RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Additional Practice Questions for Chapter S3: Normal Distribution (Solutions)

1 **8864/2007/01/Q6**

A manufacturer produces packets of margarine. The mass of margarine in a packet has a normal distribution with mean 502g and standard deviation 0.8g. Find the proportion of packets which contain less than 500g of margarine. **[2]**

The manufacturer increases the mean amount of margarine in a packet to μ g. The standard deviation remains unchanged. Only 1 packet in 1000, on average, now contains less than 500g. Find μ, correct to 1 decimal place. **[3]**

Solution
Let X be the mass of a packet of margarine in grams. Then $X \sim \mathrm{N}\left(502, 0.8^2\right)$ Required proportion= $\mathrm{P}\left(\mathrm{X} < 500\right) = 0.00621$ (3 s.f) Let Y be the mass of a packet of margarine with new mean mass μ in grams. Then $Y \sim \mathrm{N}(\mu, 0.8^2)$ Given that $\mathrm{P}(Y < 500) = 0.001$ $\mathrm{P}\left(Z < \dfrac{500-\mu}{0.8}\right) = 0.001$ From GC, $\mathrm{P}\left(Z < -3.09023\right) = 0.001$ $\dfrac{500-\mu}{0.8} = -3.09023$ $500-\mu = -2.4722$ $\mu = 502.5$ (1 d.p)

2 9205/1995/02/Q9 (modified)

The length of time which an ordinary light-bulb will last may be taken to have a normal distribution with mean 600 hours and standard deviation 100 hours. The length of time for which a new 'long life' bulb will last may be taken to have a normal distribution with mean 2000 hours and standard deviation 200 hours.

(i) One ordinary bulb is chosen at random. Find the probability that it will last for more than 450 hours. **[2]**

(ii) Two ordinary bulbs are chosen at random. Find the probability that the sum of the times for which they last will be less than 1100 hours. **[3]**

(iii) One ordinary bulb and one long-life bulb are chosen at random. Find the probability that the long-life bulb lasts for more than three times as long as the ordinary bulb. **[3]**

The total time in which 10 'long life' bulbs will last is denoted by W hours. Find the value of t (to the nearest hour) such that $\mathrm{P}(W > t) = 0.75$. **[3]**

Solution	
(i)	Let X be the length of time for which an ordinary light-bulb will last, in hours. Then $X \sim \mathrm{N}\left(600, 100^2\right)$ Let Y be the length of time for which a new 'long-life' light-bulb will last, in hours. Then $Y \sim \mathrm{N}\left(2000, 200^2\right)$ $\mathrm{P}(X > 450) = 0.933$ (3 s.f)
(ii)	$X_1 + X_2 \sim \mathrm{N}\left(2(600),\ 2(100)^2\right)$ i.e. $X_1 + X_2 \sim \mathrm{N}\left(1200,\ 20000\right)$ $\mathrm{P}(X_1 + X_2 < 1100) = 0.240$ (3 s.f)
(iii)	$Y - 3X \sim \mathrm{N}\left(2000 - 3(600),\ 200^2 + 3^2(100^2)\right)$ i.e. $Y - 3X \sim \mathrm{N}\left(200,\ 130000\right)$ $\mathrm{P}(Y - 3X > 0) = 0.710$ (3 s.f) Let $W = Y_1 + Y_2 + \ldots + Y_{10}$. Then $W \sim \mathrm{N}\left(10(2000), 10(200)^2\right)$ i.e. $W \sim \mathrm{N}\left(20000,\ 400\ 000\right)$ $\mathrm{P}(W > t) = 0.75$ From the G.C, $\mathrm{P}(W > 19573) = 0.75$ $\therefore\ t = 19573$ (to the nearest hour)

3 **Math B/1982/01/Q12 (Modified)**

[The notation $\mathrm{N}(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2.]
The school bus leaves the stop near Ben's home at X minutes past 8.00 a.m., where X is an $\mathrm{N}(20, 3^2)$ random variable. Ben reaches the bus stop at Y minutes after 8.00 a.m., where Y is an $\mathrm{N}(15, 2^2)$ random variable, X and Y being independent.
Find the probability (to three decimal places) that Ben misses the bus. **[2]**

The ride from this stop to the school lasts T minutes, where T is an $N(30, (\sqrt{7})^2)$ random variable, independent of X. The random variable W is the number of minutes before 9.00 a.m. at which the bus arrives at the school.

Express W in terms of X and T. **[1]**
Calculate the mean and variance of W and find, to three significant figures, the probability that the bus arrives at the school after 9.00 a.m. **[2]**

Solution
$X - Y \sim \mathrm{N}(5, 13)$ P(Ben misses the bus) $= \mathrm{P}(X - Y < 0)$ $= 0.083$ (3 d.p) $W = 60 - X - T$ $\mathrm{E}(W) = 60 - 20 - 30 = 10$ $\mathrm{Var}(W) = 3^2 + \sqrt{7}^2 = 16$ i.e $W \sim \mathrm{N}(10, 16)$ P(the bus arrives after 9) $= \mathrm{P}(W < 0)$ $= 0.00621$ (3 s.f.)

4 9740/2007/02/Q8

Chickens and turkeys are sold by weight. The masses, in kg, of chickens and turkeys are modeled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean mass	Standard deviation
Chickens	2.2	0.5
Turkeys	10.5	2.1

Chickens are sold at \$3 per kg and turkeys at \$5 per kg.

(i) Find the probability that a randomly chosen chicken has a selling price exceeding \$7. **[2]**

(ii) Find the probability of the event that both a randomly chosen chicken has a selling price exceeding \$7 and a randomly chosen turkey has a selling price exceeding \$55. **[3]**

(iii) Find the probability that the total selling price of a randomly chosen chicken and a randomly chosen turkey is more than \$62. **[4]**

(iv) Explain why the answer to part **(iii)** is greater than the answer to part **(ii)**. **[1]**

Solution	
(i)	Let X and Y be the mass of a chicken and turkey in kg respectively. Then $X \sim \mathrm{N}(2.2, 0.5^2)$ and $Y \sim \mathrm{N}(10.5, 2.1^2)$. Let C be the selling price, in \$, of a chicken. Then $C = 3X \sim \mathrm{N}\left(3\times 2.2,\ 3^2 \times 0.5^2\right)$, i.e. $C \sim \mathrm{N}(6.6,\ 2.25)$. Let T be the selling price, in \$, of a turkey. Then $T = 5Y \sim \mathrm{N}(5\times 10.5,\ 5^2 \times 2.1^2)$, i.e. $T \sim \mathrm{N}(52.5,\ 110.25)$. $\mathrm{P}(C > 7) = 0.395$ (3 s.f.) or $\mathrm{P}\left(X > \frac{7}{3}\right)$
(ii)	$\mathrm{P}(C > 7 \text{ and } T > 55) = \mathrm{P}(C > 7)\mathrm{P}(T > 55)$, since C and T are independent $= 0.160$ (3 s.f.) or $\mathrm{P}\left(X > \frac{7}{3}\right)\mathrm{P}\left(Y > \frac{55}{5}\right)$
(iii)	$\mathrm{E}(C+T) = \mathrm{E}(C) + \mathrm{E}(T) = 6.6 + 52.5 = 59.1$ $\mathrm{Var}(C+T) = \mathrm{Var}(C) + \mathrm{Var}(T) = 1.5^2 + 10.5^2 = 112.5$ $C + T \sim \mathrm{N}(59.1,\ 112.5)$ $\mathrm{P}(C+T > 62) = 0.392$ (3 s.f.)
(iv)	Part **(iii)** includes the cases in part **(ii)** as well as other cases. For example, $6 < C < 7$ and $T > 56$ so that $C + T > 62$ is included in part **(iii)** but not in part **(ii)**. Hence, the answer for **(iii)** would be greater than the answer in **(ii)**.

5 **HCI Prelim 9740/2014/02/Q7**

The speed of a randomly chosen passenger car travelling from point A to point B on an expressway has a normal distribution with mean 85 km/h and standard deviation 20 km/h. It is assumed that a passenger car travels at a constant speed from point A to point B.

(i) For passenger cars travelling from point A to point B, find the probability that the total speed of 3 randomly chosen cars differs from twice the speed of a randomly chosen car by at most 50 km/h. **[3]**

(ii) The speeds of 80 randomly chosen passenger cars which travelled from point A to point B are recorded. Find the probability that the total distance travelled in 5 minutes is at most 550 km. State an assumption you had made in your calculation. **[4]**

Solution	
(i)	Let S denote the speed of a randomly chosen passenger car in km/h. $S \sim \mathrm{N}(85, 20^2)$ Let $X = S_1 + S_2 + S_3 - 2S$. $\mathrm{E}(X) = 3\mathrm{E}(S) - 2\mathrm{E}(S) = \mathrm{E}(S) = 85$ $\mathrm{Var}(X) = 3\mathrm{Var}(S) + 2^2\mathrm{Var}(S) = 7\mathrm{Var}(S) = 2800$ $X \sim \mathrm{N}(85, 2800)$ $\mathrm{P}(\lvert X \rvert \le 50)$ $= \mathrm{P}(-50 \le X \le 50)$ $= 0.249$ (3 s.f.)
(ii)	Let $T = \frac{5}{60}S_1 + \frac{5}{60}S_2 + \ldots + \frac{5}{60}S_{80} = \frac{5}{60}(S_1 + S_2 + \ldots + S_{80})$ $\mathrm{E}(T) = \frac{5}{60} \times 80\mathrm{E}(S) = \frac{5}{60} \times 80 \times 85 = \frac{1700}{3}$ $\mathrm{Var}(T) = \left(\frac{5}{60}\right)^2 80\mathrm{Var}(S) = \left(\frac{5}{60}\right)^2 80 \times 20^2 = \frac{2000}{9}$ $T \sim \mathrm{N}\left(\frac{1700}{3}, \frac{2000}{9}\right)$ $\mathrm{P}(T \le 550) = 0.132$ (3 s.f.) Assume the speed of a passenger car is independent of the speed of another passenger car.

6 9758/2017/02/Q10

A small component for a machine is made from two metal spheres joined by a short metal bar. The masses in grams of the spheres have the distribution $\mathrm{N}(20, 0.5^2)$.

(i) Find the probability that the mass of a randomly selected sphere is more than 20.2 grams. **[1]**

In order to protect them from rusting, the spheres are given a coating which increases the mass of each sphere by 10%.

(ii) Find the probability that the mass of a coated sphere is between 21.5 and 22.45 grams. State the distribution you use and its parameters. **[3]**

(iii) The masses of the metal bars are normally distributed such that 60% of them have a mass greater than 12.2 grams and 25% of them have a mass less than 12 grams. Find the mean and standard deviation of the masses of metal bars. **[4]**

(iv) The probability that the total mass of a component, consisting of two randomly chosen coated spheres and one randomly chosen bar, is more than k grams is 0.75. Find k, stating the parameters of any distribution you use. **[4]**

Solution	
(i)	Let X be the mass of a sphere in grams. $X \sim \mathrm{N}(20, 0.5^2)$ $\mathrm{P}(X > 20.2) = 0.345$ (3 s.f.)
(ii)	Let Y be the mass of a coated sphere in grams. $Y = 1.1X$ $\mathrm{E}(1.1X) = 1.1(20) = 22$ $\mathrm{Var}(1.1X) = 1.1^2\,\mathrm{Var}(X) = 1.1^2(0.5^2) = 0.55^2$ $Y \sim \mathrm{N}(22, 0.55^2)$ $\mathrm{P}(21.5 < Y < 22.45) = 0.612$ (3 s.f.)
(iii)	Let M be the mass of a metal bar in grams. $M \sim \mathrm{N}(\mu, \sigma^2)$ $\mathrm{P}(M > 12.2) = 0.6$ $\mathrm{P}\left(Z > \dfrac{12.2-\mu}{\sigma}\right) = 0.6$ $\dfrac{12.2-\mu}{\sigma} = -0.25335$ $\mu - 0.25335\sigma = 12.2 \quad ...(1)$

Solution	
	$P(M<12)=0.25$ $P\left(Z<\frac{12-\mu}{\sigma}\right)=0.25$ $\frac{12-\mu}{\sigma}=-0.67449$ $\mu-0.67449\sigma=12 \quad ...(2)$ From GC, $\mu=12.320, \sigma=0.47490$ (5 s.f.) Mean of metal bar $=12.3\text{ g}$ (3 s.f.) Standard deviation of metal bar $=0.475\text{ g}$ (3 s.f.)
(iv)	$E(Y_1+Y_2+M)=22+22+12.320=56.320$ $\text{Var}(Y_1+Y_2+M)=0.55^2+0.55^2+0.47490^2=0.83053$ $\therefore Y_1+Y_2+M \sim N(56.320, 0.83053)$ $P(Y_1+Y_2+M>k)=0.75$ From GC, $k=55.7$ (3 s.f.)

7 9758/2021/02/Q10

In this question you should state the parameters of any normal distributions you use.

A company makes 3-legged wooden stools from 4 solid components - a seat in the form of a disc, and 3 legs each in the form of a long, thin cylinder. The seats and legs are bought in bulk from another company. Over a period of time it is found that the masses of the seats are normally distributed; 80% of the seats have mass less than 2.1 kg, and 15% of the seats have mass less than 1.95 kg.

(a) Find the mean mass of the seats and show that the standard deviation is 0.0799 kg, correct to 3 significant figures. [3]

The masses of the legs, in kg, follow the distribution $\mathrm{N}(1.2, 0.02^2)$.

(b) Find the expected number of legs with mass more than 1.21 kg in a randomly chosen batch of 500 legs. [2]

(c) Find the probability that the total mass of a randomly chosen seat and 3 randomly chosen legs is between 5.6kg and 5.7kg. [3]

In order to make the stools, circular holes are drilled in the seats and the legs are fitted into them. In this process, the mass of seats is modelled as being reduced by 9% and the masses of the legs are unchanged.

(d) Find the probability that the total mass of a randomly chosen drilled seat and 3 randomly chosen legs is less than 5.6 kg. [3]

The holes made in the seats have diameters, in mm, that follow the distribution $\mathrm{N}(31, 0.4^2)$ and the diameters of the legs, in mm, follow the distribution $\mathrm{N}(30.7, 0.3^2)$. If the diameter of a leg is greater than the diameter of a hole, then the leg has to be sanded down to make it fit. If the diameter of a hole is more than 0.8 mm greater than the diameter of a leg, then padding has to be added when the leg is glued to the seat.

(e) A stool is made of a randomly chosen drilled seat and 3 randomly chosen legs. The legs are paired up with the holes at random. Find the probability, that the 3 legs can be fitted without the need for any sanding or padding. [4]

Solution	
(a)	Let X be the mass of a randomly chosen seat in kg. Let μ the mean mass of the seats and σ be the standard deviation. Then $X \sim \mathrm{N}\left(\mu, \sigma^2\right)$ $P\left(X<2.1\right)=P\left(Z<\dfrac{2.1-\mu}{\sigma}\right)=0.8 \Rightarrow \dfrac{2.1-\mu}{\sigma}=0.8416212335$ $\Rightarrow \mu+0.8416212335\sigma=2.1$ $P\left(X<1.95\right)=P\left(Z<\dfrac{1.95-\mu}{\sigma}\right)=0.15 \Rightarrow \dfrac{1.95-\mu}{\sigma}=-1.03643338$ $\Rightarrow \mu-1.03643338\sigma=1.95$

Solution	
	Solving, $\mu = 2.032779812 = 2.03$ (3 s.f.) standard devidation $= \sigma = 0.0798698818 = 0.0799$ (3 s.f.)
(b)	Let Y be the mass of a randomly chosen leg in kg. Then $Y \sim \mathrm{N}\left(1.2,\ 0.02^2\right)$ Expected number of legs with mass more than 1.21 kg in a randomly chosen batch of 500 legs $= P\left(Y > 1.21\right) \times 500 = 154.2687661 \approx 154$
(c)	Let $T = X + Y_1 + Y_2 + Y_3$ $T \sim \mathrm{N}\left(2.0327798 + 1.2 + 1.2 + 1.2,\ 0.07986988^2 + 0.02^2 + 0.02^2 + 0.02^2\right)$ $\Rightarrow T \sim \mathrm{N}\left(5.6327798,\ 0.0075791977\right)$ Probability that the total mass of a randomly chosen seat and 3 randomly chosen legs is between 5.6kg and 5.7kg $= P\left(5.6 < T < 5.7\right) = 0.4267170787 = 0.427$ (3 s.f.)
(d)	Let $K = 0.9X + Y_1 + Y_2 + Y_3$ $K \sim \mathrm{N}\left(0.91 \times 2.0327798 + 1.2 + 1.2 + 1.2, 0.91^2 \times 0.07986988^2 + 0.02^2 + 0.02^2 + 0.02^2\right)$ $\Rightarrow K \sim \mathrm{N}\left(5.449829618,\ 0.0064826136\right)$ Probability that the total mass of a randomly chosen drilled seat and 3 randomly chosen legs is less than 5.6 kg $= P\left(K < 5.6\right) = 0.9689185039 = 0.969$ (3 s.f.)
(e)	Let A be the diameter of a randomly chosen hole in the seats in mm. Then $A \sim \mathrm{N}\left(31,\ 0.4^2\right)$ Let B be the diameter of a randomly chosen leg in mm. Then $B \sim \mathrm{N}\left(30.7\ 0.3^2\right)$ $A - B \sim \mathrm{N}\left(31 - 30.7,\ 0.4^2 + 0.3^2\right)$ $\Rightarrow A - B \sim \mathrm{N}\left(0.3,\ 0.5^2\right)$ Probability, that the 3 legs can be fitted without the need for any sanding or padding $= \left[P\left(0 < A - B < 0.8\right)\right]^3 = 0.1823726956 = 0.182$ (3 s.f.)

8 TJC Prelim 9758/2021/02/Q11(i)-(iii) (modified)

A group of ornithologists in a nature reserve is conducting a research on two endemic species of birds. They will tag and measure the beak lengths of the birds they have caught. After a large number of birds were tagged and measured, they found the beak lengths of the two species of birds have independent normal distributions. The table below gives the mean and standard deviation of the beak length of each species.

Species	Mean	Standard Deviation
A	40 mm	5 mm
B	60 mm	5 mm

Let A and B be the random variables, in mm, denoting the beak lengths of birds of species A and B respectively.

(i) The probability of a randomly selected bird of species A having a beak length at least l mm is at least 0.9. Find the range of values of l, correct to 1 decimal place. **[2]**

(ii) Find the value of k such that $\mathrm{P}(A \le k) = \mathrm{P}(B \ge k)$. **[2]**

(iii) One ornithologist randomly selected 3 birds of species A and 1 bird of species B. Find the probability that the difference between the total beak lengths of the birds of species A and twice the beak length of the bird of species B is at least 2 mm. **[4]**

Solution

(i)

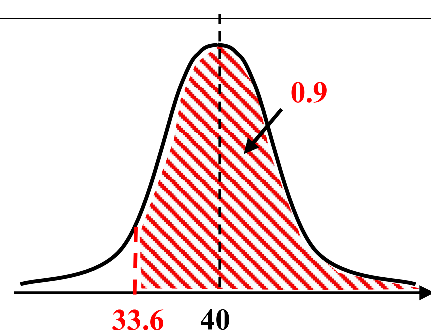
$A \sim \mathrm{N}(40, 5^2)$

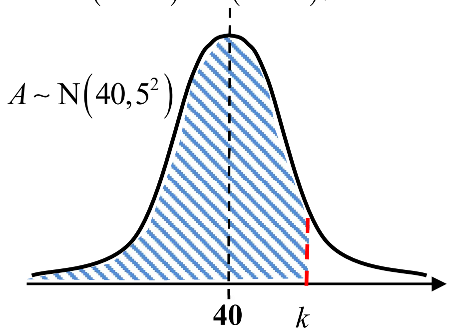
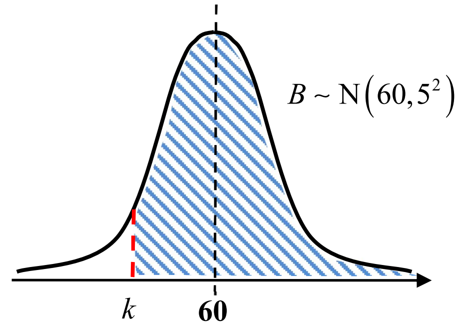
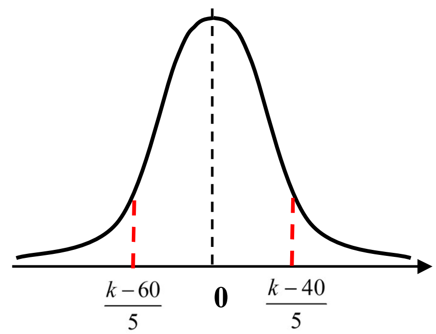
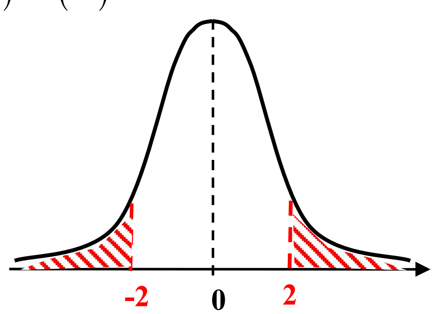
$\mathrm{P}(A \ge l) \ge 0.9$

From GC, $\mathrm{P}(A \ge 33.6) = 0.9$

From the diagram,

$0 \le l \le 33.6$ (3 s.f.)



(ii)	$A \sim \mathrm{N}\left(40, 5^2\right)$ and $B \sim \mathrm{N}\left(60, 5^2\right)$ Given $\mathrm{P}\left(A \le k\right) = \mathrm{P}\left(B \ge k\right)$, we observe that $40 < k < 60$. [from diagram]  Since standard deviation of A and B are the same, by symmetry, $k = 50$. OR: $\mathrm{P}\left(A \le k\right) = \mathrm{P}\left(B \ge k\right)$ $\mathrm{P}\left(Z \le \frac{k-40}{5}\right) = \mathrm{P}\left(Z \ge \frac{k-60}{5}\right)$ $-\left(\frac{k-60}{5}\right) = \frac{k-40}{5}$ $-k + 60 = k - 40$ $2k = 100$ $\therefore k = 50$ 
(iii)	Let $T = A_1 + A_2 + A_3 - 2B$ $\mathrm{E}\left(T\right) = 3\mathrm{E}\left(A\right) - 2\mathrm{E}\left(B\right) = 3 \times 40 - 2 \times 60 = 0$ $\mathrm{Var}\left(T\right) = 3\mathrm{Var}\left(A\right) + 2^2\mathrm{E}\left(B\right) = 3\left(5^2\right) + 4\left(5^2\right) = 175$ i.e. $T \sim \mathrm{N}\left(0, 175\right)$ Required probability $= \mathrm{P}\left(\left\|T\right\| \ge 2\right)$ $= \mathrm{P}\left(T \le -2\right) + \mathrm{P}\left(T \ge 2\right)$ $= 2\mathrm{P}\left(T \ge 2\right)$ $\because$ of symmetry $= 0.880$ (3 s.f.) 

9 RVH Prelim 9740/2014/02/Q8

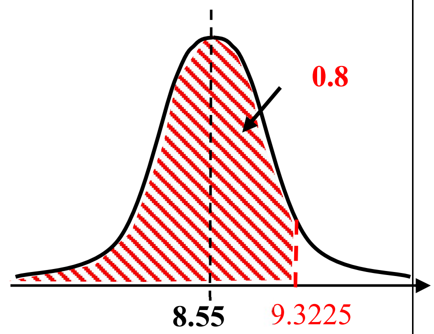
In a supermarket, the mass of a randomly chosen apple of variety A has a normal distribution with mean 120 g and standard deviation 25 g. The mass of a randomly chosen apple of variety B in the supermarket also follows a normal distribution. It is observed that 10% of the apple of variety B has mass less than 100 g while 90% of the apple of variety B has mass less than 150 g.

(i) Find the mean and standard deviation of the mass of a randomly chosen apple of variety B. **[3]**

(ii) Apples are sold by weight. Apples of variety A are sold at \$2.00 per 100 g while the apples of variety B are sold at \$3.00 per 100 g. The probability that the total cost of 2 randomly chosen apples of variety A and a randomly chosen apple of variety B is less than \$$q$, is more than 0.8. Find the least value of q. **[5]**

Another variety of apple whose mass M is of mean 100 g and standard deviation 75 g is also on sale in the supermarket. Determine with a reason, whether a normal distribution with the stated mean and standard deviation would be a valid model for M. **[1]**

Solution	
(i)	Let Y be the mass of a random chosen apple of variety B in grams. Then $Y \sim N(\mu, \sigma^2)$. Given that $\mathrm{P}(Y < 150) = 0.9$, $\mathrm{P}(Y \geq 150) = 0.1$ Since $\mathrm{P}(Y < 100) = \mathrm{P}(Y > 150) = 0.1$, applying symmetrical property of normal distribution, we have $\mu = \dfrac{100+150}{2} = 125\,\mathrm{g}$ $\mathrm{P}(Y < 100) = 0.1$ $\mathrm{P}\left(Z < \dfrac{100-\mu}{\sigma}\right) = 0.1$ From GC, $\mathrm{P}(Z < -1.2816) = 0.1$ $\dfrac{100-\mu}{\sigma} = -1.2816$ Thus, σ = 19.508 (5 s.f.)= 19.5 g (3 s.f.) $\therefore$ Mean and standard deviation of Y are 125g and 19.5g respectively.
(ii)	Let X be the mass of a random chosen apple of variety A in grams. $X \sim \mathrm{N}(120, 25^2)$ Let $V = 0.02(X_1 + X_2) + 0.03Y$ $\mathrm{E}(V) = (0.02)(2)(120) + (0.03)(125) = 8.55$ $\mathrm{Var}(V) = 0.02^2(2)(25^2) + 0.03^2(19.508^2)$=0.84251 (5s.f.)

Solution	
	$\therefore V \sim \mathrm{N}(8.55, 0.84251)$ $\mathrm{P}(V < q) > 0.8$ From GC, $\mathrm{P}\left(V < 9.3225\right) = 0.8$ From the diagram, $q > 9.3225$ The least value of q is 9.33 (nearest 2 dp).  Suppose $M \sim \mathrm{N}(100, 75^2)$. Then 99.7% of the values will lie within 100 ± 3(75) which contains a significant range of negative values. In fact, $\mathrm{P}(M < 0) = 0.0912$ (3 sf). Thus, it is not valid to assume a normal distribution for the mass of this variety of apple.

10 PJC Prelim 9740/2010/02/Q12

The weight, Y, of a Yummy cereal bar is normally distributed with mean $(120 - k)$ g and standard deviation 10 g. The weight, F, of a Fullness cereal bar is normally distributed with mean 180 g and standard deviation 20 g.

(i) Given that $\mathrm{P}(Y < 80) = \mathrm{P}(Y > 150)$, show that the value of k is 5. **[1]**

(ii) 5 Yummy cereal bars are randomly chosen. Find the probability that exactly one bar weighs lesser than the lower quartile weight and exactly one bar weighs more than the median weight. **[2]**

(iii) Find the probability that the weight of 2 randomly chosen Yummy cereal bars differs from one and a half times the weight of a randomly chosen Fullness cereal bar by at most 5g. State the assumption needed for your working. **[4]**

The Fullness cereal bars are sold at \$2 per 100g.

(iv) Find the probability that a randomly chosen Fullness cereal bar costs more than \$3.50. **[2]**

Solution	
(i)	Given that $\mathrm{P}(Y < 80) = \mathrm{P}(Y > 150)$ $\Rightarrow 120 - k = \dfrac{80+150}{2} = 115$ $\therefore\ k = 5$
(ii)	Let m and q_L be the median and lower quartile weight in grams respectively. Then $\mathrm{P}(Y > m) = \dfrac{1}{2}$ and $\mathrm{P}(Y < q_L) = \dfrac{1}{4} = \mathrm{P}(q_L \le Y \le m)$ Required probability $= \mathrm{P}(Y < q_L)\mathrm{P}(Y > m)[\mathrm{P}(q_L \le Y \le m)]^3 \dfrac{5!}{3!}$ $= \left(\dfrac{1}{4}\right)\left(\dfrac{1}{2}\right)\left(\dfrac{1}{4}\right)^3 \dfrac{5!}{3!} = \dfrac{5}{128}$ From GC, $\mathrm{P}\left(V < 9.3225\right) = 0.8$ From the diagram, $q > 9.3225$ The least value of q is 9.33 (nearest 2 dp).
(iii)	$Y \sim \mathrm{N}(115, 10^2)$ and $F \sim \mathrm{N}(180, 20^2)$ Let $W = Y_1 + Y_2 - \dfrac{3}{2}F$. Then $W \sim \mathrm{N}(-40,\ 1100)$ $\mathrm{P}\left(\lvert W\rvert \le 5\right) = \mathrm{P}\left(-5 \le W \le 5\right) = 0.0582$ (3 s.f.) Assumption: The weight of a cereal bar is independent of the weight of another cereal bar.

Solution	
(iv)	$0.02F \sim \mathrm{N}(3.6,\ 0.16)$ $\mathrm{P}(0.02F > 3.5) = 0.599$ (3 s.f)

****The End****