Name:____ ( ) *Write in BLOCK Letters*	Class: S____ TG:______

90

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2024

Additional Mathematics **4049/01**

Paper 1 22 August 2024

Secondary 4 Express / 5 Normal Academic **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Qn	1	2	3	4	5	*6a*	*6b*	7	8	9	10	11	*12a*	*12b(i)*	*12b(ii)c*
Strand	A1	A2	A6	A6	G1	C1	C2	G2	G1	C3	A5	G2	C2	C1	C2
Marks	4	6	6	6	6	*3*	*3*	8	8	8	9	11	*5*	*3*	*4*

My Target ***BEFORE***		*My Reflection:*
My Target ***AFTER***		

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\textit{Area of } \Delta = \frac{1}{2}bc \sin A$$

Answer ALL questions.

1 The maximum value of the expression $ax^2 + 10x + c$ is 7. Find a pair of possible integer values of a and c. Show your working clearly. [4]

2 The equation of a curve is $y = ax^2 + 6x + 4a - 4$, where a is a constant.

(a) In the case $a = 3$, find the set of values of x for which the curve lies completely above the line $y = 17$. [2]

(b) In the case where $a = \frac{1}{2}$, show that the line $y = 8x - 4$ is a tangent to the curve. [3]

(c) The straight line L meets the curve at one point only.
Given that L is not a tangent to the curve, what can be deduced about L? [1]

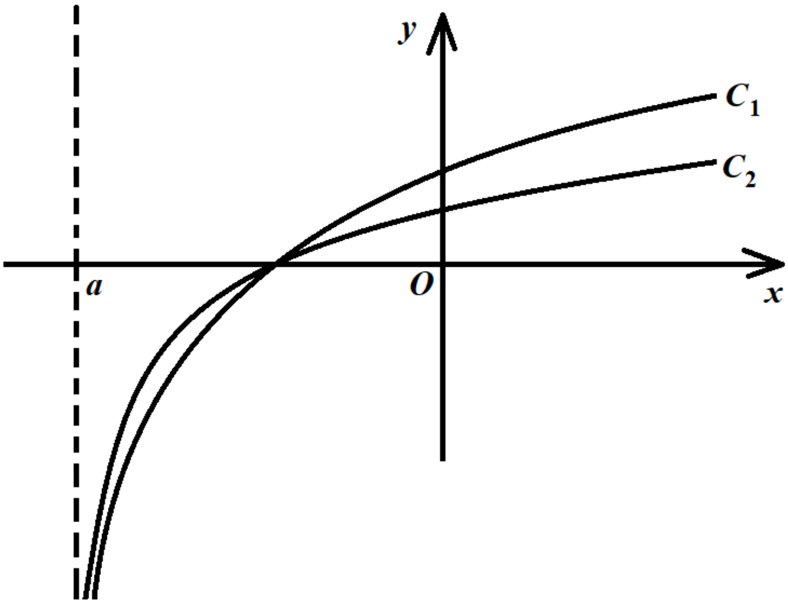
3 **(a)** Solve the equation $2^{2x} - 2^{x+3} + 4 = 0$. [3]

(b) The price of a watch was $24500 on 1 Jan 2014. The price of the watch appreciates by the formula, $P = 24\,500e^{0.02t}$, where t is the time in years after purchase of the watch and P is the price of the watch after t years. Estimate

(i) the price of the watch on 1 Jan 2024. [1]

(ii) the year in which the price of the watch is double the initial price. [2]

4 The diagram shows two curves C_1 and C_2, one of which represents $y = \lg(x+2)$ and the other $y = \ln(x+2)$.

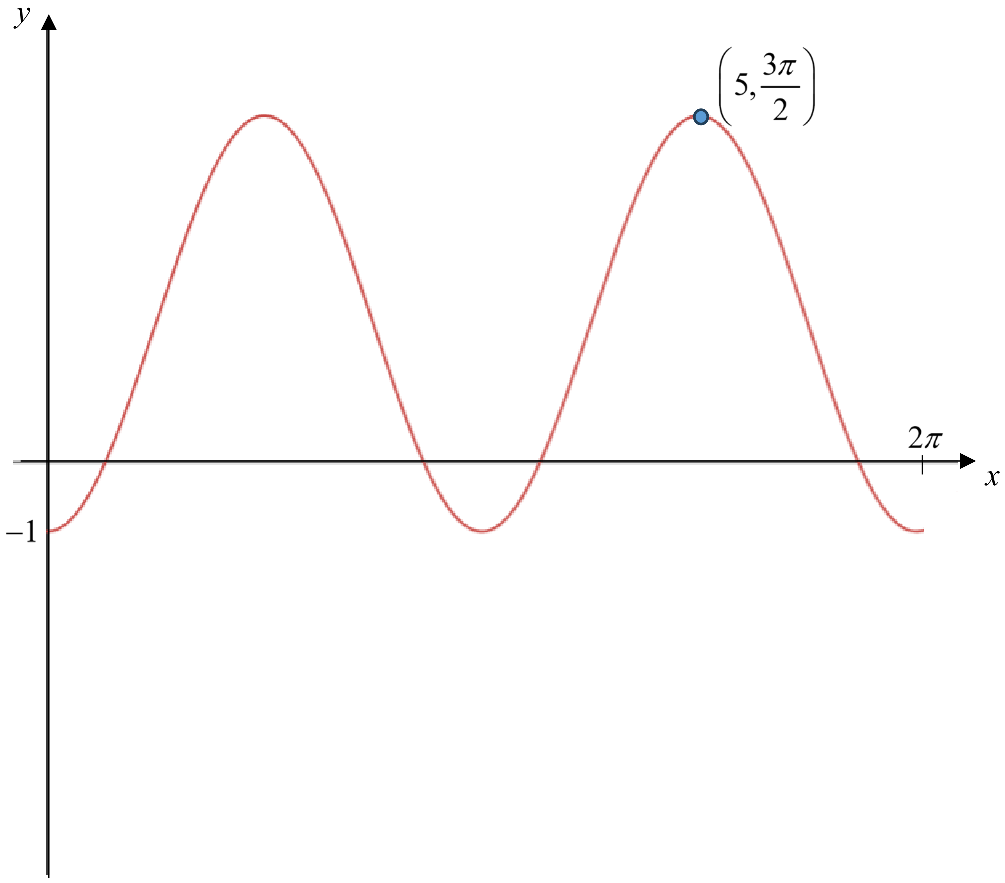


(a) State the value of a. [1]

(b) Explain briefly if C_1 represents the graph $y = \lg(x+2)$. [2]

(c) Solve the equation $2\lg(x+2) = \ln(x+2)$. [3]

5 The figure below shows part of the graph of $y = a\cos bx + c$ for $0 < x < 2\pi$.



(a) Find the value of each of the constants a, b and c. [3]

(b) On the same axes, sketch the graph of $y = 4\sin x + 1$ for $0 < x < 2\pi$. [2]

(c) Write down the number of solutions to the equation $4\sin x - a\cos bx - c + 1 = 0$. [1]

6 **(a)** Differentiate $\dfrac{3x^2 - 5x}{4x^3}$ with respect to x. [3]

(b) Integrate $\left(\dfrac{1}{3x} - \sqrt{x}\right)^2$ with respect to x. [3]

7 A rhombus *ABCD* has coordinates $A(-1,5)$, $B(p,8)$, $C(7,13)$ and $D(q,10)$.

(a) Find the value of the gradient of the line *BD*. [2]

(b) Find the coordinates of *B* and of *D*. [4]

(c) Find the area of the rhombus *ABCD*. [2]

8 **(a)** Show that $\tan 15° = 2-\sqrt{3}$. [4]

(b) Hence, express $\operatorname{cosec}^2 15°$ in the form $a+b\sqrt{3}$, where a and b are integers. [4]

9 A particle moves along a straight line, such that the acceleration is given by

$$a = t + \sin t$$

where t is the time taken after leaving the point O, from rest.

(a) Find the displacement of the particle, in terms of t. [6]

(b) Explain why the particle can never be instantaneously at rest, where $t > 0$. [2]

10 **(a)** Find the coefficient of $\frac{1}{x^8}$ in the expansion of $\left(\frac{1}{2x^2}-x\right)^{13}$. [4]

(b) **(i)** Obtain, in ascending powers of x, the first four terms in the expansion of $\left(2-\frac{x}{2}\right)^8$. [3]

(ii) Hence, estimate the value of 1.975^8 correct to 3 decimal places. [2]

11 The points $A(5,10)$ and $B(5,-6)$ lie on the circumference of circle C_1.
The line $y=12$ is a tangent to C_1.

(a) Find the radius of the circle. [2]

(b) Given also that the points A and B lie to the left of the centre, show that the coordinates of the centre of C_1 is $(11, 2)$ and write down the equation of C_1. [4]

Another circle C_2 has equation $x^2 - 16x + y^2 + 4y + 59 = 0$.

(c) Determine if the circles intersect each other or not. In the case where they do not intersect, determine whether C_2 is inside or outside C_1. [5]

12 There are different ways to find $\int x(2+3x)^3 \mathrm{d}x$. This question investigates 2 different methods.

(a) **(i)** Write down all the terms in the expansion of $(2+3x)^3$. [2]

(ii) Hence, express $\int x(2+3x)^3 \mathrm{d}x$ in the form of f(x) where f(x) is a polynomial. [3]

(b) **(i)** Differentiate $x(2+3x)^4$ with respect to x. [3]

(ii) Hence, express $\int x\left(2+3x\right)^{3}\mathrm{d}x$ in the form of g(x), where g(x) is a function of x. [3]

(c) Determine the value of $\left[\mathrm{f}(x)-\mathrm{g}(x)\right]_{0}^{2024}$. [1]

End of Paper

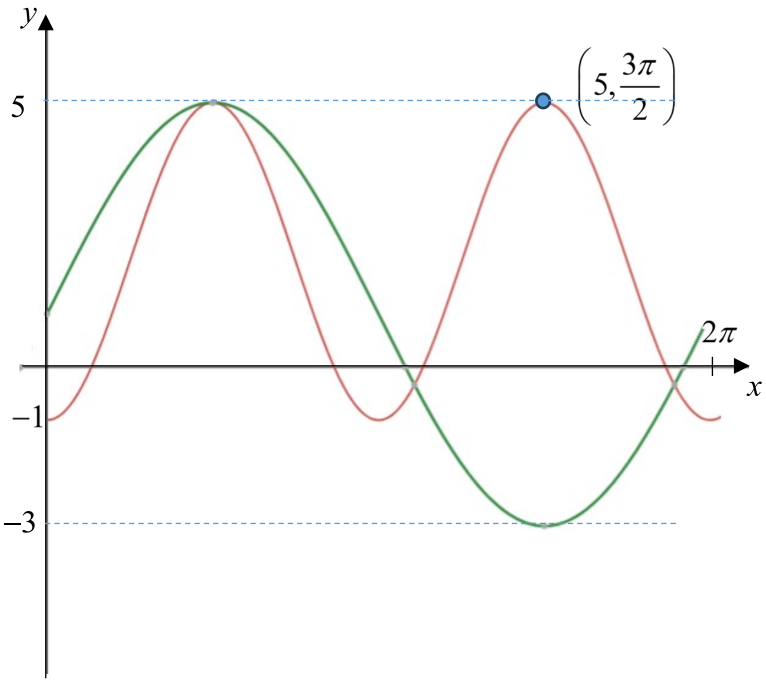
Marking Scheme

Q	Descriptor	Mark
1	$ax^2+10x+c=a\left(x^2+\frac{10}{a}x\right)+c$ $=a\left[\left(x+\frac{5}{a}\right)^2-\frac{25}{a^2}\right]+c$ $=a\left(x+\frac{5}{a}\right)^2-\frac{25}{a}+c$ $-\frac{25}{a}+c=7$ $c=\frac{25}{a}+7$ Any $a<0$: $a=-5$, $c=2$.	
OR	$y=ax^2+10x+c$ $\frac{dy}{dx}=2ax+10$ for max point, $\frac{dy}{dx}=0$ and $x<0$. $2ax+10=0$ $x=-\frac{5}{a}$ $y_{max}=7 @ x=-\frac{5}{a}$ $a\left(-\frac{5}{a}\right)^2+10\left(-\frac{5}{a}\right)+c=7$ $-\frac{25}{a}+c=7$ possible values $x=-5$, $\therefore$ $c=2$.	
OR	$ax^2+10x+c=7$ $ax^2+10x+(c-7)=0$ $b^2-4ac=0$ $10^2-4a(c-7)=0$ $c=\frac{25}{a}+7$ Any $a<0$. Let $a=-5$, $c=2$.	
A1		**4 marks**

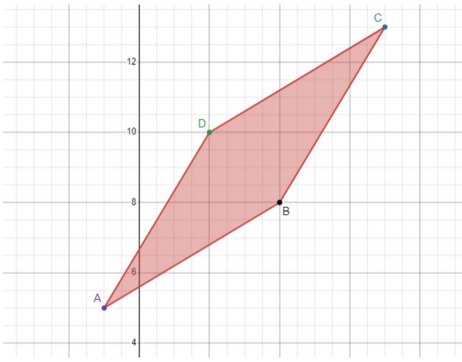
Q	Descriptor	Mark
2a	$y = ax^2 + 6x + 4a - 4 \quad a = 3 \quad y > 17.$ $3x^2 + 6x + 8 > 17$ $3x^2 + 6x - 9 > 0$ $3(x+3)(x-1) > 0$ $x < -3$ or $x > 1.$	
2b	$y = ax^2 + 6x + 4a - 4$ $a = \frac{1}{2} \quad y = 8x - 4.$ $\frac{1}{2}x^2 + 6x - 2 = 8x - 4$ $\frac{1}{2}x^2 - 2x + 2 = 0$ $b^2 - 4ac = (-2)^2 - 4\left(\frac{1}{2}\right)(2)$ $= 0$ Since $b^2 - 4ac = 0,$ $\therefore\ y = 8x - 4$ a tangent to curve.	
2c	The line L is parallel to y-axis. L is a vertical line. *(Or similar.)*	
A2		**6 marks**
3a	$2^{2x} - 2^{x+3} + 4 = 0$ Let $p = 2^x, \quad p^2 - 8p + 4 = 0$ $2^x = 0.5358...$ or $2^x = 7.464...$ $x = \frac{\ln 0.5358...}{\ln 2}...$ or $x = \frac{\ln 7.4641...}{\ln 2}$ $x = -0.9002...$ or $x = 2.8999...$ $x = -0.900$ or $x = 2.90$	
3b(i)	$P = 24\,500e^{0.02t} \qquad 01.01.2014 \to \$24\,500$ $= 24\,500e^{0.02(10)}$ $= \$29924.37$	
3b(ii)	$2(24\,500) = 24\,500e^{0.02t}$ $2 = e^{0.02t}$ $0.02t = \ln 2$ $t = 34.657...$ years In the year 2048.	
A6		**6 marks**

Q	Descriptor	Mark

4a	$a=-2$	
4b	$y=\lg(x+2)$ Let $x=5, \quad y=0.8450...$ $y=\ln(x+2)$ Let $x=5, \quad y=1..9459...$ $\therefore$ No C_1 is not $y=\lg(x+2)$.	
4c	$2\lg(x+2)=\ln(x+2)$ $\dfrac{\ln(x+2)^2}{\ln 10}=\ln(x+2)$ $\ln(x+2)^2-\ln 10\cdot\ln(x+2)=0$ $\ln(x+2)\left(2-\ln 10\right)=0$ $\ln(x+2)=0$ $x+2=e^0$ $x=-1.$	
A6		**6 marks**

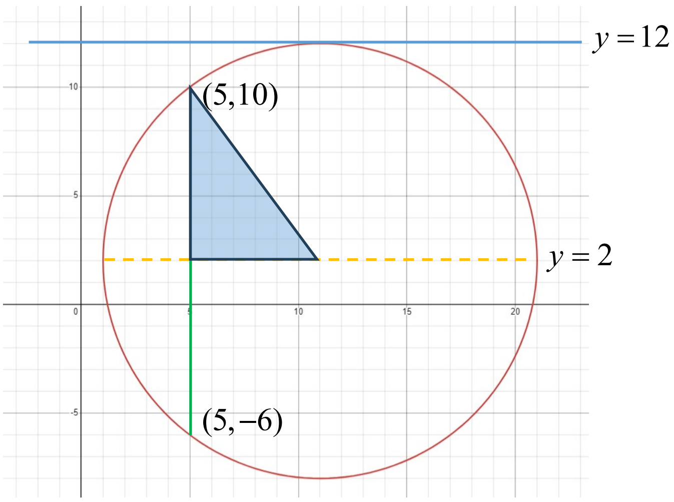
Q	Descriptor	Mark
5a	Period = π $\dfrac{2\pi}{b}=\pi$ $b=2$ Max = 5, min = -1 $c=\dfrac{5+(-1)}{2}=2$ $a=-3$ $a=-3, b=2, c=2$	
5b		
5c	$4\sin x - a\cos bx - c + 1 = 0$ $4\sin x + 1 = a\cos bx + c$ Number of solutions = number of intersections = 3	
G1		**6 marks**

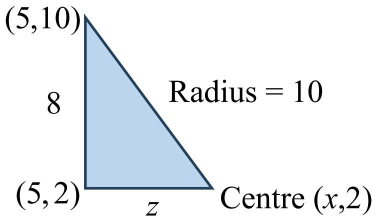
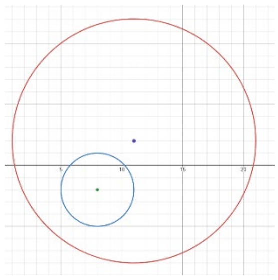
Q	Descriptor	Mark
6a	$\dfrac{3x^2-5x}{4x^3}=\dfrac{3}{4x}-\dfrac{5}{4x^2}$ $\dfrac{\mathrm{d}\left(\dfrac{3x^2-5x}{4x^3}\right)}{\mathrm{d}x}=-\dfrac{3}{4x^2}+\dfrac{5}{2x^3}$	
6b	$\int\left(\dfrac{1}{3x}-\sqrt{x}\right)^2\mathrm{d}x$ $=\int\dfrac{1}{9x^2}-\dfrac{2}{3\sqrt{x}}+x\ \mathrm{d}x$ $=-\dfrac{1}{9x}-\dfrac{4\sqrt{x}}{3}+\dfrac{x^2}{2}+c$	
C1/ C2		**6 marks**

Q	Descriptor	Mark
7a	$m_{AC} = \dfrac{13-5}{7-(-1)} = 1$ $m_{BD} = -1$ (diagonals of rhombus are $\perp$)	
7b	$m_{BD} = -1$ $\dfrac{10-8}{q-p} = -1$ $10-8 = p-q$ $p-q=2$ ---(1) midpoint of $AC = \left(\dfrac{-1+7}{2}, \dfrac{5+13}{2}\right)$ $= (3,9)$ midpoint of $BD = \left(\dfrac{p+q}{2}, \dfrac{8+10}{2}\right)$ $(3,9) = \left(\dfrac{p+q}{2}, 9\right)$ $\dfrac{p+q}{2} = 3$ $p+q=6$ ---(2) $(1)+(2): 2p = 8$ $p=4$ $q=2$ $\therefore B(4,8), D(2,10)$ 	
7c	Area of Rhombus $= \dfrac{1}{2}\begin{vmatrix} -1 & 4 & 7 & 2 & -1 \\ 5 & 8 & 13 & 10 & 5 \end{vmatrix}$ $= \dfrac{1}{2}(-8+52+70+10+10-26-56-20)$ $= \dfrac{1}{2}(32)$ $= 16 \text{ units}^2$	
OR	$AC = \sqrt{128}$ $MD = \sqrt{2}$ $A = 2 \times \dfrac{1}{2} \times \sqrt{128} \times \sqrt{2}$ $= 16 \text{ units}^2$	
G2		**8 marks**

Q	Descriptor	Descriptor	Mark
8a	$\tan 15° = \tan(60° - 45°)$ $= \dfrac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°}$ $= \dfrac{\sqrt{3}-1}{1+\sqrt{3}} \times \dfrac{1-\sqrt{3}}{1-\sqrt{3}}$ $= \dfrac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$ $= \dfrac{2\sqrt{3}-4}{-2}$ $= 2-\sqrt{3}$ (shown)	$\tan 15° = \tan(45° - 30°)$ $= \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$ $= \dfrac{1-\dfrac{\sqrt{3}}{3}}{1+\dfrac{\sqrt{3}}{3}}$ $= \dfrac{3-\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ $= \dfrac{9-6\sqrt{3}+3}{9-3}$ $= \dfrac{12-6\sqrt{3}}{6}$ $= 2-\sqrt{3}$ (shown)	
8b	$\operatorname{cosec}^2 15° = 1 + \cot^2 15°$ $= 1 + \left(\dfrac{1}{2-\sqrt{3}}\right)^2$ $= 1 + \dfrac{1}{4-4\sqrt{3}+3}$ $= 1 + \dfrac{1}{7-4\sqrt{3}}$ $= \dfrac{7-4\sqrt{3}+1}{7-4\sqrt{3}}$ $= \dfrac{8-4\sqrt{3}}{7-4\sqrt{3}} \times \dfrac{7+4\sqrt{3}}{7+4\sqrt{3}}$ $= \dfrac{56+32\sqrt{3}-28\sqrt{3}-16(3)}{49-16(3)}$ $= 8+4\sqrt{3}$		
G1			**8 marks**

Q	Descriptor	Mark
9a	$a = t + \sin t$ $v = \frac{t^2}{2} - \cos t + c$ $t = 0,\ v = 0,\ c = 1$ $v = \frac{t^2}{2} - \cos t + 1$ $s = \frac{t^3}{6} - \sin t + t + d$ $t = 0,\ \mathrm{s} = 0,\ d = 0$ $s = \frac{t^3}{6} - \sin t + t$	
9b	$v = \frac{t^2}{2} - \cos t + 1$ $1 + \frac{t^2}{2} > 1$ $\frac{t^2}{2} - \cos t + 1 > 0$ $v > 0$	
C3		**8 marks**

Q	Descriptor	Mark
10a	$\left(\frac{1}{2x^2}-x\right)^{13} = \ldots C^{13}_r\left(\frac{1}{2x^2}\right)^{13-r}(-x)^r\ldots$ $\rightarrow \ldots\left(x^{-2}\right)^{13-r}(x)^r\ldots = x^{-8}$ $-26+2r+r=-8$ $r=6$ $T_7 = C^{13}_6\left(\frac{1}{2x^2}\right)^{13-6}(-x)^6$ $=\frac{1716x^6}{128x^{14}}$ $=\frac{429}{32}\cdot\frac{1}{x^8}$ coeff of $\frac{1}{x^8}=\frac{429}{32}$.	
10bi	$\left(2-\frac{x}{2}\right)^8 = 2^8+2^7C^8_1\left(-\frac{x}{2}\right)^1+2^6C^8_2\left(-\frac{x}{2}\right)^2+2^5C^8_3\left(-\frac{x}{2}\right)^3+\ldots$ $=256-512x+448x^2-224x^3+\ldots$	
10bii	$2-\frac{x}{2}=1.975$ $x=0.05$ $\left(2-\frac{0.05}{2}\right)^8=256-512(0.05)+448(0.05)^2-224(0.05)^3+\ldots$ $=231.492$.	
A5		**9 marks**
11a	 y - coordinate of centre $=\frac{10+(-6)}{2}=2$ Radius of circle $=12-2=10$ units	

Q	Descriptor	Mark
11b	 By Pythagoras theorem, $z^2 = 10^2 - 8^2$ $z = 6$ Coordinates of centre = $(5+6, 2) = (11, 2)$ (shown) Equation of C_1: $(x-11)^2 + (y-2)^2 = 100$	
OR	$C(x, 2)$ $\sqrt{(x-5)^2 + (10-2)^2} = 10$ $(x-5)^2 = 36$ $x = 11$ or $x = -1$ (NA) Eqn of $C_1 : (x-11)^2 + (y-2)^2 = 100$	
11c	$x^2 - 16x + y^2 + 4y + 59 = 0$ $(x-8)^2 - 64 + (y+2)^2 - 4 + 59 = 0$ $(x-8)^2 + (y+2)^2 = 9$ Radius of $C_2 = 3$ Centre of C_2: $(8, -2)$ Distance between the 2 centres $= \sqrt{(11-8)^2 + (2-(-2))^2}$ $= 5$ units  Since the sum of the distance between the 2 centres and the radius of C_2 is 5 + 3 = 8 units < radius of C_1, the 2 circles will not intersect and C_2 lies inside C_1.	
G2		**11 marks**

Q	Descriptor	Mark
12ai	$8+36x+54x^2+27x^3$	
12aii	$\mathrm{f}(x)=\int x\left(8+36x+54x^2+27x^3\right)\,\mathrm{d}x$ $=\int\left(8x+36x^2+54x^3+27x^4\right)\,\mathrm{d}x$ $=4x^2+12x^3+\dfrac{27x^4}{2}+\dfrac{27x^5}{5}+c$	
12bi	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=(2+3x)^4+x(4)(3)(2+3x)^3$ $=(2+3x)^4+12x(2+3x)^3$	
12bii	$\int 12x(2+3x)^3\mathrm{d}x$ $=x(2+3x)^4-\int(2+3x)^4\,\mathrm{d}x$ $=x(2+3x)^4-\dfrac{(2+3x)^5}{3(5)}+c$ $\mathrm{g}(x)=\int x(2+3x)^3\mathrm{d}x$ $=\dfrac{x(2+3x)^4}{12}-\dfrac{(2+3x)^5}{180}+d$	
12c	0	
		12 marks