Loss in EPE = Gain in KE
(i.e. $eV = KE_{electron}$)

When electrons decelerate:
Loss in KE = Energy of photon
(i.e. $KE_{electron} = E_{photon}$)

Additional Notes

Since (i) intensity is a ratio of power and area and (ii) power is a ratio of energy and time,

$$Int = \frac{n(hf)_{photon}}{tA}$$

For a particular wavelength, the intensity depends on the number of electrons striking target metal per unit time (n/t) and atomic number of target metal.

Notes on Characteristic X-ray:

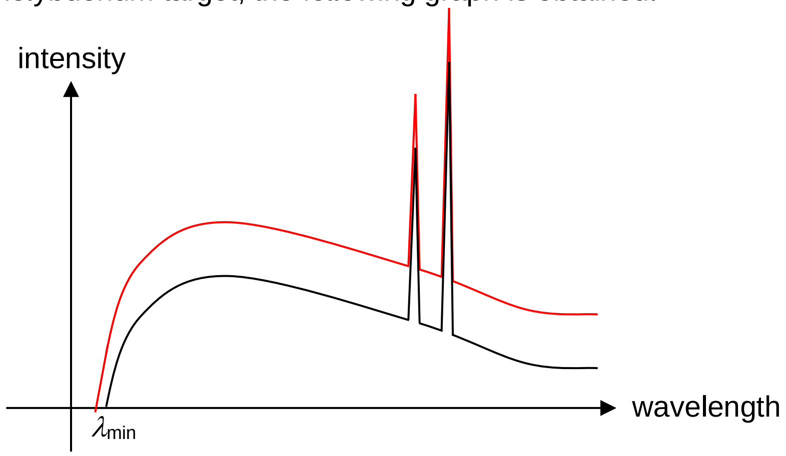
- Although the M $\rightarrow$ K transition produces a X-ray photon with greater energy than the L $\rightarrow$ K one, the probability of the former occurring is significantly lower because the M-shell is further away from the K-shell.

- Hence the K_β peak is lower than the K_α peak and the wavelength of the K_β peak is smaller than the wavelength of K_α peak.

$$\lambda_{K_\alpha} = \frac{hc}{E_2 - E_1}$$

Example 7

In the production of X-rays, electrons are accelerated through a potential of 35 kV. When they strike a molybdenum target, the following graph is obtained.



(a) Explain the effect on the graph, if any, when the speeds of the incident electrons are increased. Sketch the new graph on the same axes above.
(b) Calculate the minimum wavelength of the X-ray emitted.

Thinking Process

(a) **What are important features in the intensity-wavelength graph?**
The minimum wavelength and the characteristic peaks. In the answer, we should consider the changes, if any, in these 2 features.

How do the speeds of the incident electrons affect the X-ray spectrum?
λ_{min} is the wavelength of the most energetic X-ray photon ($E \propto 1/\lambda$). This happens when the incident electron loses all its KE to produce the single X-ray photon. As the speed increases, KE is higher when the electrons hit the target metal. If it now loses lose all its KE to produce one single X-ray photon, this photon must have greater energy i.e. shorter wavelength.

No changes to the positions of the peaks though, because the target metal is not changed.

(b) By COE, energy of X-ray photon = KE of incident electron = energy gained by incident electron when it is accelerated

Additional Notes

Solution

F The Uncertainty Principle

Werner Heisenberg proposed, in 1927, that it is impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy.

The **Heisenberg uncertainty principle** states that if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its linear momentum is made with uncertainty Δp_x, the product of the two uncertainties is restricted to

$$\Delta p_x \, \Delta x \geq h$$

Heisenberg's position-momentum uncertainty:

$$\Delta p \, \Delta x \gtrsim h$$

Δx = uncertainty in measurement of position (m)
Δp = uncertainty in measurement of momentum (kg m s^{-1})
h = Planck constant

The term 'uncertainty' in this topic is of no relation to the uncertainty of measurements in Topic 1.

- The implication of this relationship between Δx and Δp is that the greater the accuracy in determining the position of a particle, the greater the uncertainty in the measurement of its momentum and vice versa.
- Due to wave-particle duality, a particle such as an electron cannot be described as a single point moving in space. As the particle moves, it has some properties of a wave.

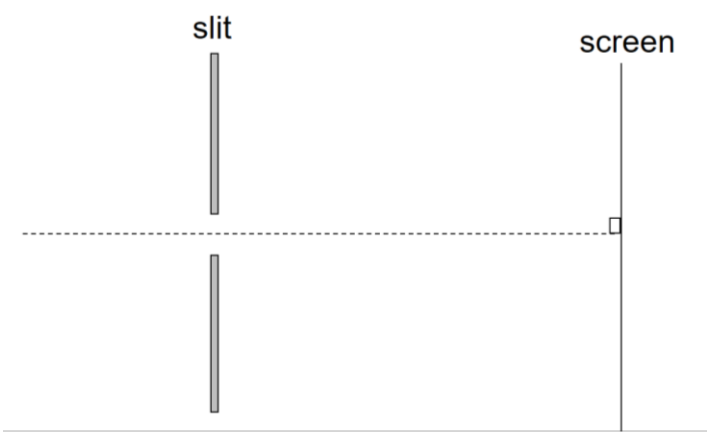
Note that when applying HUP, Δx and Δp has to be along the same axis/direction.

Example 8

In a single slit diffraction experiment of an electron beam, a diffraction pattern is observed on a suitable screen. For an electron passing through the slit, the uncertainty in its position Δx and the uncertainty in its momentum Δρ are related by the following expression

$$\Delta \rho \, \Delta x \gtrsim h$$

In the figure below, draw the positions and directions of Δρ and Δx as defined in the above equation.



Example 9

An electron moves with a constant velocity of 1.50×10^6 m s^{-1}. If its momentum is measured to a precision of 0.20%, calculate the minimum uncertainty in its position.

Solution

Additional Notes

Topic Summary

Photoelectric effect	The ejection of an electron from a metal surface when the surface is irradiated with electromagnetic radiation of a high enough frequency.
Photon	A quantum of electromagnetic energy which is dependent only on the frequency of the radiation. The amount of energy *E* of a photon is given by $E = hf = \frac{hc}{\lambda}$
Work function energy	The minimum amount of energy required to eject an electron from the metal surface.
Threshold frequency	The lowest frequency of radiation that ejects electrons from a particular metal surface.
Einstein's photoelectric equation	$E_{incident\ photon} = \phi_{irradiated\ material} + KE_{max\ (of\ emitted\ photoelectrons)}$ $hf = \phi + \frac{1}{2}mv_{max}^2$
Stopping potential	The stopping potential V_s is the minimum potential difference to reduce the photoelectric current to zero. All the kinetic energy of the most energetic electron will be converted into electrical potential energy, eV_s. $eV_s = \frac{1}{2}mv_{max}^2$
de Broglie wavelength	A particle with momentum *p* is associated with a wavelength λ give by the de Broglie equation: $\lambda = \frac{h}{p}$
Transition to higher energy levels	This phenomenon occurs when: (1) the atom absorbs a photon with energy *hf* = excitation energy, or (2) the atom collides with a particle (eg an electron) with $KE \geq$ excitation energy.
Transition to lower energy levels	A photon is released (emitted as electromagnetic radiation) with energy corresponding to the ΔE between the levels.
Absorption line spectrum	Absorption line spectrum is a continuous spectrum crossed by dark lines due to some missing frequencies and is produced when white light passes through a cooler gas or vapour.
Emission line spectrum	Emission line spectra consist of quite separate bright lines of definite wavelengths on a dark background and are given by luminous gases and vapours at low pressure.

Braking radiation (Bremsstrahlung radiation)	Bremsstrahlung (braking radiation) describes the radiation which is emitted when fast-moving electrons are rapidly slowed down as they pass through the electric field around an atomic nucleus. This effect results in the **continuous spectrum**.
Characteristic X-rays	Characteristic X-rays are emitted when an electron in an upper state (L- or M-shells) of an atom drops down to fill the vacated lower state (K-shell) that has its electron dislodged by the bombarding electrons. The pattern of the characteristic X-rays is unique to each element.
Heisenberg Uncertainty Principle	The **Heisenberg uncertainty principle** states that if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its linear momentum is made with uncertainty Δp_x, the product of the two uncertainties is restricted to $\Delta p_x \, \Delta x \geq h$ Another form of the uncertainty principle relates measurements of energy and time: $\Delta E \, \Delta t \geq h$