Name: _____ ( ) Date: _____ Class: _____

June Holiday Revision

Question 1

Find the solutions of $(5n-3)(7n+11)=13(7n+11)$.

Question 2

Solve $x^2 = 7x$.

Question 3

(a) By completing the square, express $-x^2+8x+15$ in the form of $-(x+a)^2+b$.

(b) Hence, solve $-x^2+8x+15=0$.

Question 4

Solve the equation $2(x+1)(x-2)=7$, giving your answers correct to 2 decimal places.

Question 5

A quadratic function is given by $2(3x+5)-(x+2)(2x-3)=0$. Solve the equation leaving your answer correct to 3 significant figures.

Question 6

Solve $(2x-1)^2-(x-4)^2=3x+7$, leaving your answer to 3 significant figures.

Question 7

Solve $\frac{4}{p}-\frac{5}{p+3}=\frac{p+8}{(p+3)^2}$.

Question 8

(a) Solve the equation $\frac{1}{4x^2-4x+1}+\frac{3}{2x-1}=4$.

(b) Hence, without solving, deduce the solutions for the equation $\frac{1}{(a-1)^2}+\frac{3}{a-1}=4$.

Question 9

A lorry was driven 45 km from Town P to Town Q at an average speed of y km/h. It returned from Town Q to Town P by the same route. The average speed for the return journey was 5 km/h less than the outward journey.

(a) Write down an expression, in terms of y, for the number of hours taken to travel from Town P to Town Q.

(b) Write down an expression, in terms of y, for the number of hours taken to travel from Town Q to Town P.

(c) If the return journey was 1 hour 30 minutes more than the outward journey,

(i) write down an equation in y, and show that it reduces to $y^2-5y-150=0$,

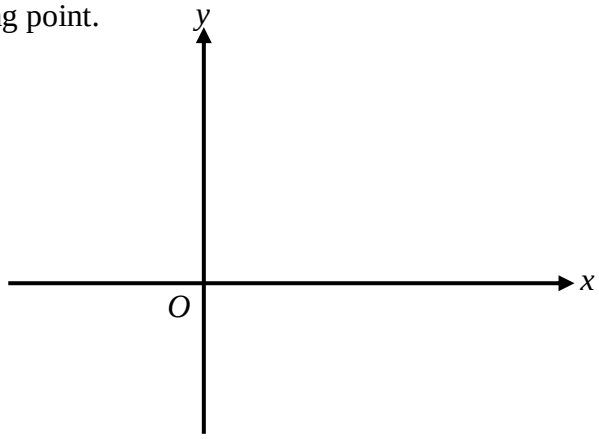
(ii) solve the equation $y^2-5y-150=0$,

(iii) find the total time the lorry was being driven.

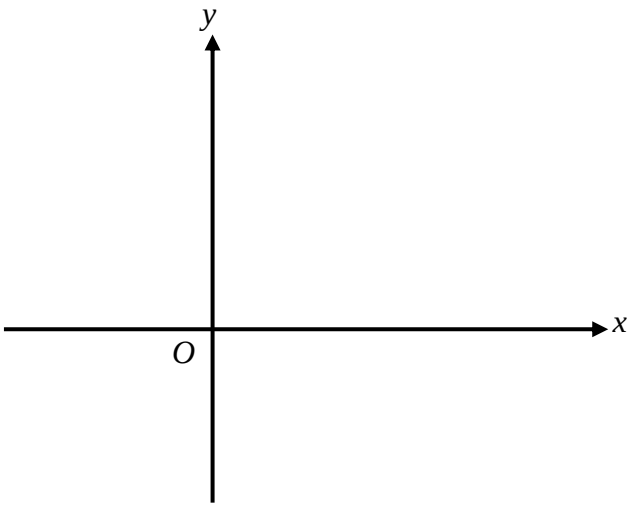
Question 10

(a) Express $y = x^2 - 6x + 4$ in the form $y = (x-a)^2 + b$.

(b) Hence, sketch the graph $y = x^2 - 6x + 4$, indicating clearly the points of intersection with the axes and the turning point.



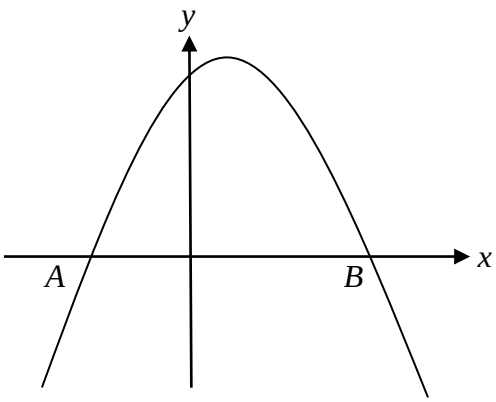
(c) Sketch the graph of $y = x(5-x)$.



(d) Write down the equation of the line of symmetry of the curve.

Question 11

The diagram shows the graph $y = -x^2 + 3x + 4$. It cuts the x-axis at A and B.



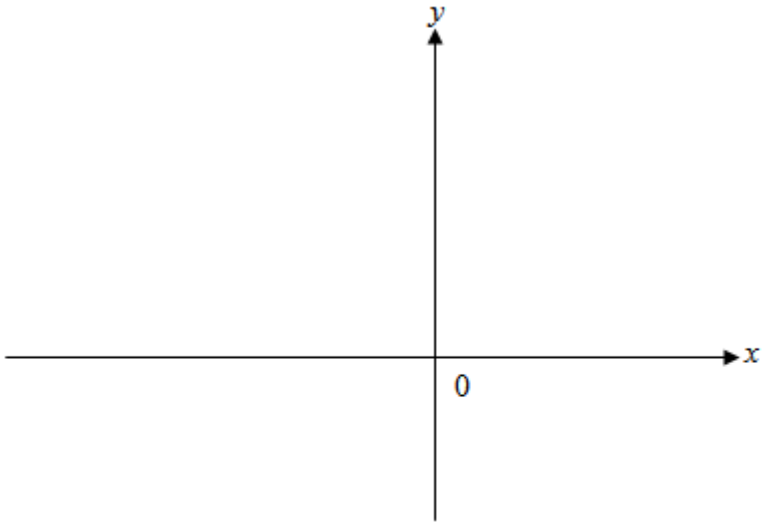
(a) Find the coordinates of A and B.

(b) Find the equation of the line of symmetry for the graph.

Question 12

(a) Sketch the quadratic graph of $y = 5-(x+2)^2$.

(b) Write down the coordinates of the maximum point of the curve $y = 5-(x+2)^2$.



Question 13

Solve the inequality $\frac{2}{3}m - \frac{m-1}{4} < 7$.

Question 14

Given that the inequality $5(x+7) \geq 6(x-2)+2(x-1)$, find the greatest possible value of x if x is

(a) an integer,

(b) a prime number.

Question 15

(a) Solve the inequalities $\frac{x-1}{3} < \frac{2x+2}{5} \leq 4$.

(b) Hence, find the perfect square values of x which satisfy the inequalities $\frac{x-1}{3} < \frac{2x+2}{5} \leq 4$

Question 16

Given that $\frac{x-11}{2} < \frac{3-2x}{5} \leq \frac{x+7}{3}$, find

(a) the range of values of x that satisfy the inequality and represent your solutions on a number line.

(b) the least integer value of x.

(c) the greatest prime value of x.

Question 17

Given that $\frac{2-x}{4} \leq 3+x < 11$ and $0 < y < 5$, where x and y are integers, find the

(a) range of values of x.

(b) greatest possible value of $(x+y)^2$.

(c) least possible value of $(x+y)(x-y)$.

Question 18

The school needs to buy new sets of tables and chairs to replace the damaged ones in the hall. The school found that at least 20 sets of tables and chairs need to be replaced. Each chair costs \$90 and each table costs \$60. The budget allocated for the replacement of furniture is \$4000 for this year.

(a) Form two inequalities in x, the number of sets of tables and chairs to be purchased.
(b) Solve the inequalities.
(c) Hence, find the maximum sets of tables and chairs that the school can buy.

Question 19

Simplify $\dfrac{8a^3b^6 \times 3\sqrt{a}}{(2ab)^3 \times (3a)^0}$ giving your answer in positive indices only.

Question 20

Simplify $\dfrac{4(a^2c)^2}{9a^2c} \div \dfrac{4a^2c}{9}$.

Question 21

Simplify $\dfrac{24m^4}{40n^3} \div \dfrac{3m^3}{20n^3} \times \dfrac{6m^2n}{(5n^2)^2}$, giving your answer in the lowest terms.

Question 22

Simplify $\dfrac{2a^2b}{7xy^3} \div \dfrac{14ab^2}{56(xy)^2}$.

Question 23

Solve $16^{3x+1} = 32^{1-x}$.

Question 24

Simplify $\dfrac{(-2c)^4}{\sqrt[3]{0.125b^6}} \div \dfrac{8\sqrt{bc}}{-2ab^3c^{-2}}$.

Question 25

Simplify $(16a^4)^{\frac{1}{2}} \times \dfrac{a^{-3}b^2}{8b^0}$, expressing your answer in positive indices only.

Question 26

Simplify $\dfrac{x^4y^{\frac{3}{5}}}{6x} \times \dfrac{12x^{-3}}{y^{-\frac{7}{5}}}$, leaving your answers in positive index form.

Question 27

Simplify $\dfrac{3a^{-3}}{\left(2a^{-\frac{1}{2}}b\right)^2} \div \dfrac{15(a^4b^3)^{-\frac{1}{2}}}{8a^{-3}\sqrt{b}}$, leave your answer in positive indices.

Question 28

If $\sqrt{a\sqrt{a}} = a^k$, find the value of k.

Question 29

Matthew has \$20 000 to invest in Company *A* or Company *B*. Company *A* offers 3.8% per annum simple interest while Company *B* offers 3.5% per annum compound interest, compounded half-yearly. Matthew wishes to invest the money for a period of 5 years. Which company should he invest in? Explain your answer.

Question 30

A printing company produces magazines, books and banners.

(a) The cost of printing a magazine in 2012 was 95 cents. The company produces 50000 magazines in 2012. Find the total cost of producing 50000 magazines in dollars.

(b) The company sold each magazine at a profit of 120%. Calculate the profit earned for selling 50000 magazines.

(c) The company borrowed \$200000 from Bank *A* which charges simple interest at 8% per annum, find the interest the company has to pay to Bank *A* after 30 months.

(d) The company borrowed the same amount from Bank *B* which charges interest at 5% per annum, compounded yearly, for 30 months. Find the amount the company could save as compared to Bank *A*.

Question 31

The world's population is predicted to reach 0.097×10^{11} by 2050.

(a) 0.097×10^{11} can be expressed as n million. Find n.

(b) The world's population was 7.2×10^{9} in 2013. Find the predicted increase in the world's population from 2013 to 2050, giving your answer in standard form.

Question 32

	China	India	USA
Population	1.3×10^{9}	1.2×10^{9}	3.1×10^{8}
Land area (km^2)	9.6×10^{6}	3.3×10^{6}	9.6×10^{6}

Use as much information from the table above as necessary to answer the following.

(a) Find the ratio of the population of USA to the population of China.

(b) How many more people live in India than in USA? Give your answer in standard form.

(c) Which country has the highest population density? Explain your answer.

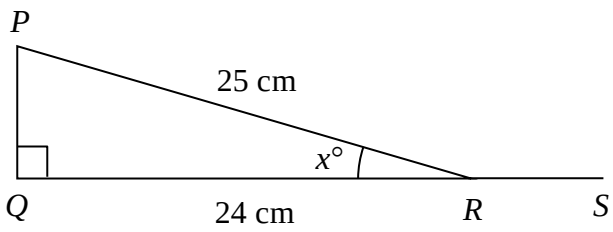
Question 33

PQR is a right-angled triangle in which $\angle PRQ = x°$, *QR* = 24 cm and *PR* = 25 cm. The point *S* lies on *QR* produced. Write down, as a fraction, the value of

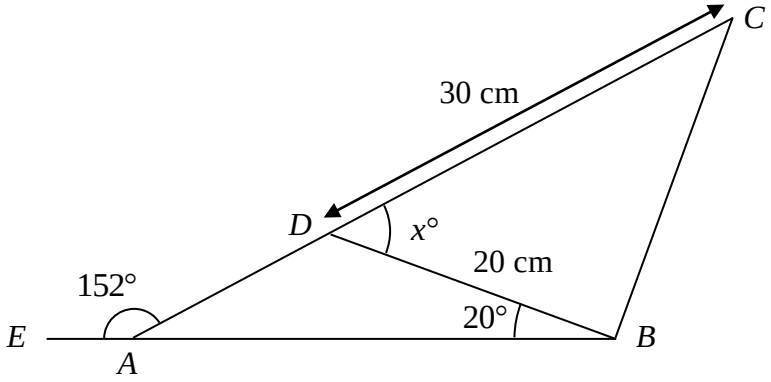
(a) $\cos\angle PRS$.

(b) $\tan(90-x)°$.

(c) $\sin(180-x)°$.



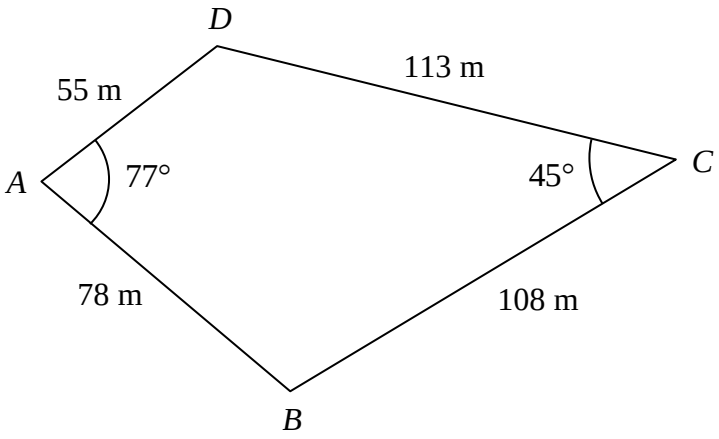
Question 34



In the figure, BD = 20 cm, CD = 30 cm, $\angle ABD = 20°$ and $\angle CAE = 152°$. Find
(a) the value of x,
(b) the area of ΔBCD, correct to 3 significant figures.

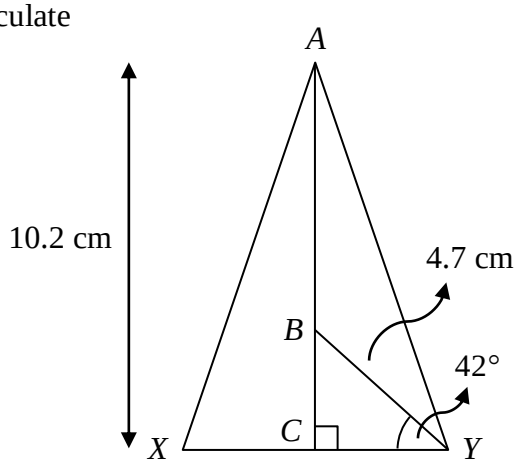
Question 35

The diagram shows a park $ABCD$. It is given that AB = 78 m, BC = 108 m, CD = 113 m, AD = 55 m, $\angle BAD = 77°$ and $\angle BCD = 45°$. Calculate the area of the park. Correct your answer to 3 significant figures.



Question 36

In the diagram, ABC and XCY are straight lines. It is given that AC = 10.2 cm, BY = 4.7 cm, $\angle BYC = 42°$ and $\angle BCY = 90°$. Calculate
(a) the length of AB,
(b) the area of ΔABY.
Give your answers correct to 3 significant figures.



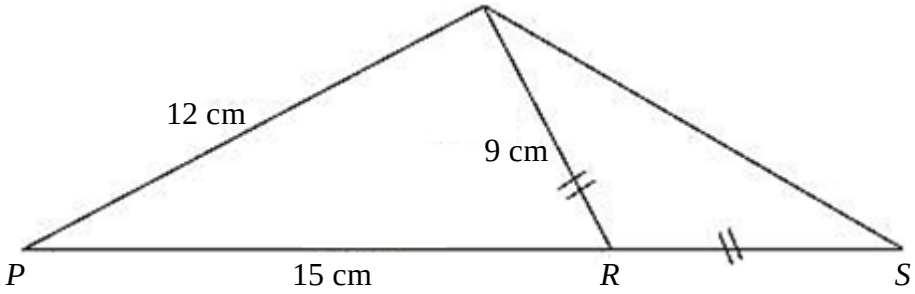
Question 37

In the diagram, PR = 15 cm, PQ = 12 cm, QR = 9 cm, QRS is an isosceles triangle and PRS is a straight line.

(a) Explain why $\angle PQR = 90°$.

(b) Hence, calculate the shortest distance from Q to PR.

(c) Find the
  (i) value of $\sin \angle QRS$.
  (ii) area of ΔQRS .



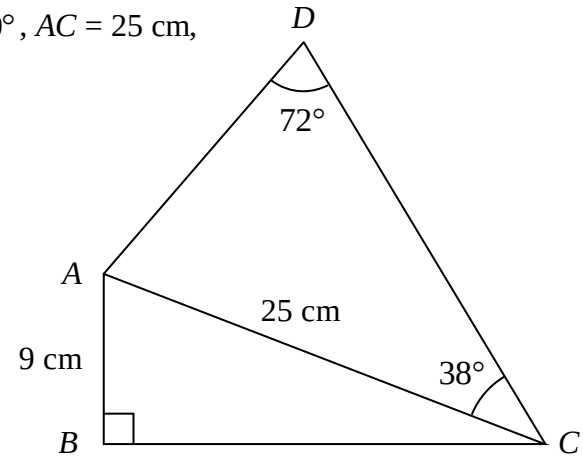
Question 38

In the figure, $ABCD$ is a quadrilateral such that $\angle ABC = 90°$, AC = 25 cm, $\angle ADC = 72°$, AB = 9 cm and $\angle ACD = 38°$. Calculate

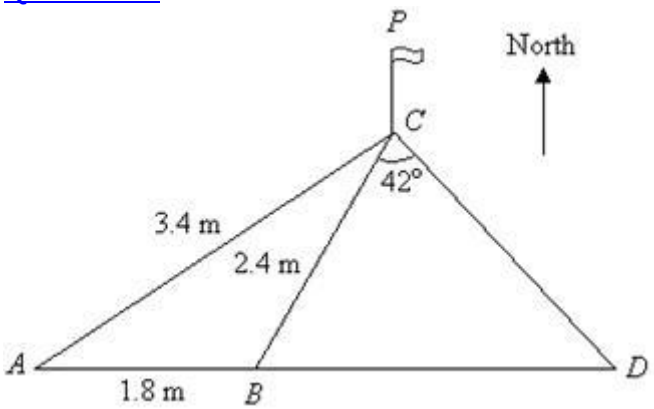
(a) the length of AD, correct to 3 significant figures,

(b) the length of BC, correct to 3 significant figures,

(c) $\angle BCD$, correct to 1 decimal place.



Question 39



Triangle ABC lies on a horizontal ground. D is on AB produced such that D is due east of. It is given that AB = 1.8 m, BC = 2.4 m, AC = 3.4 m and $\angle BCD = 42°$.

(a) Calculate
  (i) $\angle ABC$,
  (ii) the length of BD,
  (iii) the bearing of D from C.

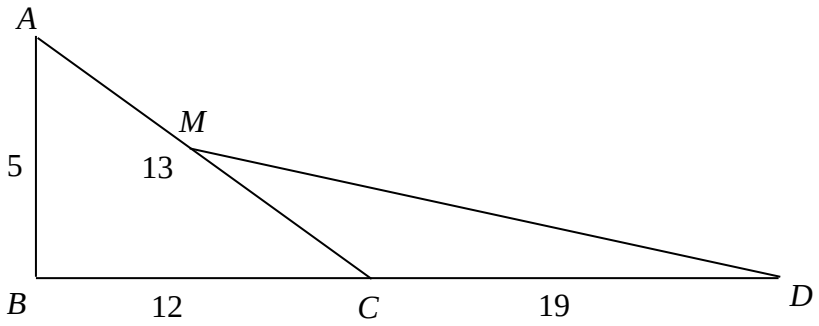
A vertical flagpole, CP, stands at C. The angle of elevation of P from B is $30°$.

(b) Find the height of the flagpole.

(c) An object moves from B to D. Calculate the greatest angle of elevation of the top of the flagpole CP when viewed from the object at any point on its path BD.

Question 40

In the diagram below, $AB = 5$ cm, $AC = 13$ cm, $BC = 12$cm and $CD = 19$ cm. M is the mid-point of AC, and the points B, C and D lie on the same line.

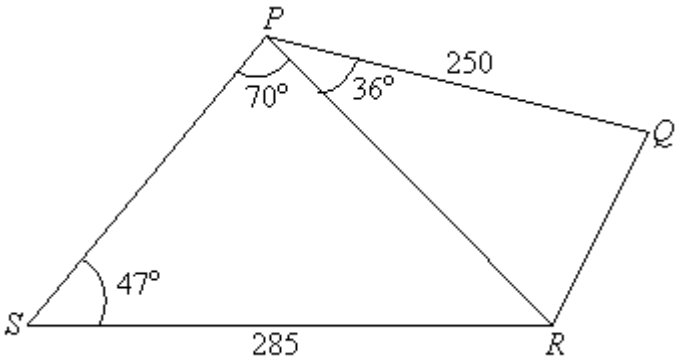


(a) Explain why triangle ABC is a right-angled triangle.
(b) Express $\cos\angle ACD$ as a fraction in its lowest terms.
(c) Hence calculate MD.

Question 41

Points P, Q, R, S lie on a horizontal ground. $PQ = 250$ m, $SR = 285$ m, $\angle QPR = 36°$, $\angle PSR = 47°$ and $\angle SPR = 70°$.

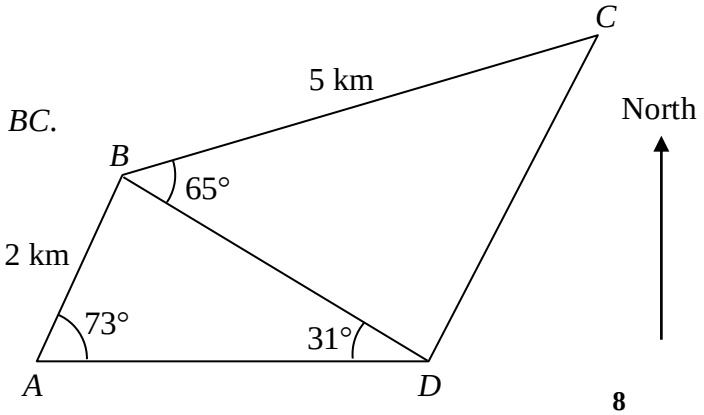
Calculate
(a) the length of PR,
(b) the length of QR,
(c) the area of triangle PSR,
(d) the shortest distance from Q to PR.



Question 42

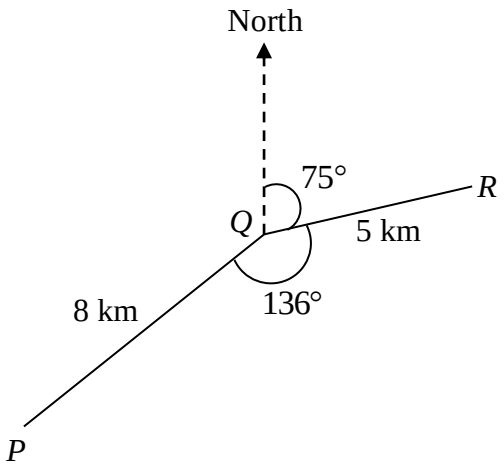
A, B, C and D are four points on a field. $\angle DAB$ is 73°, $\angle ADB$ is 31°, $\angle DBC$ is 65°, AB is 2 km and BC is 5 km. D is due east of A.

(a) Show that $BD = 3.714$ km.
(b) Calculate
  (i) CD.
  (ii) the area of triangle BCD.
  (iii) the shortest distance from D to BC.
  (iv) the bearing of B from D.



Question 43

A ship sails 8 km from P to Q. It then sails 5 km from Q to R on a bearing of $075°$.



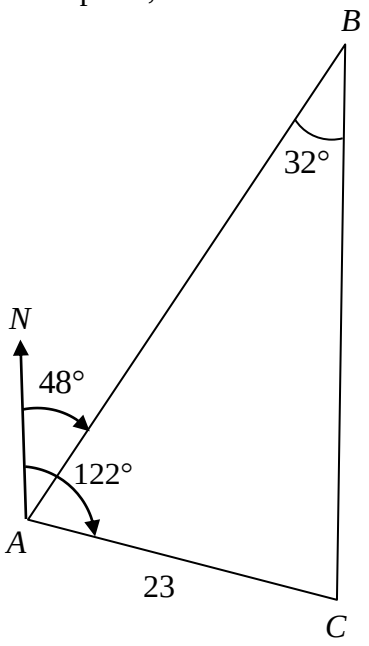
(a) Given that $\angle PQR = 136°$, calculate the
  (i) bearing of Q from P.
  (ii) distance PR.

(b) The ship finally sails from R to a position T, which is due North of Q. Given that $\angle QTR = 40°$, calculate the
  (i) distance RT.
  (ii) shortest distance between the point Q and the ship as it sails from R to T.

Question 44

The points A, B and C lie on level ground. B is on a bearing of $048°$ from A and C is on a bearing of $122°$ from A. Given that $\angle ABC = 32°$ and C is 23 m from A, find, giving your answer correct to one decimal place,

(a) the distance BC.
(b) the area of triangle ABC.
(c) the bearing of A from B.
(d) the bearing of B from C.
(e) the shortest distance from A to BC.

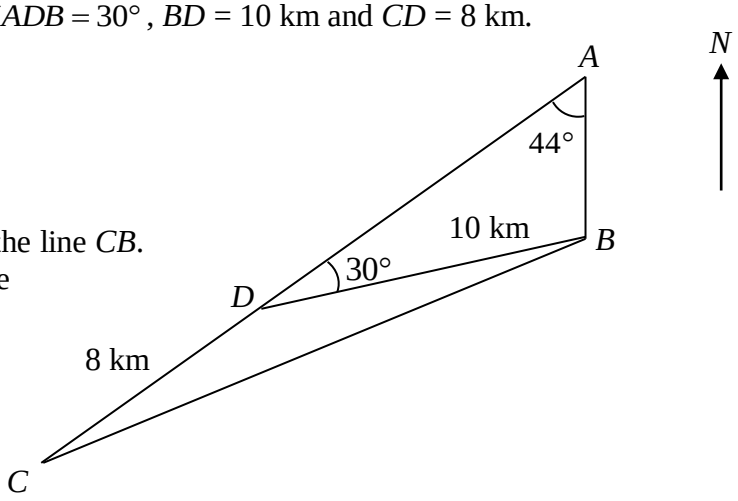


Question 45

A, *B*, *C* and *D* represent four islands in the open sea with *A* due north of *B*. *A*, *D* and *C* are on a straight line. $\angle BAD = 44°$, $\angle ADB = 30°$, BD = 10 km and CD = 8 km.

Calculate

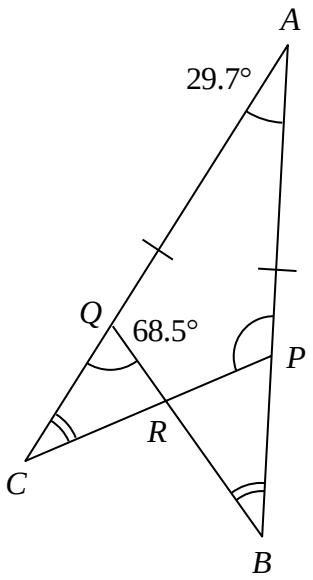
(a) the length of *BC*.
(b) the length of *AB*.
(c) the bearing of *B* from *C*.
(d) the area of triangle *BCD*.
(e) A ship travels from *C* to *B* along the line *CB*. Find the shortest distance from the ship to Island *D*.



Question 46

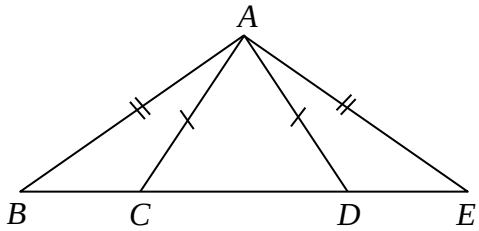
In the diagram, $\angle ABQ = \angle ACP$, $\angle BAC = 29.7°$, $\angle BQC = 68.5°$ and $AB = AC$.

(a) Identify a pair of congruent triangles and state the reason for their congruence.
(b) Calculate $\angle APC$.



Question 47

In the triangle below, *C* and *D* are on the line *BE* such that $AB = AE$ and $AC = AD$. Prove that triangle *ABC* is congruent to triangle *AED*.



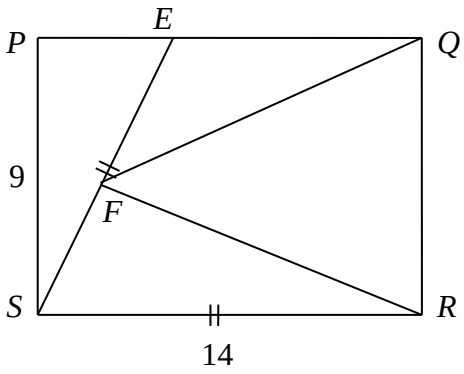
Question 48

In the diagram, $PQRS$ is a rectangle, SFE is a straight line and E lies on PQ. The line RF is perpendicular to the line SE at F. Given that $SE = SR$, $PS = 9$ cm, $SR = 14$ cm and $\angle FSR = \angle FRQ$.

(a) Identify a pair of congruent triangles and give reasons for your answer.

(b) Show that $SF = 10.7$ cm.

(c) Find the area of $EFRQ$.



Question 49

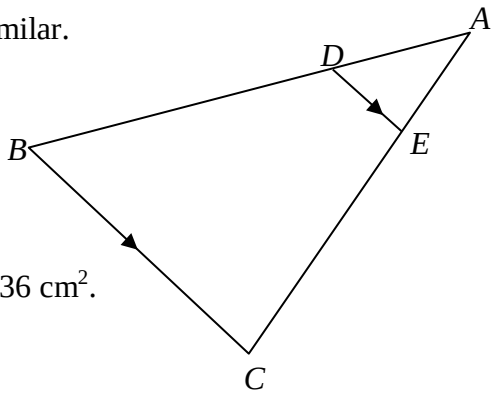
In the diagram, BC is parallel to DE.

(a) Show that triangles ABC and ADE are similar.

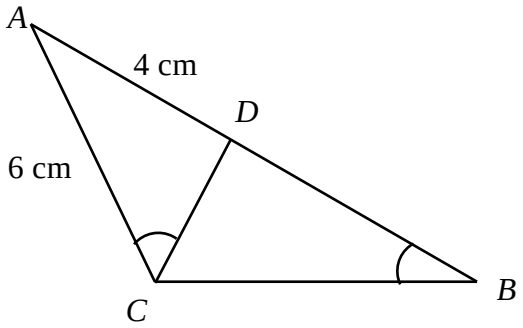
(b) Given that $BD = 2DA$, find

(i) $\frac{DE}{BC}$.

(ii) the area of triangle ADE given that the area of the trapezium $DECB$ is 136 cm^2.



Question 50



In the diagram, $\angle ABC = \angle ACD$, $AC = 6$ cm and $AD = 4$ cm,

(a) Show that ΔABC and ΔACD are similar.

(b) Calculate the length of AB.

(c) Find the ratio of $\frac{\text{area of}\Delta ACD}{\text{area of}\Delta ABC}$.

(d) Find the ratio of $\frac{\text{area of } \Delta ACD}{\text{area of } \Delta BCD}$.

[Answer Key]

Question 1

$-1\frac{4}{7}$ or $3\frac{1}{5}$

Question 2

0 or 7

Question 3

(a) $-(x-4)^2+31$ (b) 9.57 or -1.57

Question 4

2.90 or -1.90

Question 5

$x=-1.84$ or 4.34

Question 6

$x=2.55$ or -2.88

Question 7

$p=4.5$ or -4

Question 8

(a) $x=\frac{3}{8}$ or 1 (b) $a=\frac{3}{4}$ or 2

Question 9

(a) $\left(\frac{45}{y}\right)$h (b) $\left(\frac{45}{y-5}\right)$h

(c) (i) 1.5 (ii) -10 or 15 (iii) 7.5 h

Question 10

(a) $(x-3)^2-5$ (d) $x = 2.5$

Question 11

(a) $A(-1, 0), B(4, 0)$ (b) $\frac{3}{2}$

Question 12

(b) $(-2, 5)$

Question 13

$m<16\frac{1}{5}$

Question 14

(a) 16 (b) 15

Question 15

(a) $-11 < x \leq 9$ (b) 1, 4, 9

Question 16

(a) $-2\frac{4}{11} \leq x < 6\frac{7}{9}$ (b) 2 (c) 5

Question 17

(a) $-2 \leq x < 8$ (b) 121 (c) -16

Question 18

(a) $x \geq 20,\ 90x + 60x \leq 4000$ (c) 26

Question 19

$3a^{\frac{1}{2}}b^3$

Question 20

1

Question 21

$\frac{24m^3}{25n^3}$

Question 22

$\frac{8ax}{7by}$

Question 23

$\frac{1}{17}$

Question 24

$-8ab^{\frac{1}{2}}c$

Question 25

$\frac{b^2}{2a}$

Question 26

$2y^2$

Question 27

$\frac{2}{5a^3}$

Question 28

$k = \frac{3}{4}$

Question 29

He should invest in Company *A* since the total amount is greater.

Question 30

(a) \$47500 (b) \$57000 (c) \$40000 (d) \$14054.74

Question 31

(a) 9700 (b) 2.5×10^9

Question 32

(a) 31 : 130 (b) 8.9×10^8 (c) India

Question 33

(a) $-\frac{24}{25}$ (b) $\frac{24}{7}$ (c) $\frac{7}{25}$

Question 34

(a) $x = 48$ (b) $223\,\text{cm}^2$

Question 35

$6400\,\text{m}^2$

Question 36

(a) 7.06 cm (b) $12.3\,\text{cm}^2$

Question 37

(b) 7.2 cm (c) (i) 0.8 (ii) 32.4 cm^2

Question 38

(a) 16.2 cm (b) 23.3 cm (c) 59.1°

Question 39

(a) (i) 107.2° (ii) 1.77 m (iii) 155.2° (b) 1.39 m (c) 31.2°

Question 40

(b) $-\frac{12}{13}$ (c) 25.1 cm

Question 41

(a) 222 m (b) 148 m (c) 28200 m^2 (d) 147 m

Question 42

(b) (i) 4.81 km (ii) 8.41 km^2 (iii) 3.37 km (iv) 301°

Question 43

(a) (i) 031° (ii) 12.1 km (b) (i) 7.51 km (ii) 4.53 km

Question 44

(a) 41.7 m (b) 461.2 m^2 (c) 228° (d) 016° (e) 22.1 m

Question 45

(a) 17.4 km (b) 7.2 km (c) 060.7° (d) 20 km^2 (e) 2.3 km

Question 46

(a) ΔABQ and ΔACP (AAS) (b) 111.5°

Question 47

Question 48

(a) $\Delta PES = \Delta FSR$ (c) 29.7 cm^2

Question 49

(b) (i) $\frac{1}{3}$ (ii) 17 cm^2

Question 50

(b) 9 cm (c) $\frac{4}{9}$ (d) $\frac{4}{5}$

[Solution]

Question 1

$$(5n-3)(7n+11)=13(7n+11)$$
$$(5n-3)(7n+11)-13(7n+11)=0$$
$$(7n+11)(5n-3-13)=0$$
$$n=-1\frac{4}{7} \text{ or } 3\frac{1}{5}$$

Question 2

$$x^2=7x$$
$$x^2-7x=0$$
$$x(x-7)=0$$
$$x=0 \text{ or } 7$$

Question 3

(a)

$$-x^2+8x+15=-(x^2-8x-15)$$
$$=-\left[x^2-8x+\left(-\frac{8}{2}\right)^2-\left(-\frac{8}{2}\right)^2-15\right]$$
$$=-\left[x^2-8x+4^2-4^2-15\right]$$
$$=-[(x-4)^2-31]$$
$$=-(x-4)^2+31$$

(b)

$$-x^2+8x+15=0$$
$$-(x-4)^2+31=0$$
$$(x-4)^2=31$$
$$x-4=\pm\sqrt{31}$$
$$x=4\pm\sqrt{31}$$
$$=9.57 \text{ or } -1.57 \quad \text{(Correct to 3 sig. fig.)}$$

Question 4

$$2(x+1)(x-2)=7$$
$$2(x^2-x-2)=7$$
$$2x^2-2x-4-7=0$$
$$2x^2-2x-11=0$$
$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(2)(-11)}}{2(2)}$$
$$=\frac{2\pm\sqrt{4+88}}{4}$$
$$=2.90 \text{ or } -1.90$$

Question 5

$$2(3x+5)-(x+2)(2x-3)=0$$
$$6x+10-(2x^2-3x+4x-6)=0$$
$$6x+10-(2x^2+x-6)=0$$
$$6x+10-2x^2-x+6=0$$
$$-2x^2+5x+16=0$$

Comparing $-2x^2+5x+16=0$ with $ax^2+bx+c=0$, we have $a=-2$, $b=5$ and $c=16$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-5\pm\sqrt{(5)^2-4(-2)(16)}}{2(-2)}$$
$$=\frac{-5\pm\sqrt{153}}{-4}$$
$x=-1.84$ or 4.34 (Correct to 3 sig. fig.)

Question 6

$$(2x-1)^2-(x-4)^2=3x+7$$
$$4x^2-4x+1-(x^2-8x+16)=3x+7$$
$$4x^2-4x+1-x^2+8x-16-3x-7=0$$
$$3x^2+x-22=0$$

Comparing $3x^2+x-22=0$ with $ax^2+bx+c=0$, we have $a=3$, $b=1$ and $c=-22$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(3)(-22)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{265}}{6}$$
$$= 2.55 \text{ or } -2.88 \quad \text{(Correct to 3 sig. fig.)}$$

Question 7

$$\frac{4}{p} - \frac{5}{p+3} = \frac{p+8}{(p+3)^2}$$
$$\frac{4(p+3) - 5p}{p(p+3)} = \frac{p+8}{(p+3)^2}$$
$$\frac{4p + 12 - 5p}{p(p+3)} = \frac{p+8}{(p+3)^2}$$
$$\frac{-p + 12}{p(p+3)} = \frac{p+8}{(p+3)^2}$$
$$(p+3)(-p+12) = p(p+8)$$
$$-p^2 + 12p - 3p + 36 = p^2 + 8p$$
$$-p^2 + 9p + 36 = p^2 + 8p$$
$$0 = p^2 + 8p + p^2 - 9p - 36$$
$$2p^2 - p - 36 = 0$$
$$(2p - 9)(p + 4) = 0$$
$$2p - 9 = 0 \text{ or } p + 4 = 0$$
$$2p = 9 \qquad p = -4$$
$$p = 4.5$$

Question 8

(a)

$$\frac{1}{4x^2-4x+1}+\frac{3}{2x-1}=4$$

$$\frac{1}{(2x-1)^2}+\frac{3}{2x-1}=4$$

$$\frac{1+3(2x-1)}{(2x-1)^2}=4$$

$$1+3(2x-1)=4(2x-1)^2$$

$$1+6x-3=4(4x^2-4x+1)$$

$$6x-2=16x^2-16x+4$$

$$0=16x^2-16x+4-6x+2$$

$$16x^2-22x+6=0$$

$$8x^2-11x+3=0$$

$$(8x-3)(x-1)=0$$

$$8x-3=0 \text{ or } x-1=0$$

$$x=\frac{3}{8} \qquad x=1$$

(b) Comparing $\frac{1}{(a-1)^2}+\frac{3}{a-1}=4$ with $\frac{1}{(2x-1)^2}+\frac{3}{2x-1}=4$, we have $a = 2x$

When $x=\frac{3}{8}$,

$$a=2\left(\frac{3}{8}\right)$$

$$=\frac{3}{4}$$

When $x = 1$,

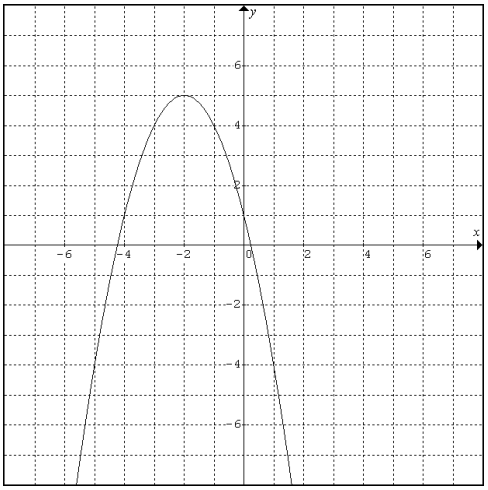
$$a=2(1)$$

$$=2$$

$$\therefore a=\frac{3}{4} \text{ or } 2$$

Question 12

(a)



Question 13

$$\frac{2}{3}m-\frac{m-1}{4}<7$$

$$12\times\left(\frac{2}{3}m-\frac{m-1}{4}\right)<12\times 7 \quad \left(\begin{array}{l}\text{The LCM of the denominators, 3 and 4, is 12.}\\ \text{Multiply both sides by 12.}\end{array}\right)$$

$$8m-3(m-1)<84$$
$$8m-3m+3<84$$
$$5m+3<84$$
$$5m<84-3$$
$$5m<81$$
$$m<\frac{81}{5}$$
$$m<16\frac{1}{5}$$

Question 14

(a)

$$5(x+7)\geq 6(x-2)+2(x-1)$$
$$5x+35\geq 6x-12+2x-2$$
$$5x+35\geq 8x-14$$
$$5x-8x\geq -14-35$$
$$-3x\geq -49$$
$$x\leq\frac{-49}{-3} \quad \left(\begin{array}{l}\text{Divide both sides by } -3 \text{ and}\\ \text{reverse the inequality sign.}\end{array}\right)$$
$$x\leq 16\frac{1}{3}$$

If x is an integer, the greatest possible value of x is 16.

(b) If x is a prime number, the greatest possible value of x is 15.

Question 15

(a) Solving both linear inequalities separately,

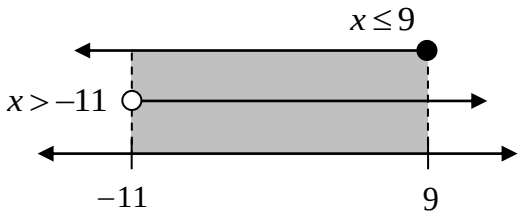
$$\frac{x-1}{3} < \frac{2x+2}{5} \qquad \text{and} \qquad \frac{2x+2}{5} \le 4$$

$$5(x-1) < 3(2x+2) \qquad 2x+2 \le 20$$

$$5x-5 < 6x+6 \qquad 2x \le 18$$

$$x > -11 \qquad x \le 9$$

Representing $x > -11$ and $x \le 9$ on a number line,



The solutions satisfying both inequalities are $-11 < x \le 9$.

(b) From (a), $-11 < x \le 9$.

Hence, the perfect square values of x which satisfies the inequalities are 1, 4 and 9.

Question 16

(a)

$$\frac{x-11}{2} < \frac{3-2x}{5}$$

$$5(x-11) < 2(3-2x)$$

$$5x-55 < 6-4x$$

$$9x < 61$$

$$x < 6\frac{7}{9}$$

$$\frac{3-2x}{5} \le \frac{x+7}{3}$$

$$3(3-2x) \le 5(x+7)$$

$$9-6x \le 5x+35$$

$$-26 \le 11x$$

$$-2\frac{4}{11} \le x$$

Question 17

(a)

$$\frac{2-x}{4} \le 3+x$$
$$2-x \le 12+4x$$
$$-2 \le x$$
$$x \ge -2$$

Or

$$3+x<11$$
$$x<8$$

$$\therefore -2 \le x < 8$$

(b) Greatest possible value of $(x+y)^2$

$$=(7+4)^2$$
$$=121$$

(c) Least possible value of $(x+y)(x-y)$

$$=x^2-y^2$$
$$=0^2-4^2$$
$$=-16$$

Question 18

(a) Let the number of sets of tables and chairs to be purchased be x.

Given that at least 20 sets of tables and chairs need to be replaced, $x \ge 20$.

Since each chair costs \$90,

$$\text{Cost of } x \text{ chairs} = x \times \$90$$
$$= \$90x$$

Since each table cost \$60,

$$\text{Cost of } x \text{ tables} = x \times \$60$$
$$= \$60x$$

Given that the budget allocated for the replacement of furniture is \$4000,

$$\text{Cost of } x \text{ chairs} + \text{Cost of } x \text{ tables} \le 4000$$
$$90x + 60x \le 4000$$

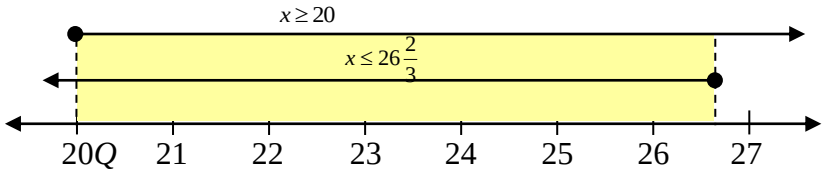
(b)

$90x + 60x \le 4000$ and $x \ge 20$

$150x \le 4000$

$x \le \dfrac{4000}{150}$

$x \le 26\dfrac{2}{3}$



Hence, the common solution as shaded is $20 \le x \le 26\dfrac{2}{3}$.

(c) The largest integer that satisfies the inequalities is 26.

Therefore, the maximum sets of tables and chairs that the school can buy is 26.

Question 19

$$\frac{8a^3b^6 \times 3\sqrt{a}}{(2ab)^3 \times (3a)^0} = \frac{8a^3b^6 \times 3\sqrt{a}}{8a^3b^3 \times 1}$$

$$= \frac{b^3 \times 3\sqrt{a}}{1}$$

$$= 3a^{\frac{1}{2}}b^3$$

Question 20

$$\frac{4(a^2c)^2}{9a^2c} \div \frac{4a^2c}{9} = \frac{4a^4c^2}{9a^2c} \times \frac{9}{4a^2c}$$

$$= 1$$

Question 21

$$\frac{24m^4}{40n^3} \div \frac{3m^3}{20n^3} \times \frac{6m^2n}{(5n^2)^2} = \frac{24m^4}{40n^3} \times \frac{20n^3}{3m^3} \times \frac{6m^2n}{25n^4}$$

$$= \frac{24m^3}{25n^3}$$

Question 22

$$\frac{2a^2b}{7xy^3} \div \frac{14ab^2}{56(xy)^2} = \frac{2a^2b}{7xy^3} \times \frac{56x^2y^2}{14ab^2}$$

$$= \frac{8ax}{7by}$$

Question 23

$$16^{3x+1} = 32^{1-x}$$
$$(2^4)^{3x+1} = (2^5)^{1-x}$$
$$12x + 4 = 5 - 5x$$
$$17x = 1$$
$$x = \frac{1}{17}$$

Question 24

$$\frac{(-2c)^4}{\sqrt[3]{0.125b^6}} \div \frac{8\sqrt{bc}}{-2ab^3c^{-2}} = \frac{(16c^4)}{(0.5b^2)} \times \frac{-2ab^3c^{-2}}{8b^{\frac{1}{2}}c}$$
$$= -8ab^{\frac{1}{2}}c$$

Question 25

$$(16a^4)^{\frac{1}{2}} \times \frac{a^{-3}b^2}{8b^0} = 4a^2 \times \frac{a^{-3}b^2}{8}$$
$$= 4a^2 \times \frac{b^2}{8a^3}$$
$$= \frac{b^2}{2a}$$

Question 26

$$\frac{x^4 y^{\frac{3}{5}}}{6x} \times \frac{12x^{-3}}{y^{-\frac{7}{5}}} = \frac{12}{6} x^{4+(-3)-1} y^{\frac{3}{5}-\left(-\frac{7}{5}\right)}$$
$$= 2y^2$$

$\boxed{a^m \times a^n = a^{m+n}}$ $\boxed{a^m \div a^n = a^{m-n}}$

$\boxed{a^0 = 1}$

Question 27

$$\frac{3a^{-3}}{\left(2a^{-\frac{1}{2}}b\right)^2} \div \frac{15(a^4b^3)^{-\frac{1}{2}}}{8a^{-3}\sqrt{b}}$$

$$= \frac{3a^{-3}}{4a^{-1}b^2} \div \frac{15a^{-2}b^{-\frac{3}{2}}}{8a^{-3}\sqrt{b}} \qquad \boxed{(ab)^m = a^m b^m} \quad \boxed{(a^m)^n = a^{mn}}$$

$$= \frac{3a^{-3}}{4a^{-1}b^2} \times \frac{8a^{-3}\sqrt{b}}{15a^{-2}b^{-\frac{3}{2}}}$$

$$= \frac{3a^{-3}}{4a^{-1}b^2} \times \frac{8a^{-3}b^{\frac{1}{2}}}{15a^{-2}b^{-\frac{3}{2}}} \qquad \boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

$$= \frac{3\times 8}{4\times 15} a^{-3+(-3)-(-1)-(-2)} b^{\frac{1}{2}-2-\left(-\frac{3}{2}\right)} \qquad \boxed{a^m \times a^n = a^{m+n}} \quad \boxed{a^m \div a^n = a^{m-n}}$$

$$= \frac{2}{5} a^{-3} \qquad \boxed{a^0 = 1}$$

$$= \frac{2}{5a^3} \qquad \boxed{a^{-n} = \frac{1}{a^n}}$$

Question 28

$$\sqrt{a\sqrt{a}} = a^k$$

$$\left(a \times a^{\frac{1}{2}}\right)^{\frac{1}{2}} = a^k \qquad \boxed{\sqrt[n]{a} = a^{\frac{1}{n}}}$$

$$\left(a^{1+\frac{1}{2}}\right)^{\frac{1}{2}} = a^k \qquad \boxed{a^m \times a^n = a^{m+n}}$$

$$\left(a^{\frac{3}{2}}\right)^{\frac{1}{2}} = a^k$$

$$a^{\frac{3}{4}} = a^k \qquad \boxed{(a^m)^n = a^{mn}}$$

$$k = \frac{3}{4}$$

Question 29

After 5 years, total amount for Company A

$$= 20\,000 + \frac{20\,000 \times 3.8 \times 5}{100}$$

$$= \$23\,800$$

After 5 years, total amount for Company B

$$= 20\,000\left(1+\frac{3.5}{200}\right)^{10}$$
$$= \$23\,788.89$$

He should invest in Company *A* since the total amount is greater.

Question 30

(a)

$$\text{Total cost of printing} = \$0.95\times 50000$$
$$= \$47500$$

(b)

$$\text{Total profit} = \frac{120}{100}\times 0.95\times 50000$$
$$= \$57000$$

(c)

$$\text{Simple Interest} = \frac{200000\times 8\times \frac{30}{12}}{100}$$
$$= \$40000$$

(d) Amount after 30 months

$$= \$200000\left(1+\frac{5}{100}\right)^{\frac{5}{2}}$$
$$= \$225945.26$$

Compound Interest = \$25945.26

$$\text{Amount Saved} = \$40000 - \$25945.26$$
$$= \$14054.74$$

Question 31

(a)

$$0.097\times 10^{11} = 9\,700\,000\,000$$
$$= 9700 \text{ million}$$

n = 9700

(b)

$$\text{Predicted increase} = 0.097\times 10^{11} - 7.2\times 10^{9}$$
$$= 2.5\times 10^{9}$$

Question 32

(c) Population density = population ÷ land area

Population density of China = 135.416

Population density of India = 363.636

Population density of USA = 32.29

Population density of India is the highest.

Question 33

(a)

$\angle PRS = 180° - x°$ (adjacent angles on a straight line)

$$\begin{aligned}\cos \angle PRS &= \cos(180 - x)° \\ &= -\cos x \\ &= -\cos \angle PRQ \\ &= -\frac{24}{25}\end{aligned}$$

(b) In right-angled ΔPQR,

$\angle QPR = 180° - 90° - x°$ (sum of angles in *triangle*)

$= 90° - x°$

Using Pythagoras' Theorem,

$25^2 = PQ^2 + 24^2$

$PQ^2 = 25^2 - 24^2$

$PQ^2 = 49$

$PQ = \pm\sqrt{49}$

$PQ = \pm 7$ cm

Since $PQ > 0$, $PQ = 7$ cm.

$$\begin{aligned}\tan(90 - x)° &= \tan \angle QPR \\ &= \frac{24}{7}\end{aligned}$$

(c)

$$\begin{aligned}\sin(180 - x)° &= \sin x \\ &= \sin \angle PRS \\ &= \frac{7}{25}\end{aligned}$$

Question 34

(a)

$\angle BAD = 180° - 152°$ (adjacent angles on a straight line)

$= 28°$

$\angle BDC = 20° + 28°$ (exterior angles of a triangle)

$= 48°$

Hence, the value of x is 48.

(b)

$$\text{Area of } \Delta BCD = \frac{1}{2} \times BD \times CD \times \sin \angle BDC$$

$$= \frac{1}{2} \times 20 \times 30 \times \sin 48°$$

$$= 223 \text{ cm}^2 \quad \text{(Correct to 3 sig. fig.)}$$

Question 35

$$\text{Area of } \Delta ABD = \frac{1}{2} \times AB \times AD \times \sin \angle BAD$$

$$= \frac{1}{2} \times 78 \times 55 \times \sin 77°$$

$$= 2090.0 \text{ m}^2 \quad \text{(Correct to 5 sig. fig.)}$$

$$\text{Area of } \Delta BCD = \frac{1}{2} \times BC \times CD \times \sin \angle BCD$$

$$= \frac{1}{2} \times 108 \times 113 \times \sin 45°$$

$$= 4314.8 \text{ m}^2 \quad \text{(Correct to 5 sig. fig.)}$$

$$\text{Area of park} = 2090.0 + 4314.8$$

$$= 6404.8$$

$$= 6400 \text{ m}^2 \quad \text{(Correct to 3 sig. fig.)}$$

Question 36

(a) In right-angled ΔBCY,

$$\sin 42° = \frac{BC}{4.7}$$

$$BC = 4.7 \sin 42°$$

$$BC = 3.1449 \text{ cm} \quad \text{(Correct to 5 sig. fig.)}$$

$$AB = 10.2 - 3.1449$$

$$= 7.0551$$

$$= 7.06 \text{ cm} \quad \text{(Correct to 3 sig. fig.)}$$

(b)

$\angle ABY = 90° + 42°$ (exterior angles of a triangle)

$= 132°$

$$\text{Area of } \Delta ABY = \frac{1}{2} \times AB \times BY \times \sin \angle ABY$$
$$= \frac{1}{2} \times 7.0551 \times 4.7 \times \sin 132°$$
$$= 12.3 \text{ cm}^2 \quad \text{(Correct to 3 sig. fig.)}$$

Question 37

(a) $12^2 + 9^2 = 15^2$

By Pythagoras' Theorem, $\angle PQR = 90°$

(b)
$$\frac{1}{2} \times 12 \times 9 = \frac{1}{2} \times 15 \times h$$
$$h = 7.2 \text{ cm}$$

(c) (ii)
$$\text{area of } \Delta QRS = \frac{1}{2} \times 9 \times 9 \times 0.8$$
$$= 32.4 \text{ cm}^2$$

Question 38

(a) In ΔACD, using Sine Rule,
$$\frac{AD}{\sin 38°} = \frac{25}{\sin 72°}$$
$$AD = \frac{25 \sin 38°}{\sin 72°}$$
$$AD = 16.2 \text{ cm} \quad \text{(Correct to 3 sig. fig.)}$$

(b) In right-angled ΔABC, using Pythagoras' Theorem,
$$25^2 = BC^2 + 9^2$$
$$BC^2 = 25^2 - 9^2$$
$$BC^2 = 544$$
$$BC = \pm\sqrt{544}$$
$$BC = \pm 23.3 \text{ cm} \quad \text{(Correct to 3 sig. fig.)}$$

Since $BC > 0$, then $BC = 23.3$ cm.

(c) In right-angled ΔABC,

$$\sin \angle BCD = \frac{9}{25}$$

$$\angle BCD = \sin^{-1}\left(\frac{9}{25}\right)$$

$$\angle BCD = 21.100° \quad \text{(Correct to 3 decimal places)}$$

$$\angle BCD = 38° + 21.100°$$
$$= 59.1° \quad \text{(Correct to 1 decimal place)}$$

Question 39

(a) (i)

$$\cos ABC = \frac{1.8^2 + 2.4^2 - 3.4^2}{2(1.8)(2.4)}$$

$$ABC = \cos^{-1}(-0.296)$$
$$= 107.2°$$

(ii)

$$\frac{BD}{\sin 42°} = \frac{2.4}{\sin 65.218}$$

$$BD = 1.77 \text{ m}$$

(iii) $90° - 65.218° = 24.782°$

$$180° - 24.782° = 155.2°$$

(b)

$$\tan 30° = \frac{x}{2.4}$$

$$x = 1.39 \text{ m}$$

(c)

$$\sin 72.78° = \frac{y}{2.4}$$

$$y = 2.292$$

$$\tan X = \frac{1.39}{2.292}$$

$$X = 31.2°$$

Question 40

(a)

$$AC^2 = 13^2$$
$$= 169$$

$$AC^2 + BC^2 = 5^2 + 12^2$$
$$= 25 + 144$$
$$= 169$$

Hence $AC^2 = AB^2 + BC^2$.

By Pythagoras theorem, triangle ABC is right-angled.

(b) $\cos \angle ACB = \frac{12}{13}$

$$\cos \angle ACD = -\cos \angle ACB$$
$$= -\frac{12}{13}$$

(c)

$$MD^2 = 6.5^2 + 19^2 - 2(6.5)(19)\left(-\frac{12}{13}\right)$$
$$MD^2 = 631.25$$
$$MD = 25.1 \text{ cm}$$

Question 42

(a) By Sine Rule,

$$\frac{BD}{\sin 73°} = \frac{2}{\sin 31°}$$
$$BD = \frac{2}{\sin 31°} \times \sin 73°$$
$$= 3.7135$$
$$= 3.714 \text{ km} \quad \text{(shown)}$$

(b) (i) By Cosine Rule,

$$CD^2 = 5^2 + 3.7135^2 - 2(5)(3.7135)\cos 65°$$
$$CD = \sqrt{23.096}$$
$$CD = 4.8058$$
$$CD = 4.81 \text{ km} \quad \text{(to 3 sig. fig.)}$$

(ii)

$$\text{Area of } \Delta BCD = \frac{1}{2}(5)(3.7135)\sin 65°$$
$$= 8.4139$$
$$= 8.41 \text{ km}^2 \quad \text{(to 3 sig. fig.)}$$

(iii)

$$\sin 65° = \frac{\text{shortest distance}}{3.7135}$$

$$\text{shortest distance} = \sin 65° \times 3.7135$$
$$= 3.3656$$
$$\approx 3.37 \text{ km}$$

(iv)

$$\text{Bearing of } B \text{ from } D = 360° - (90° - 31°)$$
$$= 301°$$

Question 43

(a) (ii)

$$PR = \sqrt{8^2 + 5^2 - 2(8)(5)\cos 136°}$$
$$= 12.106$$
$$= 12.1 \text{ km}$$

(b) (i) Using sine rule,

$$\frac{5}{\sin 40°} = \frac{RT}{\sin 75°}$$
$$RT = \frac{5 \sin 75}{\sin 40}$$
$$= 7.5136$$
$$= 7.51 \text{ km}$$

(ii) Let QS be the shortest distance.

$$\angle QRT = 180 - 40 - 75$$
$$= 65$$

$$\sin 65 = \frac{QS}{5}$$
$$QS = 5 \sin 65$$
$$= 4.5315$$
$$= 4.53 \text{ km}$$

Question 44

(a)

$$\angle BAC = 122° - 48°$$
$$= 74°$$

Using sine rule,

$$\frac{BC}{\sin 74°} = \frac{23}{\sin 32°}$$

$$BC = \frac{23\sin 74°}{\sin 32°}$$

$$= 41.72$$

$$\approx 41.7 \text{ m}$$

(b)

$$\angle BCA = 180° - 74° - 32°$$

$$= 74°$$

$$\text{Area of } \Delta ABC = \frac{1}{2}(23)(41.72)\sin 74°$$

$$= 461.2 \text{ m}^2$$

(c) The bearing of A from B

$$= 180° + 48°$$

$$= 228°$$

(d) The bearing B from C

$$= 48° - 32°$$

$$= 016°$$

(e)

$$x = 23\sin 74°$$

$$= 22.1 \text{ m}$$

Question 45

(a)

$$\angle CDB = 180° - 30°$$

$$= 150°$$

$$BC^2 = 8^2 + 10^2 - 2(8)(10)\cos 150°$$

$$BC = 17.39 \text{ km}$$

$$\approx 17.4 \text{ km}$$

(b)

$$\frac{10}{\sin 44°} = \frac{AB}{\sin 30°}$$

$$AB = \frac{10\sin 30°}{\sin 44°}$$

$$= 7.19778$$

$$\approx 7.2 \text{ km}$$

(c) Bearing of A from C

$= \angle CAB$

$= 44°$

$$\frac{10}{\sin \angle DCB} = \frac{17.39}{\sin 150°}$$

$$\angle DCB = 16.7°$$

Bearing of B from C

$= 44 + 16.7$

$= 60.7°$

$\approx 060.7°$

(d)

$$\text{Area of } \Delta BCD = \frac{1}{2} \times 10 \times 8 \sin 150°$$

$$= 20 \text{ km}^2$$

(e)

$$\frac{1}{2} \times 17.394 \times h = 20$$

$$h = 2.3 \text{ km}$$

Question 46

(a) $\angle ABQ = \angle ACP$ (given)

In ΔABQ,

$\angle AQB + \angle ABQ + 29.7° = 180°$ (sum of angles in *triangle*)

In ΔACP,

$\angle ACP + \angle APC + 29.7° = 180°$ (sum of angles in *triangle*)

$$\angle AQB + \angle ABQ + 29.7° = \angle ACP + \angle APC + 29.7°$$

$$\angle AQB + \angle ABQ = \angle ACP + \angle APC$$

$$\angle AQB + \angle APC = \angle ACP + \angle APC \quad (\text{since } \angle ABQ = \angle APC)$$

$$\angle AQB = \angle ACP$$

$AB = AC$ (given)

Hence, ΔABQ is congruent to ΔACP by AAS.

(b)

$\angle AQB + 68.5° = 180°$ (angles on a straight line)

$\angle AQB = 111.5°$

From (a),

$$\angle APC = \angle AQB$$
$$= 111.5°$$

Question 47

$AB = AE$ (given)

$\angle ABC = \angle AED$ (ΔABE is isos.)

$$\angle ACB = 180° - \angle ACD$$
$$= 180° - \angle ADC \quad (\angle ACD = \angle ADC, \Delta ACD \text{ is isos.})$$
$$= \angle ADE$$

$\therefore \Delta ABC \equiv \Delta AED$ (AAS) (proven)

Question 48

(a) $\angle FSR = \angle FRO$

$\angle PSE = \angle SRF$

$\angle EPS = \angle SFR (90°)$ (Angle)

$\angle PSE = \angle SRF$ (Angle)

$SE = SR$ (side)

$\Delta PES = \Delta FSR$

(b) Using Pythagoras Theorem, $SF = 10.7$ cm (proven)

Question 49

(a) $\angle ABC = \angle ADE$ (corr. $\angle$s)

$\angle ACB = \angle AED$ (corr. $\angle$s)

$\angle BAC = \angle DAE$ (common $\angle$)

Hence, triangles ABC and ADE are similar. (shown)

(b) (ii)

$$\frac{A}{A+136} = \left(\frac{1}{3}\right)^2$$
$$\frac{A}{A+136} = \frac{1}{9}$$
$$9A = A + 136$$
$$8A = 136$$
$$A = 17 \text{ cm}^2$$

Question 50

(a) $\angle ABC = \angle ACD$ (given)

$\angle BAC = \angle CAD$ (common)

ΔABC and ΔACD are similar by AAA similarity.

(b)

$$\frac{AB}{6} = \frac{6}{4}$$
$$AB = 9$$

(c)

$$\frac{\text{area of } \Delta ACD}{\text{area of } \Delta ABC} = \left(\frac{6}{9}\right)^2$$
$$= \frac{4}{9}$$

(d)

$$\frac{\text{area of } \Delta ACD}{\text{area of } \Delta BCD} = \frac{AD}{BD}$$
$$= \frac{4}{5}$$