RAFFLES INSTITUTION
Y5-6 PHYSICS DEPARTMENT

Tutorial 16 Electromagnetism Suggested Solutions

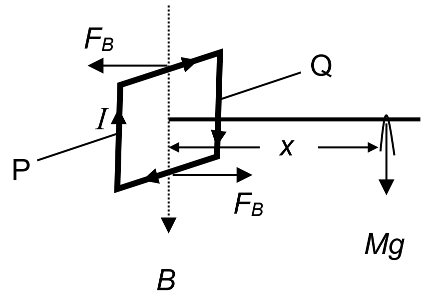
D1 **(a)** The magnetic flux density of a magnetic field is numerically equal to the force per unit length per unit current of a long straight conductor at right angles to a uniform magnetic field.

$$B = \frac{F}{IL} \qquad \text{...... (1)}$$

Taking moments about the axis PQ,
sum of anti-clockwise moments = sum of clockwise moments

$$NF_B L = Mgx$$

$$F_B = \frac{Mgx}{NL} \qquad \text{..... (2)}$$



Substituting (2) into (1),

$$B = \frac{\left(\frac{Mgx}{NL}\right)}{IL} = \frac{Mgx}{IL^2N} \quad \text{(shown)}$$

(b) Rearranging the above equation, $x = \dfrac{BIL^2N}{Mg}$

$I = \dfrac{E}{R}$ where $E =$ emf of battery and $R =$ resistance of coil

$R = \dfrac{\rho\ell}{A}$

where $\rho =$ resistivity of wire
$\ell =$ total length of wire used to make the coil with N turns
$A =$ cross-sectional area of wire

(i) When number of turns is doubled, the total length of wire required to form the coil is also doubled.
Hence, for the same emf, resistance of the coil is doubled and current flowing through it is halved.

$$\text{For } 2N,\ x' = \frac{B\left(\frac{1}{2}I\right)L^2 2N}{Mg} = \frac{BIL^2N}{Mg} = x$$

Thus, there is no change in the value of x.

(ii) If the length of each side of the coil is halved, the total length of wire required to form the coil would also be halved.
Hence, for the same emf, resistance of the coil is halved and current flowing through it is doubled.

$$\text{For } \frac{L}{2},\ x' = \frac{B(2I)\left(\frac{L}{2}\right)^2 N}{Mg} = \frac{1}{2}\left(\frac{BIL^2N}{Mg}\right) = \frac{1}{2}x$$

Thus, the value of x is half the original value.

D2 **(a)** **(i)** **1.**

$$R=\rho\frac{L}{A}=\left(4.8\times10^{-8}\right)\frac{\left(2.0\times10^{-2}\right)}{\left(5.0\times10^{-2}\right)\left(2.0\times10^{-2}\right)}=9.6\times10^{-7}\,\Omega$$

2. $V = IR = (50)\ (9.6 \times 10^{-7}) = 4.8 \times 10^{-5}\,\text{V}$

(ii) **1.** By Fleming's Left Hand Rule, the force is directed towards the left, perpendicular to the direction of the current and the direction of the magnetic field.

2. $F = BIL = (0.12)\ (50)\ (2.0 \times 10^{-2}) = 0.12\ \text{N}$

(iii)

$$\Delta p=\frac{F}{A}=\Delta h\rho g$$

$$\Delta h=\frac{F}{A\rho g}=\frac{0.12}{\left(2.0\times10^{-2}\right)^{2}\left(9.6\times10^{2}\right)(9.81)}=0.032\ \text{m}=3.2\ \text{cm}$$

(b) <u>Advantage</u>

- No moving parts are involved which leads to less wear and tear and also less noise.

<u>Disadvantage</u>

- The technique can only work with liquids that are conductive.
- The circuit needs to handle a large current which may lead to high power losses in the wiring.

D3 **(a)** By the principle of conservation of energy,

increase in KE = decrease in electric PE

$$\frac{1}{2}Mv^{2}-0=QV$$

$$v=\sqrt{\frac{2QV}{M}}\quad\text{(shown)}$$

(b) The magnetic force on the ions provides for the centripetal force.

$$BQv=\frac{Mv^{2}}{r}$$

$$r=\frac{Mv}{BQ}=\frac{M}{BQ}\sqrt{\frac{2QV}{M}}=\frac{1}{B}\sqrt{\frac{2MV}{Q}}$$

(c)

$$r=\frac{1}{B}\sqrt{\frac{2MV}{Q}}=\frac{1}{B}\sqrt{\frac{2V}{Q/M}}$$

For an ion with twice the specific charge, the radius of the circular path of the ion in the magnetic field would be reduced.

The new radius is given by $r'=\dfrac{1}{B}\sqrt{\dfrac{2V}{2\left(Q/M\right)}}=\dfrac{1}{\sqrt{2}}r=0.71\,r$

D4 (a) (i) Consider the circular motion due to the vertical component of velocity. The magnetic force provides for the centripetal force.

$$\frac{m_e v_\perp^2}{R} = Bev_\perp$$

$$R = \frac{m_e v_\perp}{Be} = \frac{\left(9.11\times10^{-31}\right)\left(6.7\times10^{6}\right)\sin 40°}{3.0\times10^{-5}\times1.60\times10^{-19}} = 0.817 \text{ m}$$

(ii) $$T = \frac{2\pi R}{v_\perp} = \frac{2\pi\left(0.817\right)}{\left(6.7\times10^{6}\right)\sin 40°} = 1.192\times10^{-6} \text{ s} = 1.19\times10^{-6} \text{ s}$$

(iii) The pitch p depends on the horizontal component of velocity.
$p = v_{//}T = \left(6.7\times10^{6}\right)\cos 40°\left(1.19\times10^{-6}\right) = 6.11 \text{ m}$ (or 6.12 m if use 1.192×10^{-6} s)

(b) If the angle θ were very small, $v_{//} \to v$ as $\cos\theta \to 1$
Now, from (a)(i), $\dfrac{R}{v_\perp} = \dfrac{m_e}{Be}$.
From (a)(ii), $T = \dfrac{2\pi R}{v_\perp} = \dfrac{2\pi m_e}{Be} = \text{constant}$
$\therefore p \approx vT$
$p = 6.7\times10^{6}\times1.19\times10^{-6} \text{ s} = 7.97 \text{ m}$
(or 7.99 m if used 1.192×10^{-6} s)

D5 (a) (i) $$R = \frac{V}{I} = \frac{4.5}{2.5} = 1.8\ \Omega$$

(ii) $$R = \rho\frac{L}{A}$$

$$L = \frac{RA}{\rho} = \frac{(1.8)\left(\pi\left(\dfrac{0.60\times10^{-3}}{2}\right)^2\right)}{\left(1.6\times10^{-8}\right)} = 31.8 \text{ m}$$

$$\text{Number of turns} = \frac{\text{Total length of wire}}{\text{Circumference}} = \frac{31.8}{8.8\times10^{-2}}$$
$= 361$ turns or 360 turns (2sf)

(iii) $$\text{Number of turns per metre} = \frac{360}{12\times10^{-2}} = 3000 \text{ (shown)}$$

(b) $$B = \mu_0 nI = \left(4\pi\times10^{-7}\right)\left(3000\right)\left(2.5\right) = 9.4\times10^{-3}\ T$$

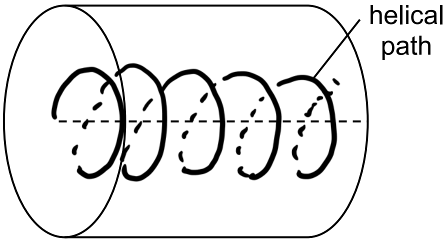
(c) (i) $$v_{\text{along axis}} = \left(4.0\times10^{7}\right)\cos 30° = 3.5\times10^{7} \text{ m s}^{-1}$$

(ii) $v_{\text{normal to axis}} = \left(4.0\times10^{7}\right)\sin30° = 2.0\times10^{7}\ \text{m s}^{-1}$

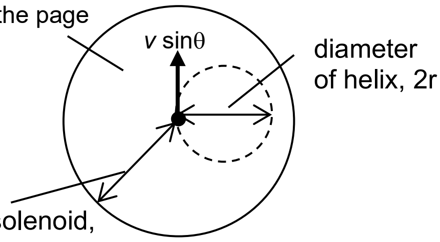
(d) The magnetic force on the electron provides for the centripetal force.
****Note: The above statement is required to explain your working.***

$$Bqv = \frac{mv^2}{r}$$
$$r = \frac{mv}{Bq}$$

(e) The electron travels in a helical path inside the solenoid. In order for it not to collide, the diameter of the helix must be less than the radius of the solenoid.



Viewing from the side

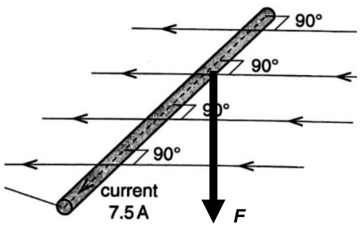


Viewing into the solenoid
(solid circle represents the solenoid, dotted circle represent the helix)

$$\begin{aligned}\text{Diameter of helix} &= 2r\\ &= 2\frac{mv_{\text{normal to axis}}}{Bq}\\ &= 2\frac{\left(9.11\times10^{-31}\right)\left(2.0\times10^{7}\right)}{\left(9.4\times10^{-3}\right)\left(1.6\times10^{-19}\right)}\\ &= 0.024\ \text{m} \quad > 0.014\ \text{m (radius of solenoid)}\end{aligned}$$

Hence, the electron will collide with the wall of the solenoid.

D6 **(a)** **(i)**



(ii) $\frac{F}{L} = BI = \left(5.2\times10^{-2}\right)\left(7.5\right) = 0.39\ \text{N m}^{-1}$

(b) **(i)** No. of electrons per unit length of wire $= \left(7.8\times10^{28}\right)\left(1.5\times10^{-6}\right)$

$$= 1.17\times10^{23}\ \text{m}^{-1}$$

$$\text{Force on each electron} = \frac{\text{Force per unit length}}{\text{No. of electrons per unit length}} = \frac{0.39}{1.17\times10^{23}} = 3.3\times10^{-24}\ \text{N} \quad \text{(shown)}$$

(ii) Force on each electron $= Bqv$

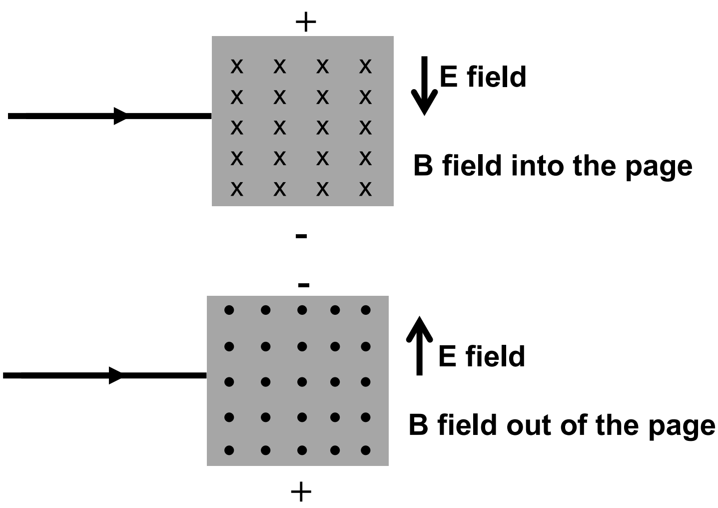
$$v = \frac{\left(3.3\times10^{-24}\right)}{\left(5.2\times10^{-2}\right)\left(1.6\times10^{-19}\right)} = 4.0\times10^{-4}\ \text{m s}^{-1}$$

Alternative method:

$I = nAvq$ where $n =$ number density

$$v = \frac{7.5}{\left(7.8\times10^{28}\right)\left(1.5\times10^{-6}\right)\left(1.60\times10^{-19}\right)} = 4.0\times10^{-4}\ \text{m s}^{-1}$$

D7 **(a)** **(i)**



(ii) increase in KE = decrease in EPE

$$\frac{1}{2}mv^2 - 0 = Q\left(\Delta V\right)$$

$$v = \sqrt{\frac{2Q\Delta V}{m}}$$

$$v = \sqrt{\frac{2\left(1.60\times10^{-19}\right)\left(1400\right)}{\left(3.32\times10^{-26}\right)}} = 1.16\times10^{5}\text{m s}^{-1}$$

(iii) For the ions to be undeflected,

$F_B = F_E$

$Bqv = Eq$

$v = \frac{E}{B}$

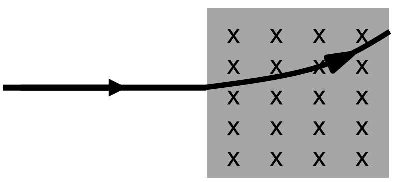
$B = \frac{6.2 \times 10^3}{1.16 \times 10^5} = 0.0534\text{T}$

(b) **(i)** Charge on neon ions are now +2*e*, since each atom loses two electrons.

From **(a) (ii)**, $v = \sqrt{\frac{2q\Delta V}{m}}$ $\quad v' = \sqrt{\frac{2(2e)\Delta V}{m}} = \sqrt{2}\ v = 1.41v$

The speed of the ions would be greater by a factor of 1.41.

(ii) Since the speed of the ions increases, they would experience a larger force due to the magnetic field. The ions would no longer pass through undeflected but are deflected upwards in the region of the two fields.



B field into the plane of the paper

* The curved path is neither parabolic nor circular.

D8 **(a)** increase in KE = decrease in electric PE

$\frac{1}{2}mv^2 - 0 = Q(\Delta V)$

$v = \sqrt{\frac{2(1.60 \times 10^{-19})(10000)}{(2.04 \times 10^{-25})}} = 1.25 \times 10^5 \text{ m s}^{-1}$

(b) **(i)** $F = qE = q\frac{V}{d} = (1.60 \times 10^{-19})\frac{(400)}{(2.0 \times 10^{-2})} = 3.2 \times 10^{-15} \text{ N}$

(ii) magnetic force = electric force

$F_B = F_E$

$B_1 qv = Eq$

$v = \frac{E}{B_1}$

where v is the speed of the ions exiting the velocity selector

E is the electric field strength within the velocity selector

B_1 is the magnetic flux density within the velocity selector (perpendicular to E)

(iii) $B_1 = \frac{E}{v} = \frac{\left(400/2.0 \times 10^{-2}\right)}{(177 \times 10^3)} = 0.113 \text{ T}$

(c) **(i)** The magnetic force acting on an ion is always directed perpendicular to its instantaneous velocity, and this force is constant in magnitude and always pointing towards a fixed point. This leads to a uniform circular motion, where the direction of velocity changes but not its magnitude.

(ii) The magnetic force thus provides the centripetal force for the ion to move in a circular path.

$$B_2qv = \frac{mv^2}{r}$$

$$r = \frac{mv}{B_2q} \Rightarrow r \propto m$$

(iii)

Dist betw 2 pts of impact, $d = d_{tin120} - d_{tin118}$

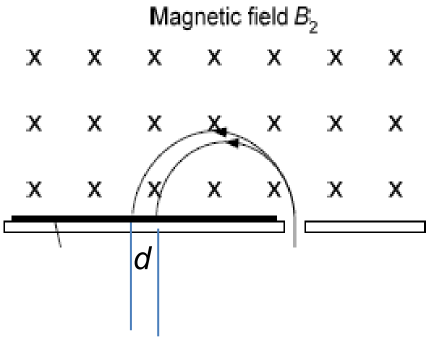
$$= 2r_{tin120} - 2r_{tin118}$$

$$= 2\frac{v}{B_2q}(m_{tin120} - m_{tin118})$$

$$= 2\frac{(177\times10^3)}{(0.75)(1.60\times10^{-19})}\times$$

$$\left[(2.04\times10^{-25})-(2.01\times10^{-25})\right]$$

$$= 8.85\times10^{-3}\text{ m} = 8.85\text{ mm}$$



Challenging Question

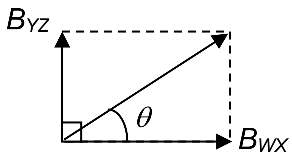
C1 **(a)**

$$B_{WX} = \frac{\mu_0 I_{WX}}{2\pi r} = \frac{4\pi\times10^{-7}\times9.0}{2\pi\times50\times10^{-3}} = 3.6\times10^{-5}\text{ T (parallel to CB)}$$

$$B_{YZ} = \frac{\mu_0 I_{YZ}}{2\pi r} = \frac{4\pi\times10^{-7}\times12.0}{2\pi\times50\times10^{-3}} = 4.8\times10^{-5}\text{ T (upward)}$$

$$B_{res} = \sqrt{B_{WX}^2 + B_{YZ}^2} = \left(\sqrt{3.6^2+4.8^2}\right)\times10^{-5} = 6.0\times10^{-5}\text{ T}$$

$$\theta = \tan^{-1}\frac{B_{YZ}}{B_{WX}} = \tan^{-1}\frac{4.8\times10^{-5}}{3.6\times10^{-5}} = 53.1^{o}$$



view from left hand

(b) WX must be arranged in such a way that it would produce a downward magnetic field at P, i.e. WX must be parallel to YZ with the current in the same direction as YZ.
It must be placed a distance *d* from P such that

$$B_{WX} = B_{YZ}$$

$$\frac{\mu_0 I_{WX}}{2\pi d} = \frac{\mu_0 I_{YZ}}{2\pi r}$$

$$d = \frac{I_{WX}}{I_{YZ}}r = \frac{9.0}{12.0}\times50 = 37.5\text{ mm}$$

Hence, the perpendicular distance between WX and YZ is 87.5 mm.