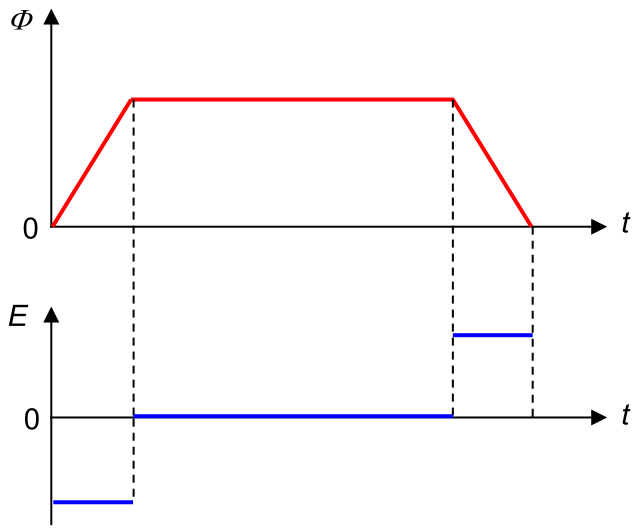
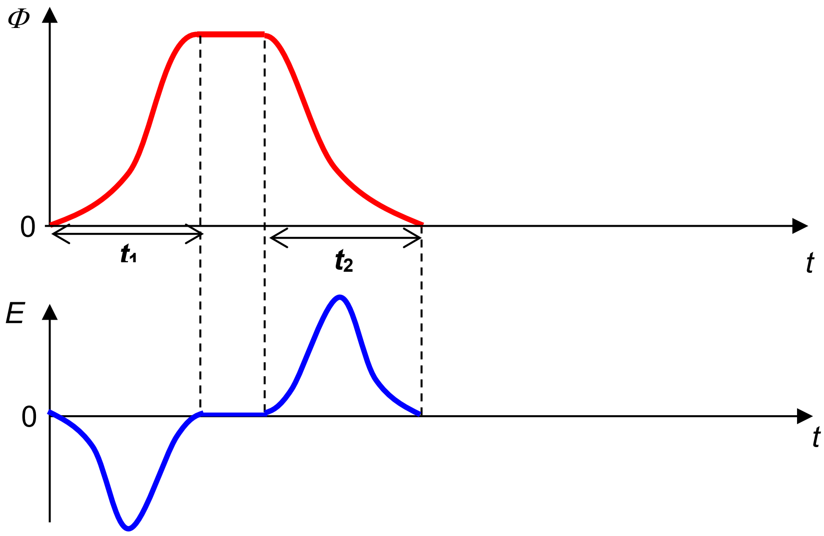
RAFFLES INSTITUTION
PHYSICS DEPARTMENT

Tutorial 17 Electromagnetic Induction Suggested Solutions

D1 **(a)**

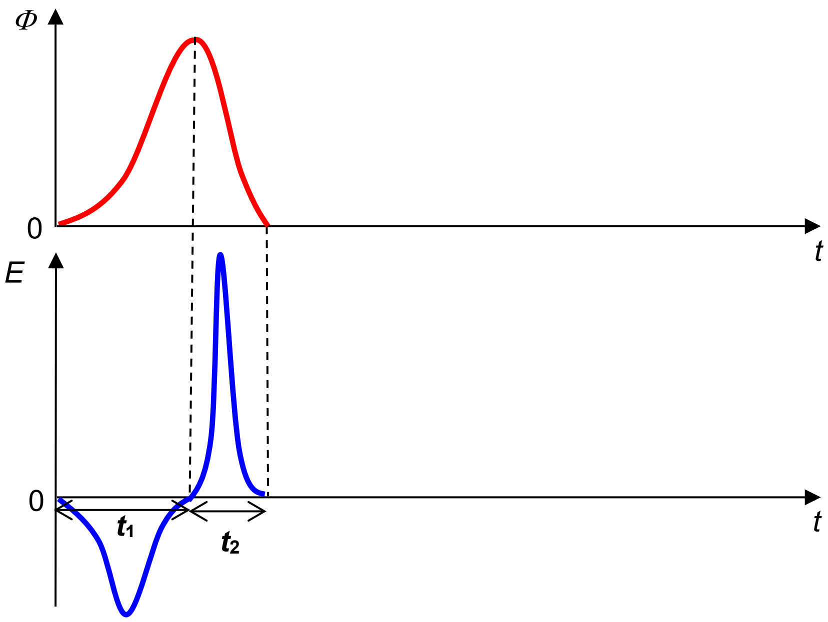


(b)



- $t_2 = t_1$
- magnitude of peak emf is equal in both durations
- Area under *E*-*t* graph is equal for the duration t_1 and the duration t_2

(c)



- $t_2 < t_1$
- magnitude of peak emf is larger in the duration t_2
- Area under *E*-*t* graph is equal for the duration t_1 and the duration t_2

D2 **(a)** Faraday's Law states that the induced e.m.f. E is directly proportional to the rate of change of magnetic flux linkage through the coil.

Mathematically, $E = -\dfrac{d\Phi}{dt}$

Therefore, $\Delta\Phi = -\int E\, dt$

The area under the E-t graph is obtained by integrating E over a particular time duration.

From the expression above, the magnitude of the area under the curve ($\left|-\int E\, dt\right|$) gives the change in magnetic flux linkage($\Delta\Phi$) through the coil.

(b) $|\Delta\Phi| = \text{Area under curve}$

$= \text{number of squares} \times \text{area of one square}$

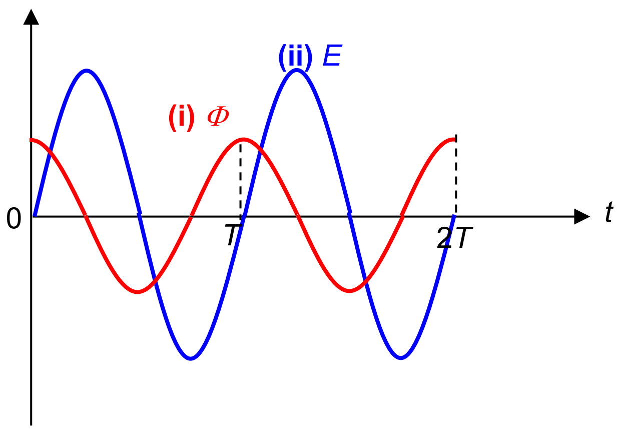
$= 9.5 \times \left(50 \times 10^{-3}\right)\left(10 \times 10^{-3}\right)$

$= 4.75 \times 10^{-3}\ \text{Wb} \approx 4.8 \times 10^{-3}\ \text{Wb}$

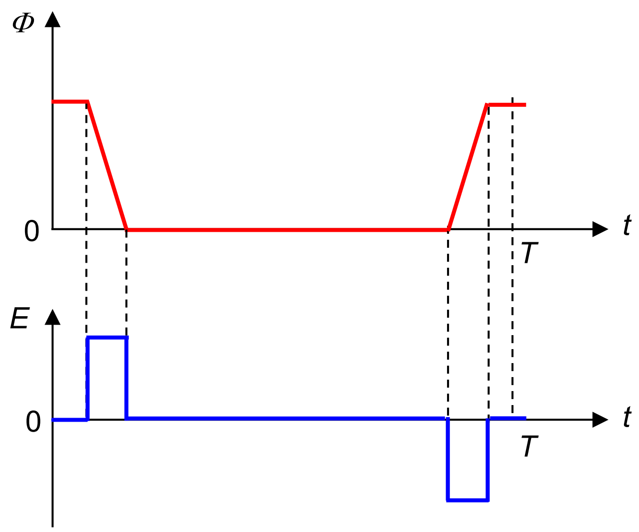
(c) $\Delta\Phi = \left|\Phi_{final} - \Phi_{initial}\right| = |0 - NBA|$

$$\therefore B = \frac{\Delta\Phi}{NA} = \frac{4.75 \times 10^{-3}}{(100)\pi\left(\dfrac{34 \times 10^{-3}}{2}\right)^2} = 0.052\ \text{T}$$

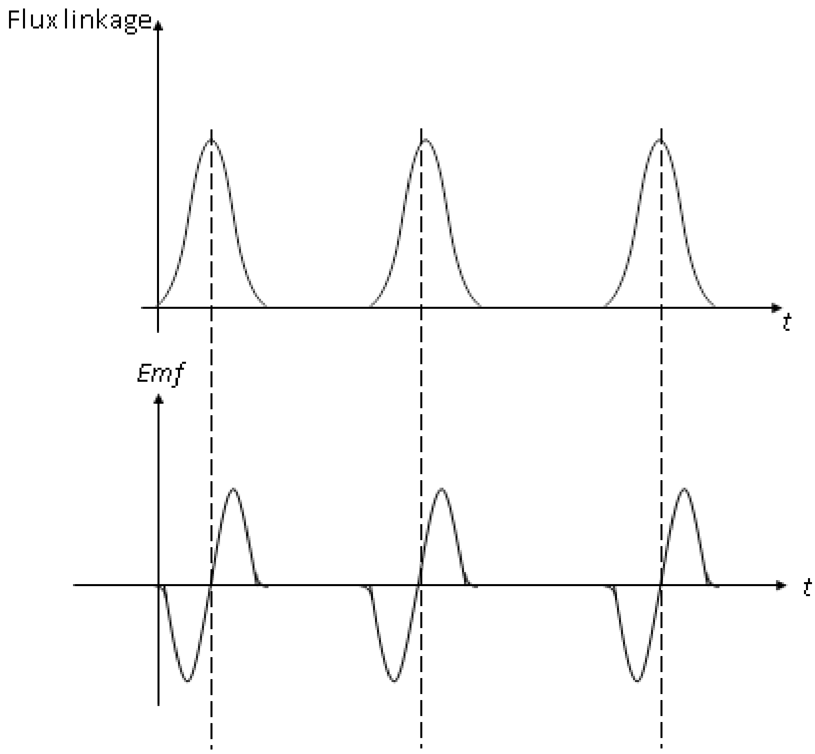
D3 **(a)**



(b)



D4 **(a)**



***Note:**
- the actual shape/equation of the variation is not as important as recognizing the **general trend** of the variation
- i.e. magnetic flux linkage increases to a maximum before decreasing to zero for a while before the next magnet passes through; the emf induced is in opposite directions when the magnet enters and leaves the position of the coil

(b) When a magnet approaches the coil, the magnetic flux linkage through the coil increases and an e.m.f. would be induced in the coil according to Faraday's Law. According to Lenz's Law, the induced e.m.f. would be negative so as to oppose the increasing flux linkage.

When the magnet moves away from the coil, the magnetic flux linkage through the coil decreases. Induced e.m.f. would be positive so as to oppose the decreasing flux linkage.

The magnitude of the e.m.f. is the greatest where the magnetic flux linkage changes at the greatest rate, i.e., at the points of inflection of the Φ - t graph. The induced e.m.f. is zero at the turning points of the Φ - t graph where gradient is zero.

In between the magnets, the flux linkage of the coil remains zero for a certain duration of time. The induced e.m.f. is therefore zero during this period.

D5 **(a)** Magnitude of maximum induced emf, $E = NA\dfrac{dB}{dt}$

$$= 500 \times 7.25 \times 10^{-4} \times \frac{(1.6-0.0)\times 10^{-2}}{0.0060-0.0000}$$

$$= 0.9667 \text{ V}$$

Maximum current, $I = \dfrac{\text{max induced } E}{\text{total resistance}}$

$$= \frac{0.9667}{(10.0+5.0)}$$

$$= 0.0644 \text{ A}$$

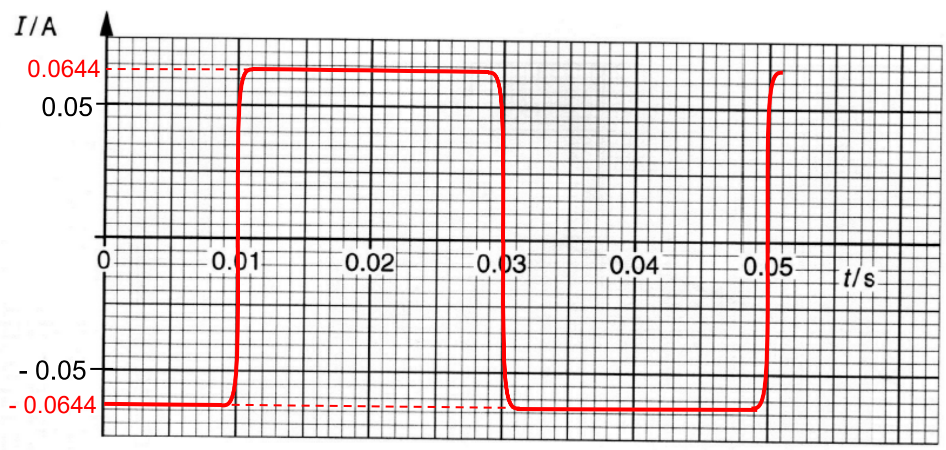
<u>Explanation (not required in solution):</u> Maximum current passes through R when the emf induced is maximum.

$$|E| = \left|-\frac{d\Phi}{dt}\right| = \left|-NA\frac{dB}{dt}\right|$$

Maximum E is induced when the gradient of the B-t graph $\dfrac{dB}{dt}$ is a maximum.

Find gradient of the straight part only, do not use t = 0.01 s as the gradient there is zero and not maximum.

(b) $I \propto E \propto -\dfrac{dB}{dt}$



D6 **(a)** **(i)**

$$E = BLv$$
$$= 0.50 \times 0.50 \times 5.0$$
$$= 1.25 \text{ V}$$

$$I = \frac{E}{R}$$
$$= \frac{1.25}{\left(\frac{5}{6} \times 0.30\right)}$$
$$= 5.0 \text{ A}$$

* Note that current only flows through the 50 cm length of the rod.

(ii) Upwards from Q to P.
(P would be at higher potential with respect to Q)

(iii) As PQ moves across the magnetic field, an induced force pointing towards the left will act on PQ, due to the induced current flowing in PQ.

This force, of magnitude BIL, will cause PQ to decelerate.

To keep PQ in constant velocity, a force of the same magnitude acting to the right must be applied on PQ, so that there will be no resultant force acting on the rod.

By Newton's 1st Law,

$$|\text{Applied force}| = |\text{Force due to induced current in PQ}|$$
$$= BIL$$
$$= 0.50 \times 5.0 \times 0.50$$
$$= 1.25 \text{ N}$$

(iv) Rate at which mechanical work is done by applied force

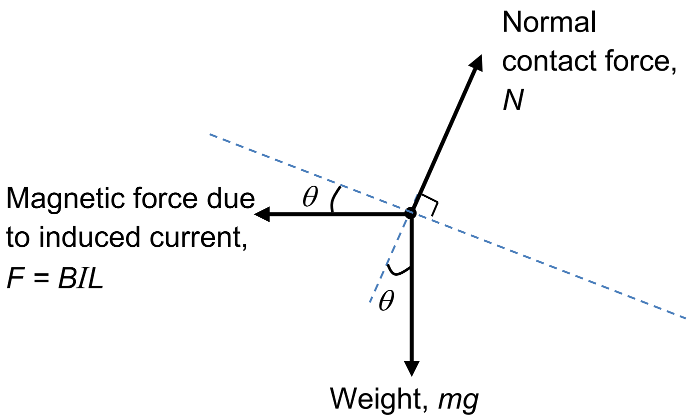
$$= Fv$$
$$= 1.25 \times 5.0$$
$$= 6.25 \text{ W}$$

Rate at which thermal energy is dissipated

$$= I^2 R$$
$$= (5.0)^2 \times \left(\frac{5}{6} \times 0.30\right)$$
$$= 6.25 \text{ W}$$

The rate at which mechanical work is done by the applied force is equal to the rate at which thermal energy is dissipated in the circuit.

(b) (i)



(ii) As the rod moves down the slope, an emf E is induced across it.

$$E = (B\cos\theta)Lv$$

where $B\cos\theta$ is the component of the magnetic flux density which is perpendicular to the direction of motion of the rod.

Since there is a closed circuit, current I flows through the circuit of resistance R (assume resistance of the wire is very small).

The rod now acts as a current carrying conductor placed in a magnetic field. A magnetic force F is induced on the rod and its direction is deduced using Fleming's Left Hand Rule (as shown in the figure).

The rod reaches terminal velocity when the component of the magnetic force upwards along the slope is equal to the component of the weight downwards along the slope (i.e., no resultant force along the slope).

$$\text{component of } F \text{ along the slope} = \text{component of } mg \text{ along the slope}$$

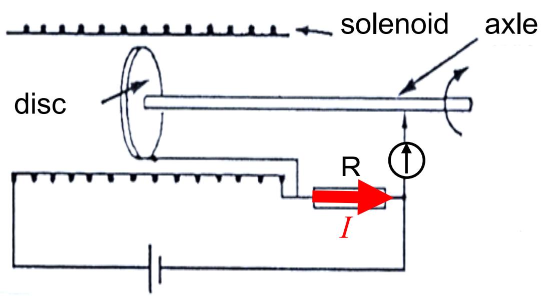
$$BIL\cos\theta = mg\sin\theta$$

$$B\left(\frac{BLv\cos\theta}{R}\right)L\cos\theta = mg\sin\theta$$

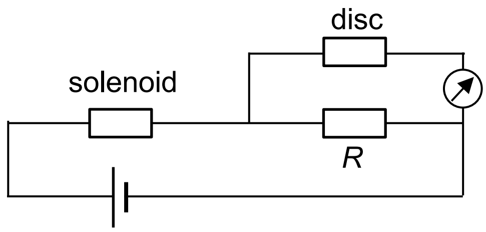
$$\frac{\left(B^2L^2\cos^2\theta\right)v}{R} = mg\sin\theta$$

$$v = \frac{mgR}{B^2L^2\cos^2\theta}\sin\theta$$

D7 **(a)**



When the disc is not rotating, the circuit diagram is:

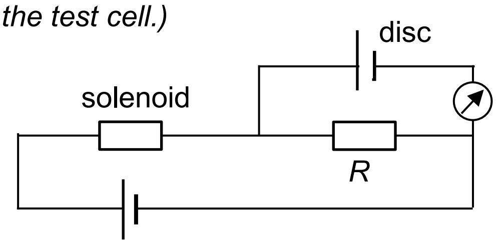


(b) Before the disc rotates, the d.c. source drives a current through the disc and galvanometer.

When the disc rotates, an e.m.f. is generated between the axle and edge of the disc. The induced emf is opposite in polarity to the p.d. across R and its magnitude is dependent on the rate of rotation. Hence, the magnitude of deflection of the galvanometer would start to decrease.

When the disc spins fast enough, the galvanometer registers no deflection because the magnitude of the e.m.f. induced in the rotating disc is equals to the magnitude of the p.d. across the resistor R.

*(*recall the potentiometer in DC circuits. At balance point, the galvanometer reading is zero because the pd across the potentiometer wire is equal to the terminal pd across the test cell.)*



(c) When the galvanometer registers no deflection,

p.d. across $R =$ induced emf in the disc

$$IR = BAf$$

$$R = \frac{BAf}{I}$$

$$= \frac{(\mu_o nI)Af}{I}$$

$$= \mu_o nAf \qquad \text{(shown)}$$

D8 **(a)** As the disc falls towards the magnet, its surface is cutting the magnetic field lines of the magnet. By Faraday's law, an induced e.m.f. is produced, and hence eddy current flows in the disc. The eddy current produces heat (due to joule heating) and this results in an increase in temperature of the disc.

(b) By Lenz's law, the induced current will flow in a direction to oppose the motion of the disc, i.e., the disc is subjected to a repulsive upward force which retards its downward motion. The resultant force experienced by the disc becomes smaller and the downward acceleration becomes smaller than the acceleration of free fall.

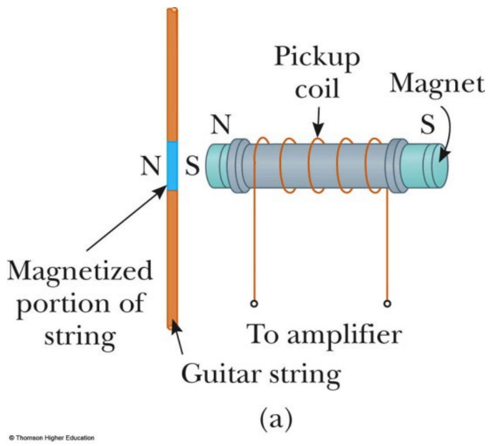
D9 **(a)** The magnitude of the magnetic flux in the coil increases from zero to a steady value.

(b) There is a change in magnetic flux linkage in the coil as the magnetic flux increases. Hence, according to Faraday's Law, an emf is induced in the coil.

(c) According to Lenz's Law, the induced emf would be in a direction opposite to the applied emf of the battery. This causes a delay in the flow of current through the bulb.
After the current in the coil reaches a steady value, the induced emf would be zero.

(d) If the iron core is replaced by a wooden core, the magnitude of the magnetic flux in the coil would decrease. Hence, the change in magnetic flux linkage would also be less than before, giving rise to a smaller induced emf, which opposes the applied emf. As a result, the delay would be shorter.

D10



- The small cylindrical magnet magnetises the steel string.
- When the magnetised steel string is plucked, it vibrates like a little magnet and changes the magnetic flux linkage through the coil.
- An alternating e.m.f. and hence an alternating current will be induced in the coil, producing an electric signal across the terminals of the coil.

Challenging Question

C1. The area of a circuit is a rectangle with a semi-circular part that is changing as the semi-circular loop is rotated. If we let the area of the rectangle be *A*, then the area of the circuit at any time is: $A - \frac{1}{2}\pi r^2 \cos\omega t$

Hence, the induced e.m.f. is:

$$|E| = \left|-\frac{d\Phi}{dt}\right|$$

$$= \left|\frac{d}{dt}\left[B\left(A - \frac{1}{2}\pi r^2 \cos\omega t\right)\right]\right|$$

$$= \left|0 - \frac{1}{2}B\pi r^2 \omega \sin\omega t\right|$$

Clearly, the peak value is $\frac{1}{2}B\pi r^2\omega$.

Hence,

$$\frac{1}{2}(0.80)\pi(0.20)^2\omega = 0.31$$

$$\omega = 6.17 \text{ rad s}^{-1}$$