RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Additional Practice Questions for Chapter S2A: Discrete Random Variables (Solutions)

1 JJC Prelim 9233/2005/02/Q12

A bag contains five red and two black counters.

(a) Mary draws four counters from the bag, one by one at random and without replacement. The random variable X is used to denote the number of red counters drawn by Mary. Show that $\mathrm{P}(X=2)=\frac{2}{7}$ and find the expectation of X. **[4]**

(b) Peter pays \$1 each time to draw a counter at random from the bag, one by one and with replacement until a black counter is obtained. Find the amount of money he should receive when a black counter is drawn in order for the game to be fair. **[3]**

[You may assume $1+2x+3x^2+4x^3+\ldots=\frac{1}{(1-x)^2}$ for $|x|<1$.]

(a) Let X be the number of red counters drawn. X can take values 2, 3 or 4.

P(X = 2) = P(2 R & 2 B in any order)

$$=\frac{5}{7}\times\frac{4}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{4!}{2!2!} \quad =\frac{2}{7} \quad \text{[Shown]}$$

P(X = 3) = P(3 R & 1B in any order)

$$=\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{4!}{3!} \quad =\frac{4}{7}$$

P(X =4) = P(4 R)

$$=\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4} \quad =\frac{1}{7}$$

$$\mathrm{E}(X)=2\times\frac{2}{7}+3\times\frac{4}{7}+4\times\frac{1}{7}=\frac{20}{7}$$

(b) For fair game,

E(amt. received) = E(amt. paid)

$$=\$1\times\frac{2}{7}+\$2\times\frac{5}{7}\times\frac{2}{7}+\$3\times\left(\frac{5}{7}\right)^2\times\frac{2}{7}+\$4\times\left(\frac{5}{7}\right)^3\times\frac{2}{7}+\ldots.$$

$$=\$\frac{2}{7}[1+2\times\frac{5}{7}+3\times\left(\frac{5}{7}\right)^2+4\times\left(\frac{5}{7}\right)^3+\ldots.]$$

$$=\$\frac{2}{7}\left[\frac{1}{(1-\frac{5}{7})^2}\right]$$

$$=\$3.50$$

2 NYJC Prelim 9233/2005/02/Q12

A biased cubical die has faces numbered 1, 2, 3, 4, 5, 6 and X denotes the number obtained when the die is tossed once with probabilities $\mathrm{P}(X = x) = kx^2$. Find the value of k. **[2]**

Joyce tosses the die and notes the number x obtained. At each toss, she will be given \$$(3x)$ if $x = 1, 2, 3$, and \$$(x)$ if $x = 4, 5, 6$. Calculate the expected amount Joyce will receive after ten tosses. **[4]**

$$\sum_{x=1}^{6} kx^2 = 1 \quad \Rightarrow \quad 91k = 1 \quad \Rightarrow \quad k = \frac{1}{91}$$

Let T be the amount she gets after one toss.

t	3	6	9	4	5
x	1	2 or 6	3	4	5
$\mathrm{P}(T = t)$	$\frac{1}{91}$	$\frac{2^2}{91}+\frac{6^2}{91}=\frac{40}{91}$	$\frac{9}{91}$	$\frac{16}{91}$	$\frac{25}{91}$

$$\mathrm{E}(T) = 3\left(\frac{1}{91}\right) + 6\left(\frac{40}{91}\right) + 9\left(\frac{9}{91}\right) + 4\left(\frac{16}{91}\right) + 5\left(\frac{25}{91}\right) = \frac{513}{91}$$

Hence expected amount she gets after ten tosses

$$\mathrm{E}(T_1 + T_2 + \ldots + T_{10}) = 10\mathrm{E}(T) = \frac{5130}{91} = \$56.37 \text{ (nearest cent)}$$

3 CJC Prelim 9233/2006/04/Q23(modified)

John is given 4 keys of which only one can unlock the door. He tries the keys randomly (without replacement) to unlock the door. The random variable X denotes the number of tries John takes to unlock the door.

(i) Show that $\mathrm{P}(X = 3) = \frac{1}{4}$.

(ii) Tabulate the probability distribution of X.

(iii) Write down $\mathrm{E}(X)$ and find $\mathrm{Var}(X)$. Hence find $\mathrm{E}(X + 2)$ and $\mathrm{Var}(X + 2)$.

(i) $\mathrm{P}(X = 3) = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

(ii) Let X be the number of tries John takes to open the door.

x	1	2	3	4
$\mathrm{P}(X = x)$	$\frac{1}{4}$	$\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}=\frac{1}{4}$

(iii) By symmetry, $\mathrm{E}(X) = \frac{5}{2}$

$$\mathrm{E}(X^2) = \frac{1}{4} + 2^2\left(\frac{1}{4}\right) + 3^2\left(\frac{1}{4}\right) + 4^2\left(\frac{1}{4}\right) = 7.5$$

$$\mathrm{Var}(X) = \mathrm{E}\left(X^2\right) - \left[\mathrm{E}\left(X\right)\right]^2 = 7.5 - (2.5)^2 = \frac{5}{4}$$

$$\begin{aligned}\mathrm{E}(X+2) &= \mathrm{E}(X) + 2 \\ &= \frac{5}{2} + 2 \\ &= \frac{9}{2}\end{aligned}$$

$$\mathrm{Var}\left(X+2\right) = \mathrm{Var}\left(X\right) = \frac{5}{4}$$

4 TJC Prelim 9233/2005/02/Q12(modified)

Urn A contains 5 black balls and 3 red balls. Urn B contains 4 black balls and 4 red balls. An experiment is conducted in the following manner:

A fair die is tossed. If it shows 1 or 2, two balls are drawn from urn A without replacement. Otherwise, one ball is drawn from each urn. Let X be the number of red balls drawn in one experiment. Show that $\mathrm{P}(X=0)=\frac{55}{168}$. **[1]**

Find the probability distribution of X. Find the exact values of expectation and variance of X. **[6]**

Let X be the number of red balls drawn. Then X takes values 0, 1, 2.

$\mathrm{P}\left(X=0\right)$

= P(fair die shows 1 or 2 and 2 black balls are drawn from urn A)

+P(fair die shows 3, 4, 5 or 6 and 1 black ball is drawn from each of urns A and B)

$=\frac{1}{3}\times\frac{5}{8}\times\frac{4}{7}+\frac{2}{3}\times\frac{5}{8}\times\frac{4}{8}=\frac{55}{168}$ (shown)

$\mathrm{P}\left(X=2\right)=\frac{1}{3}\times\frac{3}{8}\times\frac{2}{7}+\frac{2}{3}\times\frac{4}{8}\times\frac{3}{8}=\frac{9}{56}$ & $\mathrm{P}\left(X=1\right)=1-\mathrm{P}\left(X=0\right)-\mathrm{P}\left(X=2\right)=\frac{43}{84}$

x	0	1	2
$\mathrm{P}\left(X=x\right)$	$\frac{55}{168}$	$\frac{43}{84}$	$\frac{9}{56}$

Hence, $\mathrm{E}\left(X\right)=0\times\frac{55}{168}+1\times\frac{43}{84}+2\times\frac{9}{56}=\frac{5}{6}$

$\mathrm{E}\left(X^2\right)=0^2\times\frac{55}{168}+1^2\times\frac{43}{84}+2^2\times\frac{9}{56}=\frac{97}{84}$

$\mathrm{Var}\left(X\right)=\mathrm{E}\left(X^2\right)-\left(\mathrm{E}\left(X\right)\right)^2=\frac{97}{84}-\left(\frac{5}{6}\right)^2=\frac{29}{63}$

5 RVHS Prelim 9758/2018/02/Q5

Two fair six-sided dice are thrown. The random variable X is the smaller of the two scores if they are different, and their common value if they are the same.

(i) Show that $\mathrm{P}(X=2)=\frac{1}{4}$ and find the probability distribution of X. **[2]**

(ii) Hence find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. **[2]**

A game is played with Ivan and Jon taking turns to throw the two dice each. Ivan throws the dice first and the player who first obtain the value of X equals to 2 wins the game. If Ivan wins the game, Jon pays him $7. Otherwise, Ivan pays Jon $10.

(iii) Find the player who has the higher expected gain. Justify your answer. **[3]**

(i) Using the following possibility table, we have:

2nd die (columns); **1st die** (rows)

	1	**2**	**3**	**4**	**5**	**6**
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

From the table, $\mathrm{P}\left(X=2\right)=\frac{9}{36}=\frac{1}{4}$

Probability distribution of X:

x	1	2	3	4	5	6
$\mathrm{P}\left(X=x\right)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

(ii)

$$\mathrm{E}(X)=\sum_{x=1}^{6}x\mathrm{P}\left(X=x\right)=\frac{91}{36}\quad\text{(or 2.53)}$$

$$\mathrm{Var}(X)=\mathrm{E}(X^2)-\left(\mathrm{E}(X)\right)^2$$

$$=\left(\sum_{x=1}^{6} x^2\mathrm{P}\left(X=x\right)\right)-\left(\sum_{x=1}^{6} x\mathrm{P}\left(X=x\right)\right)^2$$

$$=\frac{301}{36}-\left(\frac{91}{36}\right)^2$$

$$=\frac{2555}{1296}\quad \text{(or 1.97)}$$

(iii)

$$\text{P(Ivan wins)} =\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)+\ldots$$

$$=\frac{\frac{1}{4}}{1-\left(\frac{3}{4}\right)^2}=\frac{4}{7}$$

E(gain) for Ivan $=\frac{4}{7}\times 7+\frac{3}{7}\times(-10)=-\$0.29$

Since Ivan is expecting to lose 29 cents (i.e. Jon is expecting to gain 29 cents), Jon has a higher expected gain.

Alternatively,

$$\text{P(Jon wins)} =\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)+\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)+\ldots$$

$$=\frac{\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)}{1-\left(\frac{3}{4}\right)^2}=\frac{3}{7}$$

E(gain) for Jon $=\frac{3}{7}\times 10+\frac{4}{7}\times(-7)=\$0.29$

Since Jon is expecting to win 29 cents (i.e. Ivan is expected to lose 29 cents), he has a higher expected gain.

6 HCI Prelim 9758/2019/02/Q10 (Part)

In a game with a 4-sided fair die numbered 1 to 4 on each face, the score for a throw is the number on the bottom face of the die. A player gets to choose either option A or option B.

Option A: The player rolls the die once. The score x is the amount of money \$$x$ that the player wins.

Option B: The player rolls the die twice. The first score is x and the second score is y. If $y > x$, the player wins \$$2xy$, but if $y < x$, the player loses \$$(x - y)$. Otherwise, he neither wins nor loses any money.

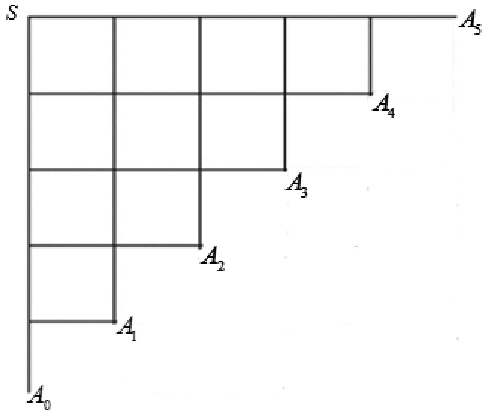
(i) Find the expected amounts won by a player in one game when playing option A and when playing option B. Show that option B is a better option. [5]

(ii) Suggest why a risk averse player would still choose option A. [1]

(iii) Show that the variance of the amount won by a player in one game when playing option A is $\$^2 1.25$. [2]

<table>
<tr>
<td>(i)</td>
<td>
Let $$W$ denote the amount won by a player in one game.
Under option A,
$$\mathrm{E}(W_A) = 1\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{4}\right) + 4\left(\frac{1}{4}\right)$$
$$= (1+2+3+4)\left(\frac{1}{4}\right)$$
$$= \frac{5}{2} = \$2.50$$
Under option B, the winning amount for each combination is listed below:
<table>
<tr><th>y
x</th><th>1</th><th>2</th><th>3</th><th>4</th></tr>
<tr><td>1</td><td>0</td><td>4</td><td>6</td><td>8</td></tr>
<tr><td>2</td><td>-1</td><td>0</td><td>12</td><td>16</td></tr>
<tr><td>3</td><td>-2</td><td>-1</td><td>0</td><td>24</td></tr>
<tr><td>4</td><td>-3</td><td>-2</td><td>-1</td><td>0</td></tr>
</table>
$$\mathrm{E}(W_B) = \left(\frac{1}{16}\right)[4+6+12+8+16+24$$
$$-1-2-3-1-2-1]$$
$$= \left(\frac{1}{16}\right)[70-10]$$
$$= \frac{15}{4} = \$3.75$$
Since $\mathrm{E}(W_B) > \mathrm{E}(W_A)$, option B is better. (shown)
</td>
</tr>
<tr>
<td>(ii)</td>
<td>Option <i>A</i> is a “sure win” option where the player would definitely gain a positive amount in all cases, whereas option <i>B</i> has a risk of losing money in some cases.</td>
</tr>
<tr>
<td>(iii)</td>
<td>
$$\mathrm{E}\left(W_A{}^2\right) = \left(1^2 + 2^2 + 3^2 + 4^2\right)\left(\frac{1}{4}\right) = \frac{15}{2}$$
$$\mathrm{Var}(W_A) = \mathrm{E}\left(W_A{}^2\right) - \left[\mathrm{E}(W_A)\right]^2 = \frac{15}{2} - \left(\frac{5}{2}\right)^2 = \frac{5}{4} = 1.25(\text{shown})$$
</td>
</tr>
</table>

7 MI Prelim 9758/2019/02/Q10



A particle moves one step each time either to the right or downwards through a network of connected paths as shown above. The particle starts at *S*, and, at each junction, randomly moves one step to the right with probability *p*, or one step downwards with probability *q*, where $q = 1 - p$. The steps taken at each junction are independent. The particle finishes its journey at one of the 6 points labelled A_i, where $i = 0,\ 1,\ 2,\ 3,\ 4,\ 5$ (see diagram). Let $\{\ X = i\ \}$ be the event that the particle arrives at point A_i.

(i) Show that $\mathrm{P}\left(X = 2\right) = 10p^2q^3$. [2]

(ii) After experimenting, it is found that the particle will end up at point A_2 most of the time. By considering the mode of *X* or otherwise, show that $\frac{1}{3} < p < \frac{1}{2}$. [4]

The above setup is a part of a two-stage computer game.

- If the particle lands on A_0, the game ends immediately and the player will not win any points.
- If the particle lands on A_i, where $i = 2$ or 4, then the player gains 2 points.
- If the particle lands on A_i, where $i = 1$, 3 or 5, then the player proceeds on to the next stage, where there is a probability of 0.4 of winning the stage. If he wins the stage, he gains 5 points; otherwise he gains 3 points.

Let *Y* be the number of points gained by the player when one game is played.

(iii) If $p = 0.4$, determine the probability distribution of *Y*. [4]

(iv) Hence find the expectation and variance of *Y*. [2]

(i)

$$\begin{aligned} \mathrm{P}(X=2) &= \mathrm{P}(\text{point } A_2) \\ &= \mathrm{P}(2 \text{ steps to the right and 3 steps downwards in any order}) \\ &= {}^5C_2 p^2 q^3 \\ &= 10p^2q^3 \text{ (Shown)} \end{aligned}$$

(ii)

Note that $\mathrm{P}(X=3) = \mathrm{P}(\text{point } A_3)$

$$= {}^5C_3 p^3 q^2 = 10p^3q^2$$

Since the particle will end up at point A_2 most of the time, then mode occurs at $X = 2$.

$\therefore \mathrm{P}(X=3) < \mathrm{P}(X=2)$

$10p^3q^2 < 10p^2q^3$

$q > p \Rightarrow 1-p > p$

$2p < 1 \Rightarrow p < 0.5$

In the same way, $\mathrm{P}(X=2) > \mathrm{P}(X=1)$

$10p^2q^3 > 5pq^4$

$2p > q$

$2p > 1-p$

$3p > 1 \Rightarrow p > \dfrac{1}{3}$

Combining both inequalities, $\dfrac{1}{3} < p < \dfrac{1}{2}$ (Shown)

(iii)

Y	0	2	3	5
$\mathrm{P}(Y=y)$	0.07776	0.4224	0.299904	0.199936

$\mathrm{P}(Y=0) = \mathrm{P}(X=0) = {}^5C_0 (0.4)^0 (0.6)^5 = 0.07776$

$$\begin{aligned} \mathrm{P}(Y=2) &= \mathrm{P}(X=i, \text{ where } i \text{ is even}) \\ &= \mathrm{P}(X=2) + \mathrm{P}(X=4) \\ &= 10(0.4)^2(0.6)^3 + {}^5C_4 (0.4)^4 (0.6)^1 = 0.4224 \end{aligned}$$

$\mathrm{P}(Y=3) = \mathrm{P}(X=i, \text{ where } i \text{ is odd and lose the game})$

$= 0.6(\mathrm{P}(X=1) + \mathrm{P}(X=3) + \mathrm{P}(X=5))$

$= 0.6\left[{}^5C_1 (0.4)^1 (0.6)^4 + {}^5C_3 (0.4)^3 (0.6)^2 + {}^5C_5 (0.4)^5 (0.6)^0\right]$

$= 0.299904$

$\mathrm{P}(Y=5) = \mathrm{P}(X=i, \text{ where } i \text{ is odd and win the game})$

$= 0.4(\mathrm{P}(X=1) + \mathrm{P}(X=3) + \mathrm{P}(X=5))$

$= 0.4\left[{}^5C_1 (0.4)^1 (0.6)^4 + {}^5C_3 (0.4)^3 (0.6)^2 + {}^5C_5 (0.4)^5 (0.6)^0\right]$

$= 0.199936$

(iv)

Using G.C.

$\mathrm{E}(Y) = 2.744192 = 2.74$ (3 sf) and $\mathrm{Var}(Y) = 1.3626^2 = 1.8567 = 1.86$ (3 sf)

8 VJC Prelim 9758/2018/02/Q8

The probability of obtaining a ‘6’ on a biased cubical dice is thrice the probability of rolling every other number on the dice.

Show that probability of obtaining a ‘6’ on the biased dice is $\frac{3}{8}$. [1]

This biased dice is put into a bag together with 3 fair cubical dice.

(i) One of the dice is chosen randomly from the bag and rolled once. Find the probability of obtaining a ‘6’. [2]

(ii) One of the dice is chosen randomly and is rolled n times.

(a) Find the probability that ‘6’ is obtained on all the n rolls. [1]

(b) Given that ‘6’ is obtained for all the n rolls, the probability that the biased dice is chosen is more than 0.95. By forming an inequality in terms of n, solve for the least value of n. [3]

(iii) One of the dice is chosen randomly from the bag and rolled once. The dice is then put back into the bag. Another dice is chosen randomly from the bag and rolled once.

For every ‘6’ obtained, the player gains \$2, otherwise the player loses \$$k$. If the game is fair, that is, expected winnings of the player is \$0, show that the value of k is 0.56. [3]

Find the variance of the player’s winnings. [2]

	Let the probability of obtaining a ‘6’ be k. $5\left(\frac{1}{3}k\right)+k=1$ $\frac{8}{3}k=1\Rightarrow k=\frac{3}{8}$
(i)	P(obtaining a ‘6’) = $\frac{1}{4}\times\frac{3}{8}+\frac{3}{4}\times\frac{1}{6}$ $=0.21875\left(\text{or }\frac{7}{32}\right)$
(ii)(a)	P(obtaining n ‘6’s) = $\frac{1}{4}\times\left(\frac{3}{8}\right)^n+\frac{3}{4}\times\left(\frac{1}{6}\right)^n$
(ii)(b)	$\frac{\frac{1}{4}\times\left(\frac{3}{8}\right)^n}{\frac{1}{4}\times\left(\frac{3}{8}\right)^n+\frac{3}{4}\times\left(\frac{1}{6}\right)^n}>0.95$ <table><tr><td>n</td><td>Probability</td></tr><tr><td>4</td><td>0.89521 < 0.95</td></tr><tr><td>5</td><td>0.95055 > 0.95</td></tr></table> Least n is 5.

(iii)

Let \$$X$ be the winnings of the player.

x	$-2k$	$2-k$	4
$\mathrm{P}(X=x)$	$\left(\frac{25}{32}\right)^2$ $=\frac{625}{1024}$	$2\left(\frac{7}{32}\right)\left(\frac{25}{32}\right)$ $=\frac{175}{512}$	$\left(\frac{7}{32}\right)^2$ $=\frac{49}{1024}$

$$\mathrm{E}\left(X\right)=-2k\left(\frac{625}{1024}\right)+\left(2-k\right)\left(\frac{175}{512}\right)+4\left(\frac{49}{1024}\right)$$

$$0=-\frac{625k}{512}+\frac{175}{256}-\frac{175k}{512}+\frac{49}{256}$$

$$\frac{25}{16}k=\frac{7}{8}$$

$$k=0.56$$

x	-1.12	1.44	4
$\mathrm{P}(X=x)$	$\frac{625}{1024}$	$\frac{175}{512}$	$\frac{49}{1024}$

$$\mathrm{Var}\left(X\right)=2.24$$

The variance of the player’s winnings is $\$^2 2.24$.

<u>Alternatively</u>

Let \$$A$ be the winnings from randomly choosing one dice from the bag and rolled once.

Let \$$B$ be the winnings of the player.

$$B=A_1+A_2$$

$$\mathrm{E}\left(B\right)=\mathrm{E}\left(A_1+A_2\right)$$
$$=2\mathrm{E}\left(A\right)$$

Given that the expected winnings is \$0,

$$\mathrm{E}\left(B\right)=0\Rightarrow 2\mathrm{E}\left(A\right)=0\Rightarrow \mathrm{E}\left(A\right)=0$$

a	-k	2
$\mathrm{P}(A=a)$	$\frac{25}{32}$	$\frac{7}{32}$

$$\frac{25}{32}\left(-k\right)+\frac{7}{32}\left(2\right)=0$$

$$\Rightarrow k=0.56 \text{ (shown)}$$

$$\mathrm{Var}\left(B\right)=\mathrm{Var}\left(A_1+A_2\right)$$
$$=2\mathrm{Var}\left(A\right)$$

a	-0.56	2
$\mathrm{P}(A=a)$	$\frac{25}{32}$	$\frac{7}{32}$

By GC, Var(A) = 1.12

Var(B) = 2.24

The variance of the player’s winnings is $\$^2 2.24$.

9 9758/2020/02/Q5

Shania and Tina are playing a game. Shania has a bag containing one green disc, r red discs and $2r$ blue discs, where $r > 1$. A red disc is worth 5 points, a blue disc is worth 2 points and the green disc is worth 0 points. Tina takes two discs from the bag at random. Tina's score is found by multiplying together the number of points for each of the two discs she takes.

(i) State Tina's possible scores. [1]

(ii) Show that the expectation of Tina's score is $\dfrac{27r-11}{3r+1}$, and find an expression for the variance. [7]

(iii) Given that the variance of Tina's score is 38, find the value of r. [2]

No.	**Suggested Solution**
(i)	0,4,10,25
(ii)	$\mathrm{P}(S=0)=\dfrac{{}^{1}C_1\,{}^{3r}C_1}{{}^{3r+1}C_2}=\dfrac{3r\times 2}{(3r+1)(3r)}=\dfrac{2}{3r+1}$ $\mathrm{P}(S=4)=\dfrac{{}^{2r}C_2}{{}^{3r+1}C_2}=\dfrac{(2r)(2r-1)}{(3r+1)(3r)}=\dfrac{\frac{2}{3}(2r-1)}{3r+1}$ $\mathrm{P}(S=10)=\dfrac{{}^{r}C_1\,{}^{2r}C_1}{{}^{3r+1}C_2}=\dfrac{r\times 2r\times 2}{(3r+1)(3r)}=\dfrac{\frac{4}{3}r}{3r+1}$ $\mathrm{P}(S=25)=\dfrac{{}^{r}C_2}{{}^{3r+1}C_2}=\dfrac{(r)(r-1)}{(3r+1)(3r)}=\dfrac{\frac{1}{3}(r-1)}{3r+1}$ Note : Good to check sum of prob =1 $\dfrac{2}{3r+1}+\dfrac{\frac{2}{3}(2r-1)}{3r+1}+\dfrac{\frac{4}{3}r}{3r+1}+\dfrac{\frac{1}{3}(r-1)}{3r+1}$ $=\dfrac{1}{3r+1}\left(2+\dfrac{4}{3}r-\dfrac{2}{3}+\dfrac{4}{3}r+\dfrac{1}{3}r-\dfrac{1}{3}\right)$ $=1$
	$\mathrm{E}(S)$ $=\sum s\mathrm{P}(S=s)$ $=\dfrac{1}{3r+1}\left(0\times 2+4\left(\dfrac{4}{3}r-\dfrac{2}{3}\right)+10\left(\dfrac{4}{3}r\right)+25\left(\dfrac{1}{3}r-\dfrac{1}{3}\right)\right)$ $=\dfrac{1}{3r+1}\left(\dfrac{16}{3}r-\dfrac{8}{3}+\dfrac{40}{3}r+\dfrac{25}{3}r-\dfrac{25}{3}\right)$ $=\dfrac{1}{3r+1}(27r-11)$

	$\mathrm{E}\left(S^2\right)$ $=\sum s^2\mathrm{P}\left(S=s\right)$ $=\dfrac{1}{3r+1}\left(4^2\left(\dfrac{4}{3}r-\dfrac{2}{3}\right)+10^2\left(\dfrac{4}{3}r\right)+25^2\left(\dfrac{1}{3}r-\dfrac{1}{3}\right)\right)$ $=\dfrac{1}{3r+1}\left(\dfrac{64}{3}r-\dfrac{32}{3}+\dfrac{400}{3}r+\dfrac{625}{3}r-\dfrac{625}{3}\right)$ $=\dfrac{1}{3r+1}\left(363r-219\right)$
	$\mathrm{Var}\left(S\right)$ $=\mathrm{E}\left(S^2\right)-\left[\mathrm{E}\left(S\right)\right]^2$ $=\dfrac{1}{3r+1}\left(363r-219\right)-\left[\dfrac{1}{3r+1}\left(27r-11\right)\right]^2$ $=\dfrac{1}{\left(3r+1\right)^2}\left[\left(363r-219\right)\left(3r+1\right)-\left(27r-11\right)^2\right]$ $=\dfrac{1}{\left(3r+1\right)^2}\left[360r^2+300r-340\right]$
(iii)	$\mathrm{Var}\left(S\right)=38$ $\dfrac{1}{\left(3r+1\right)^2}\left[360r^2+300r-340\right]=38$ $GC: r=3$

10 RJC Prelim 9233/2005/04/Q25(modified)

X denotes the random variable representing the number of thunderstorms occurring per month. The probability distribution function of X is given in the following table.

x	0	1	2	3	4
P $(X = x)$	a	0.2	0.5	0.1	b

(i) State the values of a and b which maximizes the value of E(X). **[2]**

Using these values of a and b,

(ii) Show that Var(X) = $\dfrac{101}{100}$. **[3]**

(iii) Two independent observations X_1 and X_2 are taken of X. Find $\mathrm{P}(X_1 - X_2 \geq 1)$. Hence, or otherwise, find $\mathrm{P}\left(\left|X_1 - X_2\right| \geq 1\right)$. **[5]**

(i) $\mathrm{E}(X)=0(a)+1(0.2)+2(0.5)+3(0.1)+4(b)$

$a=0,\ b=0.2$

(ii) $\mathrm{E}(X)=1(0.2)+2(0.5)+3(0.1)+4(0.2)=2.3$

$\mathrm{E}(X^2)=1^2(0.2)+2^2(0.5)+3^2(0.1)+4^2(0.2)=6.3$

$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2=6.3-(2.3)^2=\frac{101}{100}$ (shown)

(iii)

$X_1 - X_2$	0	1	2	3	4
0	0	1	2	3	4
1	− 1	0	1	2	3
2	− 2	− 1	0	1	2
3	− 3	− 2	− 1	0	1
4	− 4	− 3	− 2	− 1	0

$$\begin{aligned}
&\mathrm{P}(X_1-X_2\geq 1)\\
&=\mathrm{P}(X_1=4,X_2=0)+\mathrm{P}(X_1=4,X_2=1)+\mathrm{P}(X_1=4,X_2=2)+\mathrm{P}(X_1=4,X_2=3)\\
&\quad+\mathrm{P}(X_1=3,X_2=0)+\mathrm{P}(X_1=3,X_2=1)+\mathrm{P}(X_1=3,X_2=2)\\
&\quad+\mathrm{P}(X_1=2,X_2=0)+\mathrm{P}(X_1=2,X_2=1)\\
&\quad+\mathrm{P}(X_1=1,X_2=0)\\
&=0.2(0.8)+0.1(0.7)+0.5(0.2)+0.2(0)\\
&=\frac{33}{100}
\end{aligned}$$

$$\begin{aligned}
\mathrm{P}(\,|\,X_1-X_2\,|\geq 1) &=1-\mathrm{P}(\,|\,X_1-X_2\,|=0)\\
&=1-\mathrm{P}(X_1-X_2=0)\\
&=1-\left(1-2\times\frac{33}{100}\right)\\
&=\frac{33}{50}
\end{aligned}$$

11 TMJC Prelim 9758/2021/02/Q8

A bag contains 3 white balls and r red balls, where $r \geq 2$. Balls are drawn one at a time, at random from the bag and without replacement. The random variable X is the number of white balls drawn until 2 red balls are drawn.

(i) Show that $\mathrm{P}(X=1)=\dfrac{6r(r-1)}{(r+3)(r+2)(r+1)}$, and find the probability distribution of X. [4]

(ii) Find the expectation of X and show that the variance of X can be expressed in the form $\dfrac{6\mathrm{g}(r)}{(r+2)(r+1)^2}$, where $\mathrm{g}(r)$ is a quadratic polynomial to be determined. [5]

(iii) Given that the variance of X is $\dfrac{11}{16}$, find the value of r. [1]

<table>
<tr>
<td>(i)</td>
<td>
X can take values 0, 1, 2, 3

$$\mathrm{P}(X=0)=\frac{r(r-1)}{(r+3)(r+2)}$$
$$\mathrm{P}(X=1)=\frac{3r(r-1)\times 2}{(r+3)(r+2)(r+1)}=\frac{6r(r-1)}{(r+3)(r+2)(r+1)}$$
$$\mathrm{P}(X=2)=\frac{3\times 2\times r(r-1)\times 3}{(r+3)(r+2)(r+1)r}=\frac{18(r-1)}{(r+3)(r+2)(r+1)}$$
$$\mathrm{P}(X=3)=\frac{3\times 2\times 1\times r(r-1)\times 4}{(r+3)(r+2)(r+1)r(r-1)}=\frac{24}{(r+3)(r+2)(r+1)}$$

<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$\mathrm{P}(X=x)$</td><td>$\frac{r(r-1)}{(r+3)(r+2)}$</td><td>$\frac{6r(r-1)}{(r+3)(r+2)(r+1)}$</td><td>$\frac{18(r-1)}{(r+3)(r+2)(r+1)}$</td><td>$\frac{24}{(r+3)(r+2)(r+1)}$</td></tr>
</table>
</td>
</tr>
<tr>
<td>(ii)</td>
<td>
$$\mathrm{E}(X)=\frac{6r(r-1)}{(r+3)(r+2)(r+1)}+\frac{36(r-1)}{(r+3)(r+2)(r+1)}+\frac{72}{(r+3)(r+2)(r+1)}$$
$$=\frac{6\,(r^2+5r+6)}{(r+3)(r+2)(r+1)}$$
$$=\frac{6\,(r+2)(r+3)}{(r+3)(r+2)(r+1)}$$
$$=\frac{6}{r+1}$$

$$\mathrm{E}(X^2)=\frac{6r(r-1)}{(r+3)(r+2)(r+1)}+\frac{72(r-1)}{(r+3)(r+2)(r+1)}+\frac{216}{(r+3)(r+2)(r+1)}$$
$$=\frac{6(r^2+11r+24)}{(r+3)(r+2)(r+1)}$$
</td>
</tr>
</table>

	$=\dfrac{6(r+3)(r+8)}{(r+3)(r+2)(r+1)}$ $=\dfrac{6(r+8)}{(r+2)(r+1)}$ $\text{Var}(X)=\text{E}(X^2)-[\text{E}(X)]^2$ $\text{Var}(X)=\dfrac{6(r+8)}{(r+2)(r+1)}-\left(\dfrac{6}{r+1}\right)^2$ $=\dfrac{6(r+8)(r+1)-36(r+2)}{(r+2)(r+1)^2}$ $=\dfrac{6(r^2+3r-4)}{(r+2)(r+1)^2}$ (shown) $\therefore \text{g}(r)=r^2+3r-4$
(iii)	Given that $\text{Var}(X)=\dfrac{11}{16}$ $\dfrac{6(r^2+3r-4)}{(r+2)(r+1)^2}=\dfrac{11}{16}$ Solve using GC, $r=7$

THE END