RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Additional Practice Questions for Chapter S1B: Probability (Solutions)

1 CJC Prelim 9740/2011/02/Q6(b)

For events A and B, it is given that $\mathrm{P}(A|B')=\frac{2}{3}$, $\mathrm{P}(A\cap B)=\frac{1}{10}$ and $\mathrm{P}(B)=\frac{2}{5}$.

Find

(i) $\mathrm{P}(A\cap B')$ **[1]**

(ii) $\mathrm{P}(A)$ **[1]**

(iii) State whether A and B are independent, giving a reason for your answer. **[1]**

[**(i)** $\frac{2}{5}$ **(ii)** $\frac{1}{2}$ **(iii)** A and B are not independent]

Solution

(i) $\mathrm{P}(A|B')=\frac{\mathrm{P}(A\cap B')}{1-\mathrm{P}(B)}$

$\Rightarrow \frac{2}{3}=\frac{\mathrm{P}(A\cap B')}{1-\frac{2}{5}}$

$\Rightarrow \mathrm{P}(A\cap B')=\frac{2}{5}$

(ii) $\mathrm{P}(A)=\mathrm{P}(A\cap B)+\mathrm{P}(A\cap B')=\frac{1}{10}+\frac{2}{5}=\frac{1}{2}$

(iii) $\mathrm{P}(A\cap B)=\frac{1}{10}$

$\mathrm{P}(A)P(B)=\frac{1}{2}\left(\frac{2}{5}\right)=\frac{1}{5}\neq\frac{1}{10}\Rightarrow$ A and B are not independent.

or $\mathrm{P}\left(A|B'\right)=\frac{2}{3}\neq \mathrm{P}(A)=\frac{1}{2}\Rightarrow$ A and B are not independent.

2 9233/2003/02/Q25

In the first stage of a computer game, the player chooses, at random, one of 5 icons, only one of which is correct. If the correct icon is chosen then, in the second stage, the player chooses, at random, one of 8 icons, only one of which is correct. If an incorrect icon is chosen in the first stage then, in the second stage, the player chooses, at random, one of 10 icons; only one of which is correct.

The events A and B are defined as follows:

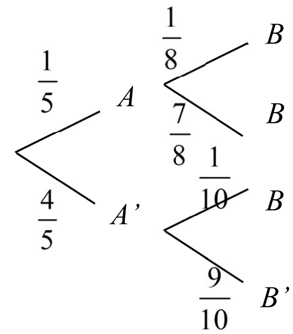
A : the first icon chosen is correct.
B : the second icon chosen is correct.

Find **(i)** $\mathrm{P}(A\cap B)$, **(ii)** $\mathrm{P}(B)$, **(iii)** $\mathrm{P}(A\cup B)$, **(iv)** $\mathrm{P}(A\,|\,B)$. **[1][3][2][2]**

[**(i)** $\frac{1}{40}$ **(ii)** $\frac{21}{200}$ **(iii)** $\frac{7}{25}$ **(iv)** $\frac{5}{21}$]

Solution

(i) $\mathrm{P}(A\cap B)=\left(\frac{1}{5}\right)\left(\frac{1}{8}\right)=\frac{1}{40}$.



(ii) $\mathrm{P}(B)=\mathrm{P}(A\cap B)+\mathrm{P}(A'\cap B)$

$$=\frac{1}{40}+\left(\frac{4}{5}\right)\left(\frac{1}{10}\right)=\frac{21}{200}.$$

(iii) $\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$$=\frac{1}{5}+\frac{21}{200}-\frac{1}{40}=\frac{7}{25}.$$

(iv) $\mathrm{P}(A|B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}=\dfrac{\frac{1}{40}}{\frac{21}{200}}=\dfrac{5}{21}$.

3 **9233/J1997/02/Q3**

In each turn of a game, a player throws a fair die repeatedly until he obtains a number that has already appeared in that turn; the player must therefore have at least 2 throws but not more than 7 throws.

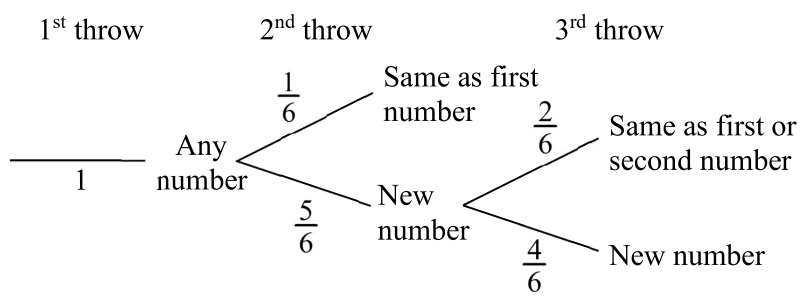
(i) Find the probability that the player has fewer than 4 throws in a turn.

(ii) Find the conditional probability that the player has exactly 2 throws in a turn, given that he has fewer than 4 throws in that turn.

(iii) Find the probability that the player has exactly 5 throws in a turn.

[**(i)** $\frac{4}{9}$ **(ii)** $\frac{3}{8}$ **(iii)** $\frac{5}{27}$]

Solution



(i) P(fewer than 4 throws in a turn)
=P(exactly 2 throws in a turn)+P(exactly 3 throws in a turn)
$=\frac{1}{6}+\frac{5}{6}\times\frac{2}{6}=\frac{4}{9}$

(ii) P(exactly 2 throws$\mid$fewer than 4 throws in that turn) $= \dfrac{\text{P(exactly 2 throws AND fewer than 4 throws in that turn)}}{\text{P(fewer than 4 throws in that turn)}}$ $= \dfrac{\text{P(exactly 2 throws)}}{\text{P(fewer than 4 throws in that turn)}} = \dfrac{\frac{1}{6}}{\frac{4}{9}} = \dfrac{3}{8}$ **(iii)** P(exactly 5 throws in a turn) $= \left(\dfrac{6}{6}\right)\left(\dfrac{5}{6}\right)\left(\dfrac{4}{6}\right)\left(\dfrac{3}{6}\right)\left(\dfrac{4}{6}\right) = \dfrac{5}{27}$

4 **SAJC Prelim 9740/2013/02/Q6**

A teacher conducted a survey on a large number of students to determine the choice of colours for painting the school hall from 3 colour options of white, green and blue. Of the students surveyed, 40% were boys and 60% were girls. Of the boys, 50% chose white, 20% chose green and the rest chose blue. Of the girls, 25% chose white, 45% chose green and the rest chose blue.

Draw a probability tree diagram to illustrate the above information. **[1]**

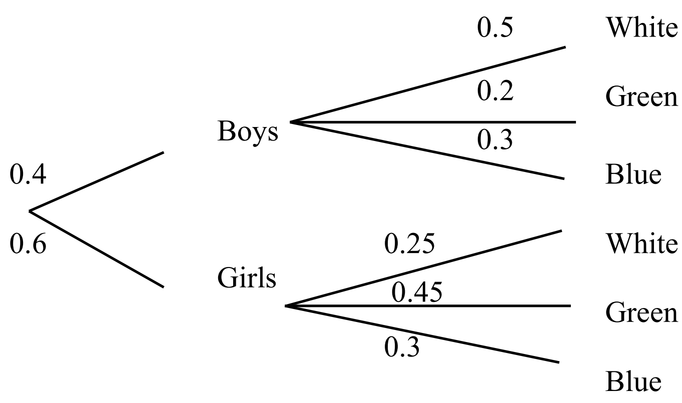
(i) One student is randomly selected. Find the probability that the student chose white. **[1]**

(ii) Two students are randomly selected. Find the probability that the two students are of the same gender or chose different colours (or both). **[3]**

(iii) Three girls are randomly selected. Find the probability that exactly 1 girl chose white, given that none of them chose blue. **[3]**

[**(i)** 0.35 **(ii)** $\dfrac{1067}{1250}$ **(iii)** $\dfrac{1215}{2744}$]

Solution



(i) P(student chose white) $= (0.4\times 0.5)+(0.6\times 0.25) = 0.35$ or $\dfrac{7}{20}$

(ii) Required probability

= 1 – P(Students are of different gender and chose same colour)

$$= 1 - 2\begin{bmatrix}(0.4\times0.5\times0.6\times0.25)+(0.4\times0.2\times0.6\times0.45)\\ +(0.4\times0.3\times0.6\times0.3)\end{bmatrix} = \frac{1067}{1250}$$

(iii) P(exactly 1 girl chose white | none of the girls chose blue)

$$= \frac{\text{P(exactly 1 girl chose white and none of the girls chose blue)}}{\text{P(none of the girls chose blue)}}$$

$$= \frac{\text{P(1 out of the 3 girls chose white and the other 2 chose green)}}{\text{P(none of the girls chose blue)}} = \frac{{}^{3}C_{1}(0.25)(0.45)(0.45)}{(0.7)(0.7)(0.7)} = \frac{1215}{2744}$$

5 **RI Prelim 9740/02/Q10**

The probability that a hockey team wins any match is $\frac{1}{2}$ and the probability that it loses any match is $\frac{1}{6}$. Three points are awarded for a win, one point for a draw and no point for a defeat. The team plays four matches and the outcome of each match is independent of the other matches.

(i) Show that the probability that the team has exactly one draw and exactly one defeat is $\frac{1}{6}$. **[1]**

Find the probability that the team

(ii) wins the first match and goes on to win exactly one other match, **[3]**

(iii) wins exactly one match, given that it obtains four points. **[4]**

[**(ii)** $\frac{3}{16}$ **(iii)** $\frac{9}{11}$]

Solution

Given P(win) $=\frac{1}{2}$ and P(defeat) $=\frac{1}{6}$ $\Rightarrow$ $\text{P(draw)} = 1-\frac{1}{2}-\frac{1}{6}=\frac{1}{3}$

(i) P(exactly 1 draw and 1 defeat) = P(1 draw, 1 defeat, 2 wins)

$$= \frac{1}{3}\times\frac{1}{6}\times\left(\frac{1}{2}\right)^2\times\frac{4!}{2!} = \frac{1}{6} \text{ (shown)}$$

(ii) P(wins the first match and goes on to win exactly one other match)

$= \frac{1}{2}\times$ P(wins one other match and loses or draws in the other 2)

$$= \frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}\right)^2\frac{3!}{2!}\right] = \frac{3}{16}$$

(iii) P(wins exactly one match | obtains four pts)

$$=\frac{\text{P(wins exactly one match and obtains four points)}}{\text{P(obtains four points)}}$$

$$=\frac{\text{P(1 win, 1 draw, 2 defeats)}}{\text{P(1 win, 1 draw, 2 defeats)}+\text{P(4 draws)}}$$

$$=\frac{\frac{1}{2}\times\frac{1}{3}\times\left(\frac{1}{6}\right)^2\times\frac{4!}{2!}}{\frac{1}{2}\times\frac{1}{3}\times\left(\frac{1}{6}\right)^2\times\frac{4!}{2!}+\left(\frac{1}{3}\right)^4}$$

$$=\frac{\frac{1}{18}}{\frac{1}{18}+\frac{1}{81}}=\frac{9}{11}$$

6 **AJC Prelim 9740/2008/02/Q7 modified**

3 machines A, B and C produce 25%, 35% and 40% respectively of the golf balls manufactured by a factory. These balls are either yellow or white. Of the balls produced by A and B, 20% and 30% respectively are yellow. It is known that the probability of picking a yellow ball is 0.355.

(a) If 3 balls are picked randomly, find the probability that

(i) at least 2 are produced by machine B **[2]**

(ii) at least 1 is a yellow ball **[2]**

(iii) at least 1 is yellow given that all the balls picked are from machine A. **[2]**

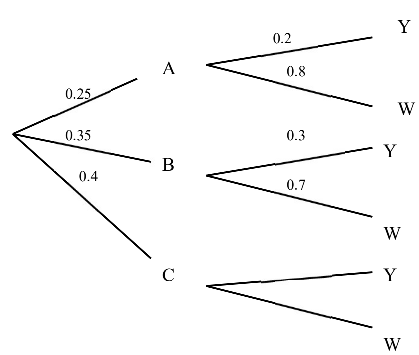
(b) Due to malfunction of machine A, 10% of the balls produced by machine A is faulty and the event of a ball produced by A is faulty is independent of its colour. Find the proportion of white balls that are faulty and produced by machine A. **[3]**

[**(a)(i)** 0.28175 **(ii)** 0.732 **(iii)** 0.488 **(b)** 0.0310]

Solution

(a) (i) $\text{P(at least 2 from B)}=\text{P}(\text{BBB}^c)\frac{3!}{2!}+\text{P(BBB)}$

$$=\left(0.35^2\right)\left(0.65\right)\left(3\right)+0.35^3=0.28175 \text{ (exact)}$$



(ii) P(at least 1 is yellow) = 1 − P(WWW)

$$=1-(1-0.355)^3=0.732 \text{ (3sf)}$$

(iii) Using reduced sample space,

$$\text{P(at least 1 is yellow | all from A)}=1-0.8^3=0.488$$

OR

P(at least 1 is yellow|all from A)

$$= \frac{3\text{P}(\text{A}_\text{y}\text{A}_\text{w}\text{A}_\text{w})+3\text{P}(\text{A}_\text{y}\text{A}_\text{y}\text{A}_\text{w})+\text{P}(\text{A}_\text{y}\text{A}_\text{y}\text{A}_\text{y})}{\text{P}(\text{AAA})}$$

$$= \frac{3\left[(0.25)(0.2)\right]\left[(0.25)(0.8)\right]^2+3\left[(0.25)(0.2)\right]^2(0.25)(0.8)+\left[(0.25)(0.2)\right]^3}{0.25^3} = 0.488$$

(b) P(ball is faulty and produced by A| it is white)

$$= \frac{\text{P}\left(\text{ball is faulty and white and produced by machine A}\right)}{\text{P(ball is white)}}$$

$$= \frac{0.1\times 0.25\times 0.8}{1-0.355}$$

$= 0.0310$ (3sf)

7 RVHS Prelim 9740/2010/02/Q10

(a) A student threw a fair die four times and recorded the number obtained for each throw. Find the probability that

(i) at least two of the four numbers obtained are the same, **[2]**

(ii) exactly two of the four numbers obtained are the same given that the number 6 appeared at least once. **[4]**

(b) Events A and B are such that $\text{P}(B') = \frac{1}{2}$, $\text{P}(A' \cap B') = \frac{7}{20}$ and $\text{P}(B|A) = \frac{4}{7}$. Find the probability $\text{P}(A)$. **[3]**

[**(a) (i)** $\frac{13}{18}$ **(ii)** $\frac{360}{671}$ **(b)** $\frac{7}{20}$]

Solution

(a)(i) P(at least 2 of the 4 numbers are the same)

= 1 – P(all the 4 numbers are different)

$$= 1-\left(\frac{6}{6}\right)\left(\frac{5}{6}\right)\left(\frac{4}{6}\right)\left(\frac{3}{6}\right) = \frac{13}{18}$$

OR

P(at least 2 of the 4 numbers are the same)

= P(exactly 2 same, and another 2 same) + P(exactly 2 same and 2 different) + P(exactly 3 same) + P(all 4 same)

$$= \frac{4!}{2!2!}\left(\frac{1}{6}\right)^4\binom{6}{2}+\frac{4!}{2!}\left(\frac{1}{6}\right)^4\binom{6}{3}\binom{3}{1}+\frac{4!}{3!}\left(\frac{1}{6}\right)^4\binom{6}{2}\binom{2}{1}+\left(\frac{1}{6}\right)^4\binom{6}{1}$$

$$= \frac{13}{18}$$

(a)(ii) P(exactly 2 of the 4 numbers are same | at least one 6)

$$= \frac{\text{P(2 out of the 4 numbers are same and at least 1 six)}}{\text{P(at least one six)}}$$

$$= \frac{\text{P(six appear twice)} + \text{P(six appear once)}}{\text{P(at least one six)}}$$

$$= \frac{\text{P}(6,6,B,C) + \text{P}(A,A,6,B)}{1 - \text{P(no six)}}$$

$$= \frac{\frac{4!}{2!}\left(\frac{1}{6}\right)^4 \binom{5}{2} + \frac{4!}{2!}\left(\frac{1}{6}\right)^4 \binom{5}{2}\binom{2}{1}}{1 - \left(\frac{5}{6}\right)^4} = \frac{360}{671}$$

(b) $\text{P}(B|A) = \frac{4}{7} \Rightarrow \frac{\text{P}(A \cap B)}{\text{P}(A)} = \frac{4}{7} \Rightarrow \text{P}(A \cap B) = \frac{4}{7}\text{P}(A)$

$$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$$

$$1 - \text{P}(A' \cap B') = \text{P}(A) + \left[1 - \text{P}(B')\right] - \frac{4}{7}\text{P}(A)$$

$$1 - \frac{7}{20} = \text{P}(A) + \left(1 - \frac{1}{2}\right) - \frac{4}{7}\text{P}(A)$$

$$\frac{3}{20} = \frac{3}{7}P(A)$$

$$\therefore \text{P}(A) = \frac{7}{20}$$

8 **9233/2001/02/Q11**

(a) Events A and B are such that $\text{P}(A) = \frac{1}{3}$, $\text{P}(B \mid A) = \frac{1}{3}$ and $\text{P}(A' \cap B') = \frac{1}{6}$.

Find
(i) $\text{P}(A \cup B)$, **(ii)** $\text{P}(B)$. **[2][3]**

(b) A man writes 5 letters, one each to A, B, C, D and E. Each letter is placed in a separate envelope and sealed. He then addresses the envelopes, at random, one each to A, B, C, D and E.

(i) Find the probability that the letter to A is in the correct envelope and the letter to B is in an incorrect envelope. **[1]**

(ii) Find the probability that the letter to A is in the correct envelope, given that the letter to B is in an incorrect envelope. **[3]**

(iii) Find the probability that both of the letters to A and B are in incorrect envelopes. **[5]**

[**(a)(i)** $\frac{5}{6}$ **(ii)** $\frac{11}{18}$; **(b)(i)** $\frac{3}{20}$ **(ii)** $\frac{3}{16}$ **(iii)** $\frac{13}{20}$]

Solution

(a) (i) From Venn diagram, $\mathrm{P}(A\cup B)=1-\mathrm{P}(A'\cap B')=1-\frac{1}{6}=\frac{5}{6}$

(ii) $\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B\mid A)=\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$

$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$\therefore \mathrm{P}(B)=\mathrm{P}(A\cup B)-\mathrm{P}(A)+\mathrm{P}(A\cap B)=\frac{5}{6}-\frac{1}{3}+\frac{1}{9}=\frac{11}{18}$

(b) Let X denote the event that letter to X is in the correct envelope for $X\in\{A,B,C,D,E\}$.

(i) $\mathrm{P}(A\cap B')=\frac{1}{5}\times\frac{3}{4}=\frac{3}{20}$

(ii) $\mathrm{P}(A\mid B')=\frac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=\frac{\frac{3}{20}}{\frac{4}{5}}=\frac{3}{16}$

(iii) $\mathrm{P}(A'\cap B')=1-\mathrm{P}(A\cup B)=1-\left[\mathrm{P}(B)+\mathrm{P}(A\cap B')\right]=1-\left[\frac{1}{5}+\frac{3}{20}\right]=\frac{13}{20}$

OR $\mathrm{P}(A'\cap B')=\mathrm{P}(B')\times\mathrm{P}(A'\mid B')=\frac{4}{5}\left(1-\frac{3}{16}\right)=\frac{13}{20}$

OR $\mathrm{P}(B')=\mathrm{P}(A\cap B')+\mathrm{P}(A'\cap B')$

$\therefore \mathrm{P}(A'\cap B')=\mathrm{P}(B')-\mathrm{P}(A\cap B')=\frac{4}{5}-\frac{3}{20}=\frac{13}{20}$

9 SAJC Prelim 9740/2010/02/Q7b

A teacher brings 4 black, 3 blue, 2 red and 1 green markers to the classroom for each of his lessons. Unknown to him, the probabilities that a black, blue, red or green marker is out of ink are $p, \frac{1}{2}p$, $\frac{1}{4}p$ and $\frac{1}{8}p$ respectively, where $0<p<1$.

(i) Find, in terms of p, the probability that a randomly chosen marker from his set of ten markers is out of ink. **[2]**

(ii) In the classroom, the first marker he tries out is out of ink. Find, in terms of p, the probability that the next marker he tries out is a red marker that is also out of ink. **[3]**

(iii) After numerous lessons, the teacher realized that, in general, the first marker he tries out works at least 7 out of 10 times. Find the range of possible values of p. **[2]**

[**(i)** $\frac{49}{80}p$ **(ii)** $\frac{47}{882}p$ **(iii)** $0<p\le\frac{24}{49}$]

Solution

10 markers in total: 4 black, 3 blue, 2 red and 1 green

(i) P(a randomly chosen marker is out of ink)

$$=\left(\frac{4}{10}\times p\right)+\left(\frac{3}{10}\times\frac{1}{2}p\right)+\left(\frac{2}{10}\times\frac{1}{4}p\right)+\left(\frac{1}{10}\times\frac{1}{8}p\right)=\frac{49}{80}p$$

(ii) P(2nd marker is red and out of ink|1st marker is out of ink)

$$=\frac{\begin{array}{l}[\text{P}\left(1^{\text{st}}\text{ marker is red and out of ink and }2^{\text{nd}}\text{marker is red and out of ink}\right)\\ +\text{P}\left(1^{\text{st}}\text{ marker is not red and out of ink and }2^{\text{nd}}\text{marker is red and out of ink}\right)]\end{array}}{\text{P}\left(1^{\text{st}}\text{marker is out of ink}\right)}$$

$$=\frac{\left(\frac{2}{10}\times\frac{1}{4}p\right)\left(\frac{1}{9}\times\frac{1}{4}p\right)+\left[\frac{4}{10}p+\left(\frac{3}{10}\times\frac{1}{2}p\right)+\left(\frac{1}{10}\times\frac{1}{8}p\right)\right]\left(\frac{2}{9}\times\frac{1}{4}p\right)}{\frac{49}{80}p}=\frac{47}{882}p$$

(iii)

$$\text{P(a randomly chosen marker works)}\geq\frac{7}{10}$$

$$1-\frac{49}{80}p\geq\frac{7}{10}$$

$$\frac{49}{80}p\leq\frac{3}{10}$$

$$0<p\leq\frac{24}{49}$$

10 RI Y6 CT2 9740/2008/Q10

[In this question, give all your answers correct to 3 decimal places.]

A delegation of 20 athletes comprises 9 basketball players, 6 volleyball players and 5 tennis players. Among them are 5 male basketball players, 6 male volleyball players and 2 male tennis players.

A committee of four people is to be formed from these 20 athletes.

(i) Find the probability that exactly three basketball players and one tennis player are chosen. **[2]**

(ii) Find the probability that 2 female athletes are chosen given that at least one volleyball player is chosen. **[4]**

(iii) Determine if the 2 events "2 female athletes are chosen" and "at least one volleyball player is chosen" are independent, justifying your answer. **[2]**

[**(i)** 0.087 **(ii)** 0.311]

Solution

Sports	**No. of Males**	**No. of Females**	**Subtotal**
Basketball	5	4	9
Volleyball	6	0	6
Tennis	2	3	5
Total	13	7	20

(i) Let W be the event that exactly three basketball players and one tennis player are chosen.

$$\mathrm{P}(W)=\frac{{}^{9}C_3\,{}^{5}C_1}{{}^{20}C_4} \text{ or } \frac{9}{20}\times\frac{8}{19}\times\frac{7}{18}\times\frac{5}{17}\times\frac{4!}{3!}=\frac{28}{323}\approx 0.087(3 \text{ d.p.})$$

(ii) Let X be the event that 2 female athletes are chosen and Y be the event that at least one volleyball player is chosen

$\mathrm{P}(Y) = 1 - \mathrm{P}$(no volleyball players are chosen)

$$=1-\frac{{}^{14}C_4}{{}^{20}C_4} \text{ or } 1-\frac{14}{20}\times\frac{13}{19}\times\frac{12}{18}\times\frac{11}{17}=\frac{3844}{4845}$$

$\mathrm{P}(X \cap Y) = \mathrm{P}$ (at least 1 volleyball player is chosen and 2 other female athletes are chosen)

$$=\mathrm{P}\left(1 \text{ volleyball, 1 other male, 2 female }\right)+\mathrm{P}\left(2 \text{ volleyball, 2 female }\right)$$

$$=\frac{{}^{7}C_2\left({}^{6}C_1\times{}^{7}C_1+{}^{6}C_2\right)}{{}^{20}C_4} \text{ or } \frac{7}{20}\times\frac{6}{19}\times\frac{6}{18}\times\frac{7}{17}\times\frac{4!}{2!}+\frac{7}{20}\times\frac{6}{19}\times\frac{6}{18}\times\frac{5}{17}\times\frac{4!}{2!2!}$$

$$=\frac{21}{85}\approx 0.24706$$

So $\mathrm{P}(X \mid Y) = \dfrac{\mathrm{P}(X\cap Y)}{\mathrm{P}(Y)}=\dfrac{1197}{3844}\approx 0.311(3 \text{ d.p.})$

(iii) $\mathrm{P}(X)=\dfrac{{}^{7}C_2\,{}^{13}C_2}{{}^{20}C_4}$ or $\dfrac{7}{20}\times\dfrac{6}{19}\times\dfrac{13}{18}\times\dfrac{12}{17}\times\dfrac{4!}{2!2!}=\dfrac{1638}{4845}\neq\dfrac{1197}{3844}=\mathrm{P}(X\mid Y)$

Since $\mathrm{P}(X \mid Y) \neq \mathrm{P}(X)$, events X and Y are not independent.

OR

$\mathrm{P}(X)\times\mathrm{P}(Y) = \dfrac{1638}{4845}\times\dfrac{3844}{4845}\approx 0.268 \neq 0.24706=\mathrm{P}(X\cap Y)$,

events X and Y are not independent.

11 NYJC Prelim 9740/2013/02/Q12

A machine-operated car wash offers four different options as follows:

Option	Description	Duration
A	Wash only	2 minutes
B	Wash and dry	3 minutes
C	Bubble-wash and dry	3 minutes
D	Steam-wash and dry	4 minutes

A driver makes his choice and then proceeds to the car wash. The probability of a driver choosing Option A, Option B, Option C or Option D is given to be $\frac{5}{12}$, a, ar and ar^2 respectively, where a and r are positive constants and $r \neq 1$.

At a particular instant, there are three cars queuing to use the car wash.

(i) Show that the probability that at least one of the three drivers has chosen option C is given by $1-(1-ar)^3$. **[1]**

(ii) By considering the sum of the probabilities for all the options, deduce that $0 < a < \frac{7}{12}$ **[3]**

For the rest of the question, take $a = \frac{1}{3}$ and $r = \frac{1}{2}$.

A fourth car, car X joins the queue at the instant when the first car enters the car wash. Assuming that no time is lost between the time a car leaving the car wash and the next car starting the wash, find the probability that

(iii) it will be at least 10 minutes before car X enters the car wash and at least one of the previous three drivers chose Option C, **[2]**

(iv) it will be at least 10 minutes before car X enters the car wash, given that at least one of the previous three drivers chose Option C. **[2]**

State a necessary assumption for your calculations in **(iii)** and **(iv)**. **[1]**

[**(iii)** $\frac{11}{288}$ **(iv)** $\frac{33}{364}$]

Solution

(i) $\mathrm{P}(\geq 1 \text{ of 3 drivers choose C}) = 1 - \mathrm{P}(\text{no one choose C}) = 1-(1-ar)^3$.

(ii)
$$\frac{5}{12} + a\left(1+r+r^2\right) = 1$$
$$a\left(1+r+r^2\right) = \frac{7}{12}$$

Since $r > 0$, $1+r+r^2 > 1$.

$$\Rightarrow a(1+r+r^2) = \frac{7}{12} > a$$
$$0 < a < \frac{7}{12} \quad \text{(deduced)}$$

(iii) $\text{P}(\geq 10 \text{ min before } X \text{ enters and} \geq 1 \text{ of 3 drivers choose C})$

$$= \text{P}(\text{one car chose B, one car chose C, one car chose D})$$
$$+ \text{P}(\text{two cars chose C one car chose D})$$
$$+ \text{P}(\text{one car chose C two cars chose D})$$
$$= 3!\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)\left(\frac{1}{12}\right) + \frac{3!}{2!}\left(\frac{1}{6}\right)\left(\frac{1}{6}\right)\left(\frac{1}{12}\right) + \frac{3!}{2!}\left(\frac{1}{6}\right)\left(\frac{1}{12}\right)\left(\frac{1}{12}\right)$$
$$= \frac{11}{288}.$$

(iv) $\text{P}(\geq 10 \text{ min before } X \text{ enters} \mid \geq 1 \text{ of 3 drivers choose C})$

$$= \frac{\text{P}(\geq 10 \text{ min before } X \text{ enters and} \geq 1 \text{ of 3 drivers choose C})}{\text{P}(\geq 1 \text{ of 3 drivers choose C})}$$
$$= \frac{\frac{11}{288}}{1-\left(1-\frac{1}{6}\right)^3} = \frac{33}{364}$$

We need to assume that the choice each driver makes is independent of each other.

12 JJC Prelim 9740/2011/02/Q7

For events A and B, it is given that $\text{P}(A') = 0.3$, $\text{P}(B') = 0.6$ and $\text{P}(B \mid A') = 0.4$.

(i) Find $\text{P}(A' \cap B)$. **[2]**

(ii) Find $\text{P}(A \cap B)$. **[2]**

(iii) Determine whether A and B are independent events. **[1]**

For a third event C, it is given that A and C are mutually exclusive, and $\text{P}(B \cap C) = 0.1$.

(iv) Find the possible range of values of $\text{P}(C)$. **[2]**

[**(i)** 0.12 **(ii)** 0.28 **(iii)** Independent **(iv)** $0.1 \leq \text{P}(C) \leq 0.28$]

Solution

(i) $\text{P}(B \mid A') = 0.4$

$$\frac{\text{P}(A' \cap B)}{\text{P}(A')} = 0.4$$
$$\text{P}(A' \cap B) = 0.4 \times \text{P}(A')$$
$$= 0.4 \times 0.3 = 0.12$$

(ii) $\text{P}(B) = \text{P}(A' \cap B) + \text{P}(A \cap B)$

$$1 - 0.6 = 0.12 + \text{P}(A \cap B)$$
$$\text{P}(A \cap B) = 0.28$$

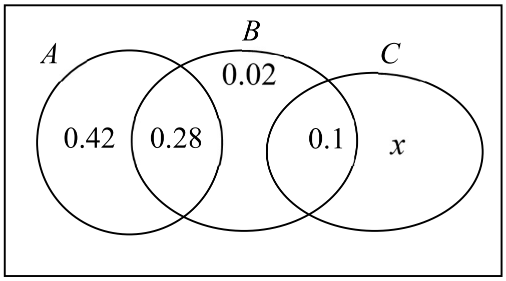
(iii) $\text{P}(A)\times\text{P}(B) = 0.7\times 0.4$

$= 0.28 = \text{P}(A\cap B)$

$\therefore$ A and B are independent events.

(iv) C and A are mutually exclusive so there should be no overlap. Therefore C is drawn as shown below.

Let x be $\text{P}(B'\cap C)$.



Since probability is not negative, $x \geq 0$.

Since total probability cannot exceed 1,

$$0.82 + x \leq 1.$$

$$\Rightarrow x \leq 0.18$$

Combining the inequalities, we have $0 \leq x \leq 0.18$.

$\therefore$ $0.1 \leq \text{P}(C) \leq 0.28$.

13 VJC Prelim 9758/2020/02/Q5

For events A and B, it is given that $\text{P}(A) = 0.6$, $\text{P}(A\cup B) = 0.7$ and $\text{P}(B \mid A) = 0.3$.

(i) Show that $\text{P}(B) = 0.28$ and determine with a reason, if A and B are independent. **[3]**

(ii) An event C is such that A and C are mutually exclusive and $\text{P}(C) > 0.25$. Find the range of values of $\text{P}(B'\cap C)$. **[3]**

[**(i)** not independent **(ii)** $0.15 < \text{P}(B'\cap C) \leq 0.3$]

Solution

(i) $\text{P}(A\cap B) = \text{P}(A)\text{P}(B \mid A) = 0.6\times 0.3 = 0.18$

$\text{P}(B) = \text{P}(A\cup B) - \text{P}(A) + \text{P}(B\cap A)$

$= 0.7 - 0.6 + 0.18$

$= 0.28$

Not independent because $\text{P}(B) = 0.28 \neq 0.3 = \text{P}(B \mid A)$

[OR $\text{P}(B\cap A) = 0.18 \neq 0.168 = 0.28\times 0.6 = \text{P}(B)\text{P}(A)$].

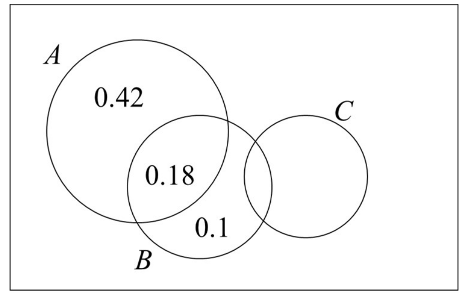
(ii)

$$\mathrm{P}(A \cup B) + \mathrm{P}(B' \cap C) \leq 1$$
$$0.7 + \mathrm{P}(B' \cap C) \leq 1$$
$$\mathrm{P}(B' \cap C) \leq 0.3$$

Also,

$$\begin{aligned} \mathrm{P}(B' \cap C) &= \mathrm{P}(C) - \mathrm{P}(B \cap C) \\ &\geq \mathrm{P}(C) - 0.1 \quad (\text{since } \mathrm{P}(B \cap C) \leq 0.1) \\ &> 0.25 - 0.1 \\ &= 0.15 \end{aligned}$$

$$\therefore 0.15 < \mathrm{P}(B' \cap C) \leq 0.3$$



14 9758/2019/02/Q8

Gerri collects characters given away in packets of breakfast cereal. There are four different characters : Horse, Rider, Dog and Bird. Each character is made in four different colours : Orange, Yellow, Green and White. Gerri has collected 56 items; the numbers of each character and colour are shown in the table.

	Orange	Yellow	Green	White
Horse	1	1	3	4
Rider	1	1	7	5
Dog	3	7	1	6
Bird	4	5	6	1

(i) Gerri puts all the items in a bag and chooses one item at random.

(a) Find the probability that this item is either a Horse or a Rider. **[1]**

(b) Find the probability that this item is a Dog or a Bird but the item is not White. **[1]**

(ii) Gerri now puts the item back in the bag and chooses two items at random.

(a) Find the probability that both of the items are Horses, but neither of the items is Orange. **[1]**

(b) Find the probability that Gerri's two items include exactly one Dog and exactly one item that is Yellow. **[3]**

(iii) Gerri has two favourites among the 16 possible colour/character combinations. The probability of choosing these two at random from the 56 items is $\frac{1}{77}$. Write down all the possibilities for favourite colour/character combinations. **[3]**

[**(i) (a)** $\frac{23}{56}$ **(b)** $\frac{13}{28}$ **(ii) (a)** $\frac{1}{55}$ **(b)** $\frac{21}{110}$]

Solution

	Orange	Yellow	Green	White	Total
Horse	1	1	3	4	9
Rider	1	1	7	5	14
Dog	3	7	1	6	17
Bird	4	5	6	1	16
Total	9	14	17	16	56

(i) **(a)** $\frac{9+14}{56}=\frac{23}{56}$

(b) $\frac{17+16-7}{56}=\frac{13}{28}$

(ii) **(a)** $\frac{8\times 7}{56\times 55}=\frac{1}{55}$

(b) Case 1: Dog is yellow, the other item is a non-yellow Horse/Rider/Bird

Probability $=\frac{7}{56}\times\frac{32}{55}\times 2$

Case 2: Dog is not yellow, the other item is a yellow Horse/Rider/Bird

Probability $=\frac{10}{56}\times\frac{7}{55}\times 2$

Required probability $=\frac{7}{56}\times\frac{32}{55}\times 2+\frac{10}{56}\times\frac{7}{55}\times 2=\frac{21}{110}$

(iii)

$\frac{a}{56}\times\frac{b}{55}\times 2=\frac{1}{77}$ where a,b refer to the number of his first and second favourites

$\therefore ab=20=1\times 20$ or 2×10 or 4×5

Reference to the table, only 4,5 is possible

Thus, possible combinations are as follow

4	5
White Horse	White Rider
White Horse	Yellow Bird
Orange Bird	White Rider
Orange Bird	Yellow Bird

15 9233/N2002/02/Q30either

A room contains n randomly chosen people.

(a) Assume that a randomly chosen person is equally likely to have been born on any day of the week. The probability that the people in the room were all born on different days of the week is denoted by P.

(i) Find P in the case $n = 3$. **[2]**

(ii) Show that $P = \dfrac{120}{343}$ in the case $n = 4$. **[1]**

(b) Assume now that a randomly chosen person is equally likely to have been born in any month of the year. Find the smallest value of n such that the probability that the people in the room were all born in different months of the year is less than $\dfrac{1}{2}$. **[4]**

(c) Assume now that a randomly chosen person is equally likely to have been born on any of the 365 days in the year. It is given that, for the case $n = 21$, the probability that the people in the room were all born on different days of the year is 0.55631, correct to 5 places of decimals. Find the smallest value of n such that the probability that at least two of the people were born on the same day of the year exceeds $\dfrac{1}{2}$. **[5]**

[**(a)(i)** 30/49 **(b)** 5 **(c)** 23]

Solution

(a)(i) n = 3, Person A, B and C.

If no restriction then 7 possibilities for A, B and C, in 7^3 possible days of the week.
If all different, then A could be born on any one of the 7 days, so 7 possibilities;
B on any 6 days, so 6 possibilities; C on any 5 days, so 5 possibilities.

Required probability = $\dfrac{7}{7}\left(\dfrac{6}{7}\right)\left(\dfrac{5}{7}\right) = \dfrac{30}{49}$ OR $\dfrac{{}^7C_3 \times 3!}{7^3} = \dfrac{{}^7P_3}{7^3} = \dfrac{30}{49}$

(ii) n = 4, we can proceed similarly as above,

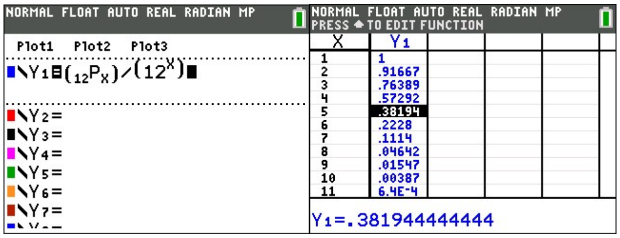
Required probability = $\dfrac{7}{7}\left(\dfrac{6}{7}\right)\left(\dfrac{5}{7}\right)\left(\dfrac{4}{7}\right) = \dfrac{120}{343}$ OR $\dfrac{{}^7C_4 \times 4!}{7^4} = \dfrac{{}^7P_4}{7^4} = \dfrac{120}{343}$

We can observe that $\dfrac{7}{7}\left(\dfrac{6}{7}\right)\left(\dfrac{5}{7}\right)\left(\dfrac{4}{7}\right) = \dfrac{7!}{7^4(7-4)!} = \dfrac{{}^7C_4 \times 4!}{7^4} = \dfrac{{}^7P_4}{7^4}$

(b) For any n such that $1 \le n \le 12$, $n \in \mathbb{Z}^+$, P (all n people born in different months)

$= \dfrac{12!}{12^n(12-n)!}$ OR $\dfrac{{}^{12}P_n}{12^n}$

For $\dfrac{{}^{12}P_n}{12^n} < \dfrac{1}{2}$,



From GC, when $n = 4$, P = 0.573 (3sf) > 0.5
When $n = 5$, P = 0.382 (3sf) < 0.5
$\therefore$ Smallest $n = 5$

(c) P(at least 2 were born on the same day)
= 1 – P(all born on different days) > 0.5
ie P (all born on different days) < 0.5
When $n = 21$, P = 0.55631
When $n = 22$, P = $0.55631 \times \dfrac{365-21}{365} = 0.52430 > 0.5$
When $n = 23$, P = $0.52430 \times \dfrac{365-22}{365} = 0.49269 < 0.5$
$\therefore$ Smallest $n = 23$

16 HCI Prelim 9740/2010/02/Q10

A public opinion poll surveyed a sample of 1000 voters. The table below shows the number of males and females supporting Party A, Party B and Party C.

	Party A	**Party B**	**Party C**
Male	200	130	70
Female	250	300	50

(a) One of the voters is chosen at random. Events A, C and M are defined as follows:
A : The voter chosen supports Party A.
C : The voter chosen supports Party C.
M : The voter chosen is a male.

Find

(i) $\mathrm{P}(A \mid M)$,

(ii) $\mathrm{P}(M' \cap C')$.

Determine whether A and M are independent. **[4]**

(b) It is given that in the sample, 20% of Party A supporters, 30% of Party B supporters and 5% of Party C supporters are immigrants.

(i) One of the voters selected from the sample at random is an immigrant. What is the probability that this voter supports Party A? **[2]**

(ii) Three voters are chosen from the sample at random.
Find the probability that there is exactly one immigrant voter who supports Party C or exactly one female who supports Party A (or both). **[4]**

[**(a) (i)** $\frac{1}{2}$ **(ii)** $\frac{11}{20}$ **(b)(i)** 0.4 **(ii)** 0.434]

Solution

	Party A	**Party B**	**Party C**	**Total**
Male	200	130	70	400
Female	250	300	50	600
Total	450	430	120	1000

(a)(i) $\mathrm{P}(A \mid M) = \frac{200}{400} = \frac{1}{2}$ OR : $\mathrm{P}(A \mid M) = \frac{\mathrm{P}(A \cap M)}{\mathrm{P}(M)} = \frac{\frac{200}{1000}}{\frac{400}{1000}} = \frac{200}{400} = \frac{1}{2}$

(a)(ii) $\mathrm{P}(M' \cap C') = \frac{250+300}{1000} = \frac{11}{20}$

Now, $P(A) = \frac{450}{1000} = \frac{9}{20}$

So, $P(A|M) = \frac{1}{2} \neq P(A)$

$\therefore A$ and M are not independent.

(b)(i) No. of immigrants in the sample $= 0.2(200+250) + 0.3(130+300) + 0.05(120) = 225$

P(voter supports Party A given voter is an immigrant)

$= \frac{\text{P(voter supports Party A \& is an immigrant)}}{\text{P(voter is an immigrant)}}$

$= \frac{\text{no. of voters who supports Party A \& are immigrants)}}{\text{no. of voters who are immigrants}}$

$= \frac{0.2 \times 450}{225} = 0.4$

(b)(ii) Number of immigrants supporting Party C $= 0.05(120) = 6$

P(exactly one immigrant voter who supports Party C or
exactly one female voter who supports Party A or both)

$= \mathrm{P}(\text{exactly 1 immigrant voter who supports party C})$

$+ \mathrm{P}(\text{exactly 1 female who supports party A}) - \mathrm{P}(\text{both})$

$= \frac{{}^{6}C_1\,{}^{994}C_2 + {}^{250}C_1\,{}^{750}C_2 - {}^{6}C_1\,{}^{250}C_1\,{}^{744}C_1}{{}^{1000}C_3} = 0.434 \text{ (3sf)}$

Alternative method:

Req'd Probability $= \frac{6}{1000}\frac{994}{999}\frac{993}{998} \times \frac{3!}{2!} + \frac{250}{1000}\frac{750}{999}\frac{749}{998} \times \frac{3!}{2!} - \frac{6}{1000}\frac{250}{999}\frac{744}{998} \times 3! = 0.434$ (3sf)

17 DHS Prelim 9740/2008/02/Q11

Three cards are drawn from an ordinary pack of 52 cards, at random and without replacement. Cards drawn have the following values:

Aces score 1 point,
Tens, Jacks, Queens and Kings score 10 points, and
Cards from two to nine score as many points as the numbers they carry (i.e. twos score 2 points, threes score 3 points, and so on).

Find the probabilities that

(i) exactly two cards each scoring more than 5 points are drawn, **[2]**

(ii) all three cards are of different suits, **[3]**

(iii) total score of the three cards is more than 28 points given that all three cards are of different suits. **[4]**

[**(i)** $\frac{496}{1105} = 0.449$ **(ii)** $\frac{169}{425} = 0.398$ **(iii)** $\frac{112}{2197} = 0.0510$]

Solution

(i) P(drawing 2 cards each scoring more than 5 points)

$$= \frac{\binom{32}{2}\binom{20}{1}}{\binom{52}{3}} = \frac{496}{1105} \qquad \text{OR} \qquad \frac{20}{52}\times\frac{32}{51}\times\frac{31}{50}\times\frac{3!}{2!} = \frac{496}{1105}$$

(ii) P(all three cards are of different suits)

$$= \binom{4}{3}\frac{\binom{13}{1}^3}{\binom{52}{3}} = \frac{169}{425} \qquad \text{OR} \qquad \binom{4}{3}\times\frac{13}{52}\times\frac{13}{51}\times\frac{13}{50}\times 3! \qquad \text{OR} \qquad \text{-} \ \frac{52}{52}\times\frac{39}{51}\times\frac{26}{50}$$

(iii) P(total score of 3 cards is more than 28 points given they are of different suits)

$$= \frac{\text{P(total score} > 28 \text{ points and cards are of diff suits)}}{\text{P(cards are of different suits)}}$$

$$= \frac{\text{P}\begin{pmatrix}\text{scores are 10, 10, 9} \\ \text{and of diff suits}\end{pmatrix} + \text{P}\begin{pmatrix}\text{scores are 10, 10, 10} \\ \text{and of diff suits}\end{pmatrix}}{\frac{169}{425}}$$

$$= \frac{\binom{4}{3}\binom{3}{1}\left(\frac{4}{52}\times\frac{4}{51}\times\frac{1}{50}\times 3!\right) + \binom{4}{3}\left(\frac{4}{52}\times\frac{4}{51}\times\frac{4}{50}\times 3!\right)}{\frac{169}{425}}$$

$$= \frac{\frac{48}{5525}+\frac{64}{5525}}{\frac{169}{425}} = \frac{\frac{112}{5525}}{\frac{169}{425}} = \frac{112}{2197} \qquad \text{OR} \qquad \frac{\left(\frac{16}{52}\times\frac{12}{51}\times\frac{8}{50}\right)+\left(\frac{16}{52}\times\frac{12}{51}\times\frac{2}{50}\times\frac{3!}{2!}\right)}{\frac{169}{425}}$$

Method 2

P(total score of 3 cards is more than 28 points given they are of different suits)

$$= \frac{\left[\binom{4}{3}\binom{3}{1}\binom{4}{1}^2\binom{1}{1}\right]+\binom{4}{3}\binom{4}{1}^3}{\binom{4}{3}\binom{13}{1}^3} = \frac{112}{2197}$$

THE END