Name:______________________________________ (   ) Class: S____. TG:______

Write in BLOCK Letters

90

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2024

Additional Mathematics **4049/02**

Paper 2 26 August 2024

Secondary 4 Express / 5 Normal Academic **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Qn	1	2	3	4	5	6	7	8	9a	9b	10a,b	10c
Strand	G2	G3	A3	G1	C1	G1	A4	C1	A2	A4	C2	C1
Marks	6	6	8	8	9	9	10	10	5	6	8	5

My Target ***BEFORE***	
My Target ***AFTER***	

My Reflection:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

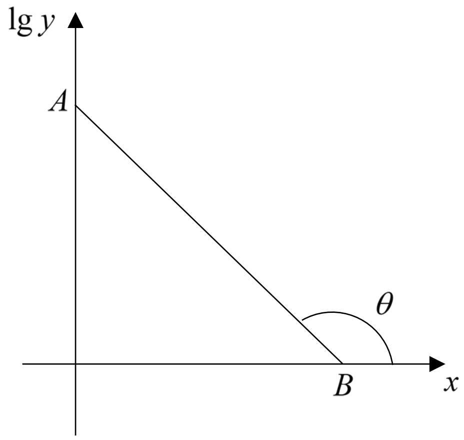
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}bc \sin A$$

Answer ALL questions.

1 The two variables x and y are related by the equation $y = 10^{-3x+2}$. The diagram shows part of a straight line graph obtained when plotting $\lg y$ against x. The straight line cuts the $\lg y$-axis and x-axis at points A and B respectively.



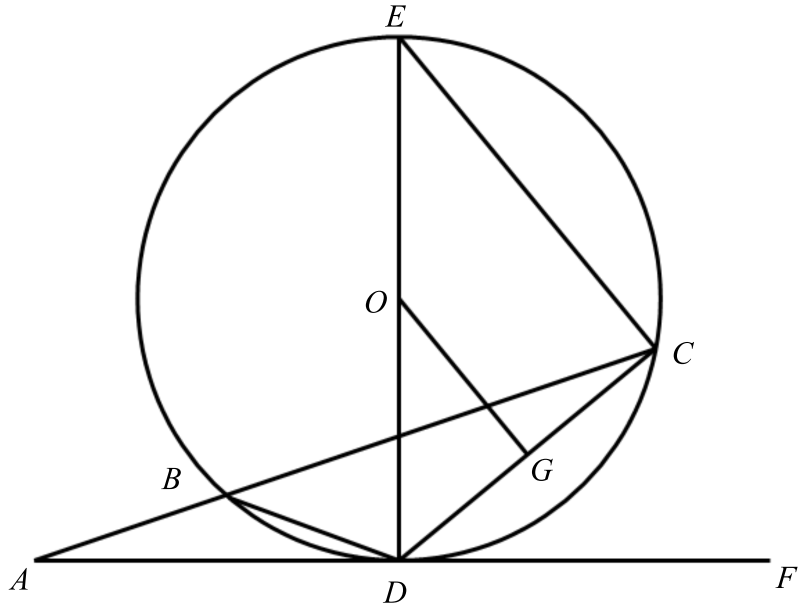
(a) Find the coordinates of A and of B. [3]

(b) Point C lies on the straight line and the x-coordinate of point C is $\frac{1}{2}$.

Find the value of y at point C. [1]

(c) Find the value of $\tan\theta$. [2]

2 In the diagram below, the line *ADF* is a tangent to the circle at *D*. *DE* is a diameter and *O* is the centre. *ABC* is a straight line intersecting the circle at points *B* and *C*.



(a) By showing that triangle *ABD* is similar to triangle *ADC*, prove that $AD^2 = AB \times AC$. [3]

(b) Given that *G* is the midpoint of *DC*, show that angle $DGO = 90°$. [3]

3 **(a)** It is given that $\dfrac{1}{4-2\sqrt{5}} = m + n\sqrt{5}$, where m and n are rational numbers.
Find the value of m and of n. [2]

(b) The sides AB and BC of a triangle, right-angled at B, are of length $\left(3\sqrt{2}+\sqrt{5}\right)$ units and $\left(\sqrt{2}+2\sqrt{5}\right)$ units respectively. Find the exact value of $\tan \measuredangle ACB$. [3]

(c) It is given that $\sqrt{a}+\sqrt{b} = \sqrt{11}+\sqrt{15}$ and $\sqrt{ab} = \sqrt{165}-\sqrt{11}$. Without using a calculator, express $a + b$ in the form $p + 2\sqrt{q}$, where p and q are integers. [3]

4 **(a)** Show that $\dfrac{\sin 2x+\cos 2x+1}{\sin x+\cos x}=2\cos x$. [3]

(b) Hence, solve $\dfrac{\sin 4\theta+\cos 4\theta+1}{\sin 2\theta+\cos 2\theta}=3\cos^2 2\theta-1$ for $-180°<\theta<180°$. [5]

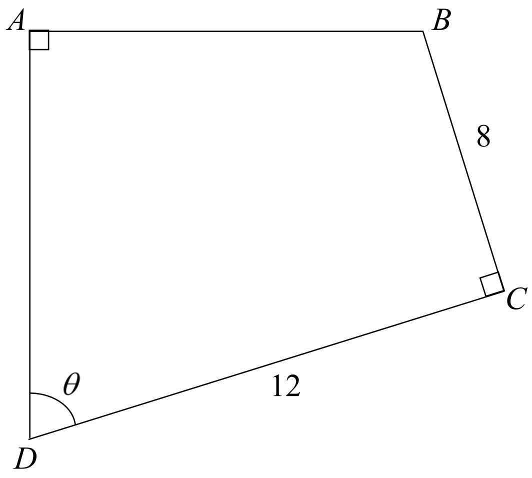
5 It is given that $x = \ln\left(t^2 + 1\right),\ t > 0$.

(a) Find $\dfrac{\mathrm{d}x}{\mathrm{d}t}$. [2]

(b) By writing t in terms of x, find $\dfrac{\mathrm{d}t}{\mathrm{d}x}$, in terms of x. [4]

(c) Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} \cdot \dfrac{\mathrm{d}t}{\mathrm{d}x} = k$, where k is an integer to be found. [3]

6 The diagram shows a quadrilateral $ABCD$ in which angle $ADC = \theta$ radians and angle $DAB =$ angle $BCD = 90°$. $BC = 8$ cm and $DC = 12$ cm.



(a) P is the perimeter of quadrilateral $ABCD$. Show that $P = 20\sin\theta + 4\cos\theta + 20$. [4]

(b) Express P in the form $R\sin(\theta+\alpha)+k$, where $R>0$ and $0<\alpha<\dfrac{\pi}{2}$. [2]

(c) State the maximum value of P and the corresponding value of θ. [2]

(d) Find the largest θ such that the perimeter of $ABCD$ is at most 30 cm. [1]

7 **(a)** Given that $3x^3 - x^2 - 3x + 7 = (x+1)(Ax+B)(x-1) + C$ for all values of x, find the value of A, of B and of C. [3]

(b) The polynomial $2x^3 + ax^2 - 10x + b$ is exactly divisible by $x - 2$ but leaves a remainder of 3 when divided by $x + 1$. Find the value of a and of b. [3]

(c) Write the polynomial $2x^3 - 3x^2 - 9x + 4$ as $(x-4)Q(x) + R$. [4]

8 A curve has the equation $y = xe^{-2x}$.

(a) Find the stationary point of the curve and determine if it is a maximum or minimum point. [5]

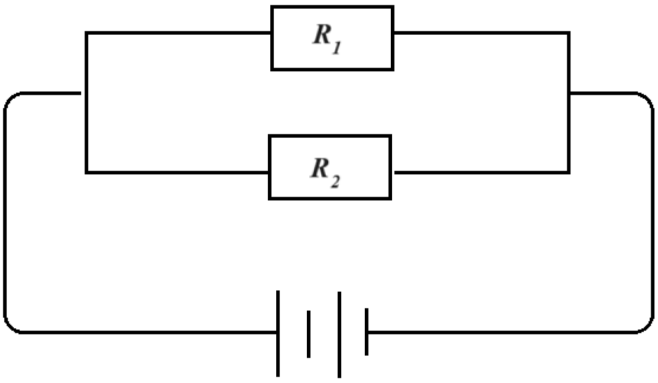
At the point where $x = 3$, find

(b) the equation of the normal to the curve. [4]

(c) the rate of change of the y-coordinate if the x-coordinate is increasing at a constant rate of 0.2 units per second. [1]

9 **(a)** The diagram shows an electric circuit with 2 resistors, R_1 and R_2. When R_1 and R_2 are connected in parallel, their resultant resistance R is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$.

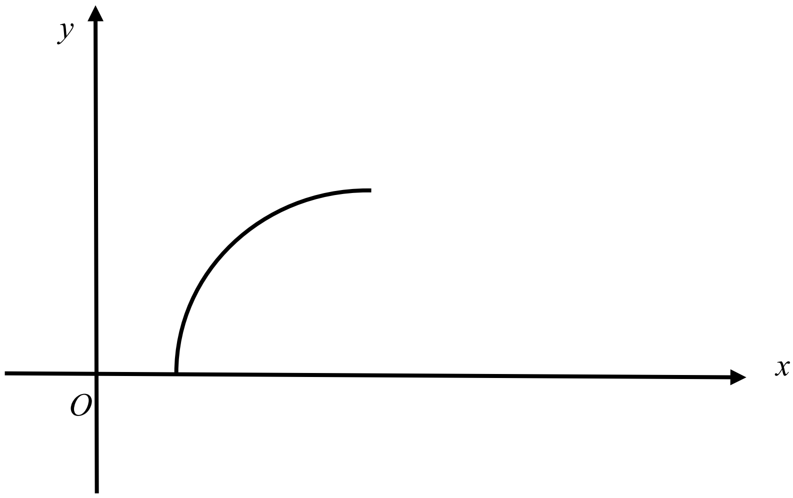
It is also given that the sum of R_1 and R_2 is 500 ohms. When they are connected in parallel, the equivalent resistance drops to 120 ohms.
Find the values of the two resistors, R_1 and R_2. [5]



(b) **(i)** Factorise $x^3 - 3x^2 + 2x - 6$. [3]

(ii) Hence express $\dfrac{8x-4}{x^3 - 3x^2 + 2x - 6}$ as partial fractions. [3]

10 The diagram below shows the curve C with the equation $y=\sqrt{x-3}$.



(a) **(i)** State the coordinates where the curve cuts the x-axis. [1]

(ii) On the same diagram, sketch the line L which has the equation $y=\frac{1}{3}x-1$, showing the x- and y-intercepts clearly. [1]

(b) Find the area of the region between C and L. [7]

[*more working space on next page*]

(c) The line M has the equation $y = \frac{1}{3}x + k$, where k is a constant.
Given that C and M intersect only once, find the value of k. [4]

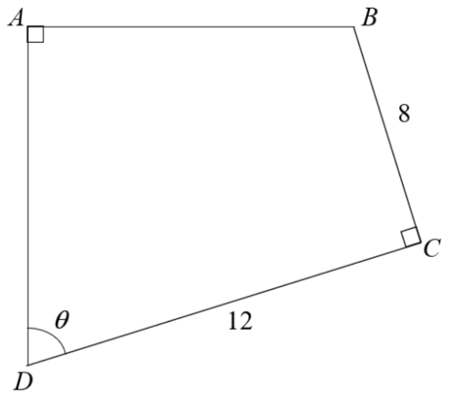
End of Paper

Marking Scheme

Q	Descriptor	Mark
1a	$y = 10^{-3x+2}$ $\lg y = \lg 10^{-3x+2}$ $\lg y = -3x + 2$ y-intercept: $A(0, 2)$ $\lg y = 0,$ $x = \frac{2}{3}$ $\therefore B\left(\frac{2}{3}, 0\right)$	
1b	$\lg y = -3\left(\frac{1}{2}\right) + 2$ $\lg y = \frac{1}{2}$ $y = 10^{\frac{1}{2}}$ $y = \sqrt{10}$	
OR	$y = 10^{-3\left(\frac{1}{2}\right)+2}$ $= 3.1622...$	
1c	$\tan(180 - \theta) = \frac{2}{\frac{2}{3}}$ $\tan(180 - \theta) = 3$ $\tan \theta = -3$	
G2		**6 marks**
2a	$\measuredangle ADB = \measuredangle ACD$ (alt seg thm) $\measuredangle A$ is a common $\measuredangle$. $\therefore$ Triangle ABD is similar to triangle ADC. $\frac{AB}{AD} = \frac{AD}{AC}$ $AD^2 = AB \times AC$ (proven)	
2b	Since G is the midpoint of DC and O is the midpoint of DE, by midpoint theorem, $OG // EC$. $\measuredangle ECD = 90°$($\measuredangle$s in semicircle) $\measuredangle DGO = 90°$(corr$\measuredangle$s, $OG // EC$)	

Q	Descriptor	Mark
OR	$DO = CO$ (radius of circle) $\measuredangle ODG = \measuredangle OCG$ (base $\measuredangle$ of isos Δ) $DG = CG$ (G is midpoint of DC) $\Delta DGO \equiv \Delta CGO$ ($S-A-S$) $\measuredangle DGO = \measuredangle CGO = 90^\circ$	
G3		**6 marks**
3a	$\frac{1}{4-2\sqrt{5}} = \frac{1}{4-2\sqrt{5}} \times \frac{4+2\sqrt{5}}{4+2\sqrt{5}}$ $= \frac{4+2\sqrt{5}}{16-20}$ $= -1-\frac{1}{2}\sqrt{5}$ $m=-1, \quad n=-\frac{1}{2}.$	
3b	$\tan \measuredangle ACB = \frac{\left(3\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{2}+2\sqrt{5}\right)}$ $= \frac{\left(3\sqrt{2}+\sqrt{5}\right)}{\left(\sqrt{2}+2\sqrt{5}\right)} \times \frac{\left(\sqrt{2}-2\sqrt{5}\right)}{\left(\sqrt{2}-2\sqrt{5}\right)}$ $= \frac{6-6\sqrt{10}+\sqrt{10}-10}{2-20}$ $= \frac{2}{9}+\frac{5}{18}\sqrt{10}.$	
3c	$\sqrt{a}+\sqrt{b} = \sqrt{11}+\sqrt{15} \quad \sqrt{ab} = \sqrt{165}-\sqrt{11}.$ $\left(\sqrt{a}+\sqrt{b}\right)^2 = \left(\sqrt{11}+\sqrt{15}\right)^2$ $a+2\sqrt{ab}+b = 11+2\sqrt{165}+15$ $a+b = 26-2\left[\sqrt{165}-\sqrt{11}\right]+2\sqrt{165}$ $= 26+2\sqrt{11}.$	
A3		**8 marks**

Q	Descriptor	Mark
4a	$LHS=\dfrac{\sin 2x+\cos 2x+1}{\sin x+\cos x}$ $=\dfrac{2\sin x\cos x+2\cos^2 x-1+1}{\sin x+\cos x}$ $=\dfrac{2\cos x(\sin x+\cos x)}{\sin x+\cos x}$ $=2\cos x=RHS$ (shown)	
4b	$\dfrac{\sin 4\theta+\cos 4\theta+1}{\sin 2\theta+\cos 2\theta}=3\cos^2 2\theta-1, \quad -360°<2\theta<360°$ $2\cos 2\theta=3\cos^2 2\theta-1$ $3\cos^2 2\theta-2\cos 2\theta-1=0$ $(3\cos 2\theta+1)(\cos 2\theta-1)=0$ $\cos 2\theta=-\dfrac{1}{3} or \cos 2\theta=1$ $\alpha=70.528°$ $2\theta=-250.528,-109.471,109.401,250.528$ $\theta=-125.3°,-54.7°,54.7°,125.3°$ $\cos 2\theta=1$ $\alpha=0$ $\theta=0°$ $\theta=-125.3°,-54.7°,0°,54.7°,125.3°$	
G1		8 marks
5a	$\dfrac{dx}{dt}=\dfrac{2t}{t^2+1}$	
5b	$e^x=t^2+1$ $t^2=e^x-1$ $t=\sqrt{e^x-1}$ $\dfrac{dt}{dx}=\dfrac{\frac{1}{2}e^x}{\sqrt{e^x-1}}=\dfrac{e^x}{2\sqrt{e^x-1}}$	
5c	$\dfrac{dx}{dt}\cdot\dfrac{dt}{dx}=\dfrac{2t}{t^2+1}\cdot\dfrac{e^x}{2\sqrt{e^x-1}}$ $=\dfrac{2t}{t^2+1}\cdot\dfrac{t^2+1}{2t}$ $=1$	
C1		9 marks

Q	Descriptor	Mark
6a	 $\cos\theta = \frac{XD}{12}$ $XD = 12\cos\theta$ $\sin\theta = \frac{BY}{8}$ $BY = 8\sin\theta$ $AB = XC - YC$ $XC = 12\sin\theta$ $YC = 8\cos\theta$ $AB = 12\sin\theta - 8\cos\theta$ $P = 12\sin\theta - 8\cos\theta + 12\cos\theta + 8\sin\theta + 8 + 12$ $= 20\sin\theta + 4\cos\theta + 20$ (shown)	
6b	$P = 20\sin\theta + 4\cos\theta + 20$ $= \sqrt{20^2+4^2}\sin\left(\theta + \tan^{-1}\frac{4}{20}\right) + 20$ $= \sqrt{416}\sin(\theta + 0.197395) + 20$ $= 4\sqrt{26}\sin(\theta + 0.197) + 20$	
6c	Max $P = 4\sqrt{26} + 20 = 40.4$ cm $\sin(\theta + 0.197395) = 1$ $\theta + 0.197395 = \frac{\pi}{2}$ $\theta = 1.37$ rad	
6d	$P = 4\sqrt{26}\sin(\theta + 0.197395) + 20 \le 30$ $\sin(\theta + 0.197395) \le 0.490290$ $\theta + 0.197395 \le 0.512422$ $\theta = 0.315$ rad	
G1		**9 marks**

Q	Descriptor	Mark
7a	$3x^3 - x^2 - 3x + 7 = (x+1)(Ax+B)(x-1)+C$ Let $x = 1,\ 3-1-3+7 = C$ $C = 6.$ Let $x = 0,\ 7 = 1(B)(-1)+6$ $B = -1.$ Let $x = 2,$ $3(2)^3 - (2)^2 - 3(2) + 7 = (2+1)(2A-1)(2-1)+6$ $21 = 6A - 3 + 6$ $A = 3.$	
7b	$\mathrm{f}(x) = 2x^3 + ax^2 - 10x + b \qquad R = 0, (x-2).\ R = 3, (x+1).$ $0 = 2(2)^3 + a(2)^2 - 10(2) + b$ $4a + b = 4 \qquad (1)$ $3 = 2(-1)^3 + a(-1)^2 - 10(-1) + b$ $a + b = -5 \qquad (2)$ $a = 3, \quad b = -8.$	
7c	$$\begin{array}{r l} & 2x^2 + 5x + 11 \\ x - 4 \big) & 2x^3 - 3x^2 - 9x + 4 \\ & \underline{\mp 2x^3 \pm 8x^2} \\ & 5x^2 - 9x \\ & \underline{\mp 5x^2 \pm 20x} \\ & 11x + 4 \\ & \underline{\mp 11x \mp 44} \\ & 48 \end{array}$$ $2x^3 - 3x^2 - 9x + 4 = (x-4)\left(2x^2 + 5x + 11\right) + 48.$	
A4		**10 marks**

Q	Descriptor	Mark
8a	$\frac{dy}{dx} = e^{-2x} + x(-2)e^{-2x}$ $= (1-2x)e^{-2x} = 0$ $x = \frac{1}{2}$ $y = \frac{1}{2e}$ $\left(\frac{1}{2}, \frac{1}{2e}\right)$ $\frac{d^2y}{dx^2} = -2e^{-2x} - 2(1-2x)e^{-2x}$ $x = \frac{1}{2}, \frac{d^2y}{dx^2} < 0$ Maximum point	
8b	$\frac{dy}{dx} = -5e^{-6}$ $y = 3e^{-6}$ $m = \frac{e^6}{5}$ $y = \frac{e^6}{5}x + c$ $3e^{-6} = \frac{3e^6}{5} + c$ $c = -242$ $y = \frac{e^6}{5}x - 242$	
8c	$\frac{dy}{dt} = -5e^{-6} \times 0.2$ $= -e^{-6}$	
C1		**10 marks**

Q	Descriptor	Mark
9a	$R_1 + R_2 = 500$ $R_1 = 500 - R_2 \quad (1)$ $\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{120} \quad (2)$ $\frac{1}{R_2} + \frac{1}{500 - R_2} = \frac{1}{120}$ $\frac{500 - R_2 + R_2}{R_2\left(500 - R_2\right)} = \frac{1}{120}$ $120 \times 500 = -R_2^{\ 2} + 500R_2$ $R_2^{\ 2} - 500R_2 + 60000 = 0$ $R_2 = 300 \text{ ohms} \quad \text{or } 200 \text{ ohms}$ $R_1 = 200 \text{ ohms} \quad \text{or } 300 \text{ ohms}$	
9b	$\text{f}(x) = x^3 - 3x^2 + 2x - 6$ $\text{f}(3) = (3)^3 - 3(3)^2 + 2(3) - 6 = 0$ $\therefore \ (x-3)$ is a factor of $\text{f}(x)$. $(x-3)(x^2 + bx + 2) = x^3 - 3x^2 + 2x - 6$ Compare coeffs of x^2 : $\quad \ldots - 3x^2 + bx^2 \ldots = -3x^2$ $b = 0$ $x^3 - 3x^2 + 2x - 6 = (x-3)(x^2 + 2).$	
9c	$\frac{8x-2}{x^3 - 3x^2 + 2x - 6.} = \frac{8x-2}{(x-3)(x^2+2)}$ $= \frac{A}{(x-3)} + \frac{Bx + C}{(x^2+2)}$ $8x - 2 = A(x^2 + 2) + (x-3)(Bx + C)$ Let $x = 3$, $\quad 8(3) - 2 = A\left(3^2 + 2\right)$ $A = 2.$ Let $x = 0$, $\quad -2 = 2(2) + (-3)(C)$ $C = 2.$ Let $x = 1$, $\quad 8 - 2 = 2(3) + (-2)(B + 2)$ $B = -2.$ $\frac{8x-2}{x^3 - 3x^2 + 2x - 6} = \frac{2}{(x-3)} + \frac{-2x+2}{(x^2+2)}.$ $= \frac{2}{(x-3)} + \frac{2-2x}{(x^2+2)}.$	
A2A4		**11 marks**

Q	Descriptor	Mark
10ai	(3, 0)	
ii	Line passes through (3,0) and (0,-1).	
10b	$\sqrt{x-3}=\frac{1}{3}x-1$ $x-3=\left(\frac{1}{3}x-1\right)^2$ $x-3=\frac{1}{9}x^2-\frac{2}{3}x+1$ $x^2-15x+36=0$ $(x-3)(x-12)=0$ $x=3, y=0$ $x=12, y=3$ Area $=\int_3^{12}\sqrt{x-3}\ \mathrm{d}x-\frac{1}{2}(9)(3)$ $=\left[\frac{(x-3)^{3/2}}{3/2}\right]_3^{12}-\frac{27}{2}$ $=\frac{2}{3}(27)-\frac{27}{2}$ $=4.5\text{units}^2$	
10c	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{x-3}}=\frac{1}{3}$ $\sqrt{x-3}=\frac{3}{2}$ $x=\frac{21}{4}$ $y=\frac{3}{2}$ $\frac{3}{2}=\frac{1}{3}\left(\frac{21}{4}\right)+k$ $k=-\frac{1}{4}$	

Q	Descriptor	Mark
OR	$\sqrt{x-3}=\frac{1}{3}x+k$ $x-3=\frac{1}{9}x^2+\frac{2}{3}kx+k^2$ $9x-27=x^2+6kx+9k^2$ $x^2+(6k-9)x+\left(9k^2+27\right)=0$ $(6k-9)^2-4(1)\left(9k^2+27\right)=0$ $(2k-3)^2-4(k^2+3)=0$ $4k^2-12k+9-4k^2-12=0$ $k=-\frac{1}{4}$	
C2 **C1**		**13 marks**