RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Tutorial S1B: Probability

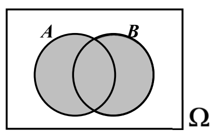
1 It is given that events A and B are independent and $\mathrm{P}(A\cup B)=\frac{5}{8}$, $\mathrm{P}(A\cap B')=\frac{7}{24}$.

Calculate **(i)** $\mathrm{P}(B)$, **(ii)** $\mathrm{P}(A)$, **(iii)** $\mathrm{P}(A\cap B)$, **(iv)** $\mathrm{P}(A'\cup B')$.

[**(i)** $\frac{1}{3}$ **(ii)** $\frac{7}{16}$ **(iii)** $\frac{7}{48}$ **(iv)** $\frac{41}{48}$]

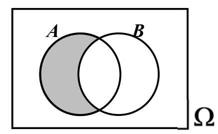
Solution

$A\cup B$



$\mathrm{P}(A\cup B)=\frac{5}{8}$

$A\cap B'$



$\mathrm{P}(A\cap B')=\frac{7}{24}$

(i) $\mathrm{P}(B)=\mathrm{P}(A\cup B)-\mathrm{P}(A\cap B')$ from diagram

$=\frac{5}{8}-\frac{7}{24}=\frac{1}{3}$

(ii) Since A and B are independent, A and B' are independent

$\mathrm{P}(A\cap B')=\mathrm{P}(A)\mathrm{P}(B')$

So $\mathrm{P}(A)=\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}=\dfrac{\frac{7}{24}}{1-\frac{1}{3}}=\frac{7}{16}$

OR

$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

$=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A)\mathrm{P}(B)$ since A and B are independent events

$\frac{5}{8}=\mathrm{P}(A)+\frac{1}{3}-\frac{1}{3}\mathrm{P}(A)$

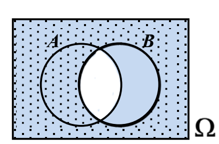
$\mathrm{P}(A)=\frac{7}{16}$

(iii) Since A and B are independent events,

$\mathrm{P}(A\cap B)=\mathrm{P}(A)\mathrm{P}(B)=\frac{7}{16}\times\frac{1}{3}=\frac{7}{48}$

(iv) $\mathrm{P}(A'\cup B')=1-\mathrm{P}(A\cap B)$

$=1-\frac{7}{48}=\frac{41}{48}$



2 **SAJC Prelim 9740/2010/02/Q7a**

A & B are two events with non-zero probability. Explain if each of the following statements is necessarily true, necessarily false, or neither necessarily true nor necessarily false.

(i) If A & B are mutually exclusive, then they are independent. **[1]**

(ii) If A & B are independent, then they are mutually exclusive. **[1]**

Solution

(i) If A & B are mutually exclusive, then $P(A \cap B) = 0$

But since $\mathrm{P}(A) \neq 0$ and $\mathrm{P}(B) \neq 0$, $P(A \cap B) = 0 \neq \mathrm{P}(A)\mathrm{P}(B)$

i.e. A and B are not independent.

Statement is necessarily false.

OR

$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)} = 0$ (since A and B are mutually exclusive)

If A and B are independent, then $\mathrm{P}(A \mid B) = \mathrm{P}(A) = 0$

contradiction!!

(ii) If A & B are independent, then $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B) > 0$

since $\mathrm{P}(A) > 0,\ \mathrm{P}(B) > 0.$

Hence, $\mathrm{P}(A \cap B) > 0 \Rightarrow A \cap B \neq \varnothing.$

$\therefore$ A and B are not mutually exclusive.

Statement is necessarily false.

3 DHS JC2 Prelim 9758/2019/02/Q7

In a school survey, a group of 80 students are asked about how much time per week (to nearest hour) they spend on their co-curricular activities (CCA). The readings are shown below:

	CCA (hours)		
	3 or less	**4 to 6**	**7 or more**
Boy	17	20	10
Girl	18	$15-k$	k

A student is selected random from the group. Defining the events as follows:

G : The student is a girl.
L : The student spends 6 hours or less weekly.
M : The student spends 4 hours or more weekly.

Find the following probabilities in terms of k.

(i) $\mathrm{P}(L' \cup M')$ **[2]**

(ii) $\mathrm{P}(G \mid L')$ **[1]**

(iii) Given that $\mathrm{P}(L \cap M) = \frac{2}{5}$, find the value of k. Hence determine if L and M are independent, justifying your answer. **[3]**

(iv) If the events G and $(L \cap M)$ are mutually exclusive, find the value of k. **[1]**

[(i) $\frac{45+k}{80}$ **(ii)** $\frac{k}{k+10}$ **(iii)** $k=3$; not independent **(iv)** $k=15$ **]**

Solution

(i)

$$\mathrm{P}(L' \cup M') = \frac{80 - n(L \cap M)}{80}$$

4 to 6 hours

$$= \frac{80-(35-k)}{80} = \frac{45+k}{80}$$

OR

$$\mathrm{P}(L' \cup M') = \mathrm{P}(L') + P(M') - \mathrm{P}(L' \cap M')$$

$$= \frac{10+k}{80} + \frac{35}{80} - 0$$

$$= \frac{45+k}{80}$$

(ii) $$\mathrm{P}(G \mid L') = \frac{P(G \cap L')}{P(L')} = \frac{k}{k+10}$$

(iii) Given $\mathrm{P}(L \cap M) = \frac{2}{5}$

From table: $\mathrm{P}(L \cap M) = \dfrac{20+(15-k)}{80} = \dfrac{35-k}{80}$

Solving, we get $k = 3$

$\mathrm{P}(L)\mathrm{P}(M) = \dfrac{67}{80} \times \dfrac{45}{80} = \dfrac{603}{1280} \neq \dfrac{2}{5}$

Since $\mathrm{P}(L \cap M) \neq \mathrm{P}(L)\mathrm{P}(M)$, L and M are not independent.

OR

$\mathrm{P}(L) = \dfrac{70-k}{80} = \dfrac{67}{80}$ and $\mathrm{P}(L \mid M) = \dfrac{35-k}{45} = \dfrac{32}{45} \neq \dfrac{67}{80}$

Since $\mathrm{P}(L) \neq \mathrm{P}(L \mid M)$, L and M are not independent.

(iv) Since $\mathrm{P}(G \cap (L \cap M)) = 0$

$\Rightarrow 15 - k = 0 \quad \therefore k = 15$

4 **9205/2003/02/Q11**

Hari, a car salesman, has found that he can make a sale to 65% of his male customers but to only 45% of his female customers. All of Hari's sales are independent. On Tuesday morning Hari has two male customers and one female customer. Find the probability that Hari makes exactly two sales. **[4]**

60% of Hari's customers are male. Find the probability that, on Wednesday morning, Hari makes a sale to his first customer. **[3]**

Find the probability that, on Thursday morning, Hari makes his first sale to his fourth customer. **[3]**

For a randomly chosen customer, find the probability that the customer is female given that Hari makes a sale to that customer. **[4]**

[0.437 125 ; 0.57 ; 0.0453; $\frac{6}{19}$]

Solution

P (makes exactly 2 sales)

=P(sale to both males, not female) + P(sale to female and only one out of two males)

$= (0\cdot 65)^2(0\cdot 55) + (0\cdot 45)(0\cdot 65)(0\cdot 35)2! = 0.437\ 125$

P(sale to first customer) $= (0.6)(0.65) + (0.4)(0.45) = 0.57$

P(first sale to fourth customer)

$= (1-0.57)^3(0.57)$

$= (0.43)^3(0.57)$

$= 0.0453$ (3sf)

P(customer is female | sale to the customer)

$$= \frac{\text{P(customer is female and Hari makes a sale)}}{\text{P(Hari makes a sale)}}$$

$$= \frac{(0.4)(0.45)}{0.57} = \frac{6}{19}$$

5 9758/2020/02/Q8

In a game, a computer randomly chooses 12 shapes from 11 circles and 17 rectangles. The number of rectangles chosen is denoted by R.

(i) Show that $\mathrm{P}(R = 1) < \mathrm{P}(R = 2)$. **[2]**

The number of rectangles available is now increased by r. The computer randomly chooses 12 shapes from the 11 circles and $(17 + r)$ rectangles. The probability that 4 rectangles are chosen is now 15 times the probability that 3 rectangles are chosen.

(ii) Find the value of r. **[5]**

[**(ii)** 6]

Solution

(i)

$$\mathrm{P}(R=1)=\frac{\binom{17}{1}\binom{11}{11}}{\binom{28}{12}}=\frac{17}{30421755}=\frac{1}{1789515}$$

$$\mathrm{P}(R=2)=\frac{\binom{17}{2}\binom{11}{10}}{\binom{28}{12}}=\frac{17\times16\times11}{2\times30421755}=\frac{1496}{30421755}=\frac{88}{1789515}>\frac{1}{1789515}=\mathrm{P}(R=1)$$

(ii)

$$\frac{\binom{17+r}{4}\binom{11}{8}}{\binom{28+r}{12}}=15\times\frac{\binom{17+r}{3}\binom{11}{9}}{\binom{28+r}{12}}$$

$$\Rightarrow \binom{17+r}{4}\binom{11}{8}=15\binom{17+r}{3}\binom{11}{9}$$

$$\Rightarrow \frac{(17+r)(16+r)(15+r)(14+r)}{1\times2\times3\times4}\times165=15\times\frac{(17+r)(16+r)(15+r)}{1\times2\times3}\times55$$

$$\Rightarrow 14+r=20$$

$$\Rightarrow r=6$$

6 **9740/2009/02/Q7**

A company buys p % of its electronic components from supplier A and the remaining $(100-p)$ % from supplier B. The probability that a randomly chosen component supplied by A is faulty is 0.05. The probability that a randomly chosen component supplied by B is faulty is 0.03.

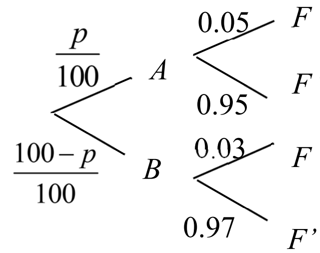
(i) Given that $p = 25$, find the probability that a randomly chosen component is faulty. **[2]**

(ii) For a general value of p, the probability that a randomly chosen component that is faulty was supplied by A is denoted by f (p). Show that $\mathrm{f}(p) = \dfrac{0.05p}{0.02p+3}$.

Prove by differentiation that f is an increasing function for $0 \le p \le 100$, and explain what this statement means in the context of the question. **[6]**

[0.035]

Solution



(i) P(component is faulty) $= \mathrm{P}(A \cap F) + \mathrm{P}(B \cap F)$

$= 0.25 \times 0.05 + 0.75 \times 0.03$

$= 0.035$

(ii) f(p)

= P(supplied by A | component is faulty)

$$= \frac{\text{P(component is supplied by A and is faulty)}}{\text{P(component is faulty)}}$$

$$= \frac{\frac{p}{100} \times 0.05}{\frac{p}{100} \times 0.05 + \frac{100-p}{100} \times 0.03}$$

$$= \frac{0.05p}{0.05p + 3 - 0.03p}$$

$$= \frac{0.05p}{0.02p+3} \quad \text{(shown)}$$

$$\mathrm{f}'(p) = \frac{(0.02p+3)0.05 - 0.05p(0.02)}{(0.02p+3)^2} = \frac{0.15}{(0.02p+3)^2} > 0 \text{ for all } 0 \le p \le 100.$$

Hence f is an increasing function for $0 \le p \le 100$. (shown)

This means that as the company buys more components from supplier A, the probability that a randomly chosen component that is faulty was supplied by supplier A increases.

NOTE:

The question requires the candidate to "prove by differentiation". It would be wrong to sketch a graph to show that it is increasing.

7 **MI PU3 9758/2021/02/Q7**

Two fair six-sided dice are thrown in a random experiment. For the first die, the six faces are numbered 1, 2, 3, 4, 5 and 6. For the second die, the six faces are numbered 2, 3, 4, 5, 6 and 7. The score is the sum of the numbers shown on the two dice if the two numbers are different. If the two numbers are the same, then the score is one more than the sum.

Define the following events:

A: The score obtained is 13.
B: The score obtained is at least 10.
C: The score obtained is divisible by 3.

(i) Find $\mathrm{P}(A)$. **[1]**

(ii) Determine if the events A and B are independent, showing your working clearly. **[2]**

(iii) Identify two events from events A, B and C that are mutually exclusive, explaining your answer clearly. **[2]**

(iv) Construct an event D, an outcome of the above random experiment, such that $\mathrm{P}(D \mid C) = \frac{1}{11}$, justifying your answer clearly. **[3]**

[**(i)** $\frac{1}{18}$ **(ii)** not independent]

Solution

Score	**1**	**2**	**3**	**4**	**5**	**6**
2	3	5	5	6	7	8
3	4	5	7	7	8	9
4	5	6	7	9	9	10
5	6	7	8	9	11	11
6	7	8	9	10	11	13
7	8	9	10	11	12	13

(i) From the table, $\mathrm{P}(A) = \frac{2}{36} = \frac{1}{18}$.

(ii) From the table, $\mathrm{P}(B) = \frac{10}{36} = \frac{5}{18}$ and $\mathrm{P}(A \cap B) = \frac{2}{36} = \frac{1}{18}$.

1) $\mathrm{P}(A) \times \mathrm{P}(B) = \frac{1}{18} \times \frac{5}{18} = \frac{5}{324}$

Since $\mathrm{P}(A \cap B) \neq \mathrm{P}(A) \times \mathrm{P}(B)$,
A and B are not independent.

Similar but easier alternative:
$A \subseteq B \Rightarrow A \cap B = A$
So $\mathrm{P}(A \cap B) = \mathrm{P}(A)$
Since $\mathrm{P}(B) < 1, \mathrm{P}(A \cap B) \neq \mathrm{P}(A) \times \mathrm{P}(B)$

2) Alternative:
$\mathrm{P}(B \mid A)=1$ since score is 13 (event A) implies score is at least 10 (event B)
Since $\mathrm{P}(B) \neq 1$, $\mathrm{P}(B \mid A) \neq \mathrm{P}(B)$, A and B are not independent

(iii) Since 13 (outcome of A) is not divisible by 3 (outcome of C), A and C are mutually exclusive.

(iv) Method 1:
Since $\mathrm{P}(D|C) = \dfrac{\mathrm{P}(C \cap D)}{\mathrm{P}(C)} = \dfrac{1}{11}$ and $\mathrm{P}(C) = \dfrac{11}{36}$,
$\mathrm{P}(C \cap D) = \dfrac{1}{11}\,\dfrac{11}{36} = \dfrac{1}{36}$.
Hence, D can be the event of obtaining a score of 3 OR 12.

Method 2:
There are 11 outcomes in event C.
For $\mathrm{P}(D|C) = \dfrac{1}{11}$, event D must have exactly one outcome from event C. Hence, D can be the event of obtaining a score of 3 OR 12.

8 **9233/2000/02/Q11**

In a probability experiment, three containers have the following contents.

A jar contains 2 white dice and 3 black dice.
A white box contains 5 red balls and 3 green balls.
A black box contains 4 red balls and 3 green balls.

One die is taken at random from the jar. If the die is white, two balls are taken from the white box, at random and without replacement. If the die is black, two balls are taken from the black box, at random and without replacement. Events W and M are defined as follows.

W : A white die is taken from the jar.
M : One red ball and one green ball are obtained.

Show that $\mathrm{P}(M \mid W) = \dfrac{15}{28}$. **[2]**

Find, giving each of your answers as an exact fraction in its lowest terms,

(i) $\mathrm{P}(M \cap W)$, **(ii)** $\mathrm{P}(W \mid M)$, **(iii)** $\mathrm{P}(W \cup M)$. **[1][5][3]**

All the dice and balls are now placed in a single container, and four objects are taken at random, each object being replaced before the next one is taken. Find the probability that one object of each colour is obtained. **[3]**

[**(i)** 3/14, **(ii)** 5/13, **(iii)** 26/35, 243/5000]

Solution

$\mathrm{P}(M|W)$ = P(obtain red and green ball | white dice is taken from jar)

= P(obtain red and green ball | white box is picked)

= P(obtain red and green ball from white box)

$= \dfrac{{}^5C_1 \times {}^3C_1}{{}^8C_2}$ or $\left(\dfrac{5}{8}\right)\left(\dfrac{3}{7}\right)2!$

$= \dfrac{15}{28}$ (shown)

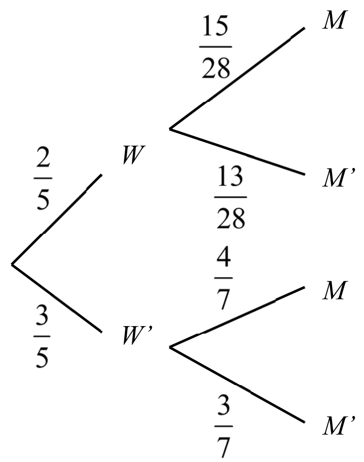
$\mathrm{P}(M|W')$ = P(obtain red and green ball from black box)

$= \dfrac{{}^4C_1 \times {}^3C_1}{{}^7C_2}$ or $\left(\dfrac{4}{7}\right)\left(\dfrac{3}{6}\right)2!$

$= \dfrac{4}{7}$

A jar contains 2 white dice and 3 black dice.
A white box contains 5 red balls and 3 green balls.
A black box contains 4 red balls and 3 green balls.
W : A white die is taken from the jar.
M : One red ball and one green ball are obtained.

We found $\mathrm{P}\left(M \mid W\right)=\dfrac{15}{28}$ and $\mathrm{P}\left(M \mid W'\right)=\dfrac{4}{7}$



A jar contains 2 white dice and 3 black dice.
A white box contains 5 red balls and 3 green balls.
A black box contains 4 red balls and 3 green balls.
W : A white die is taken from the jar.
W : One red ball and one green ball are obtained.

(i) $\mathrm{P}\left(M \cap W\right)=\left(\dfrac{2}{5}\right)\left(\dfrac{15}{28}\right)=\dfrac{3}{14}$

(ii) $\mathrm{P}\left(M\right)=\mathrm{P}\left(M \cap W\right)+\mathrm{P}\left(M \cap W'\right)$

$$=\frac{3}{14}+\left(\frac{3}{5}\right)\left(\frac{4}{7}\right)=\frac{39}{70}$$

$$\mathrm{P}\left(W \mid M\right)=\frac{\mathrm{P}\left(M \cap W\right)}{\mathrm{P}\left(M\right)}=\frac{3/14}{39/70}=\frac{5}{13}$$

(iii) $\mathrm{P}\left(W \cup M\right)=1-\mathrm{P}\left(W' \cap M'\right)=1-\left(\dfrac{3}{5}\right)\left(\dfrac{3}{7}\right)=\dfrac{26}{35}$

OR:

$$\mathrm{P}\left(W \cup M\right)=\mathrm{P}\left(W\right)+\mathrm{P}\left(M\right)-\mathrm{P}\left(W \cap M\right)$$

$$=\frac{2}{5}+\frac{39}{70}-\frac{3}{14}=\frac{26}{35}.$$

When all the dice and balls are placed in a single container, there are a total of 20 objects, which consist 2 white dice, 3 black dice, 9 red balls and 6 green balls

P(one object of each colour picked, out of the 4 objects drawn with replacement)

$$=\frac{2}{20}\times\frac{3}{20}\times\frac{9}{20}\times\frac{6}{20}\times 4!$$

$$=\frac{243}{5000}$$

9 **IJC Prelim 9740/2010/02/Q7**

Box A contains 6 balls numbered 1, 2, 2, 2, 5, 7.
Box B contains 4 balls numbered 1, 4, 4, 7.
Box C contains 3 balls numbered 3, 4, 6.

One ball is removed at random from each box.

(i) Show that the probability that each of the three numbers obtained is greater than 3 is $\frac{1}{6}$. **[1]**

(ii) Find the probability that each of the three numbers obtained is greater than 3 or the sum of the three numbers obtained is 13 (or both). **[4]**

All the balls are now placed in a single container.

A game is played by a single player. The player takes balls, one by one and with replacement, from the container, continuing until either a number 1 results or a prime number results. The player wins if the number on the last ball chosen is 1 and loses otherwise. Find the probability that a player wins a particular game. **[3]**

[**(i)** $\frac{1}{6}$ **(ii)** $\frac{5}{24}$; $\frac{2}{9}$]

Solution

(i) $\text{P(all are greater than 3)} = \left(\frac{2}{6}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{1}{6}$

(ii) Let X be the event where each of the 3 numbers is greater than 3, and
Let Y be the event where sum of the 3 numbers is equal to 13.

$$\begin{aligned}\mathrm{P}(Y) &= \mathrm{P}((2,7,4),(5,4,4)) \\ &= \left(\frac{3}{6}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{6}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) \\ &= \frac{5}{72}\end{aligned}$$

$$\begin{aligned}\mathrm{P}(X\cap Y) &= \mathrm{P}\big((5,4,4)\big) \\ &= \left(\frac{1}{6}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) \\ &= \frac{2}{72}\end{aligned}$$

$$\begin{aligned}\text{Required probability} &= \mathrm{P}(X\cup Y) \\ &= \mathrm{P}(X)+\mathrm{P}(Y)-\mathrm{P}(X\cap Y) \\ &= \frac{1}{6}+\frac{5}{72}-\frac{2}{72} \\ &= \frac{5}{24}\end{aligned}$$

All the balls are now placed in a single container.

x	1	2	3	4	5	6	7
No. of balls labeled x	2	3	1	3	1	1	2

For the player to win the game, the player has to pick a ball labelled "1". There are 2 balls labelled "1".

For the game to continue, the player has to pick a ball labelled "4" or "6" (i.e not "1" or not prime). There are 4 balls labelled "4" or "6".

$$\begin{aligned}\text{P(a player wins a particular game)} &= \frac{2}{13}+\left(\frac{4}{13}\right)\left(\frac{2}{13}\right)+\left(\frac{4}{13}\right)^2\left(\frac{2}{13}\right)+\ldots \\ &= \frac{2}{13}\left(1+\frac{4}{13}+\left(\frac{4}{13}\right)^2+\ldots\right) \\ &= \frac{2}{13}\left(\frac{1}{1-\frac{4}{13}}\right) \\ &= \frac{2}{9}\end{aligned}$$

OR P(a player wins) $=\frac{2}{13}+\left(\frac{4}{13}\right)$ P(a player wins)

Re-arrange and done.

OR "Reduced sample space approach":

P(player wins) = P(ball drawn is "1" | ball drawn is either a "1" or "prime") $=\frac{2}{9}$

10 9758/2018/02/Q7

The events A, B and C are such that $\mathrm{P}(A) = a$, $\mathrm{P}(B) = b$ and $\mathrm{P}(C) = c$. A and B are independent events. A and C are mutually exclusive events.

(i) Find an expression for $\mathrm{P}(A' \cap B')$ and hence prove that A' and B' are independent events. **[2]**

(ii) Find an expression for $\mathrm{P}(A' \cap C')$. Draw a Venn diagram to illustrate the case when A' and C' are also mutually exclusive events. (You should not show event B on your diagram). **[2]**

You are now given that A' and C' are **not** mutually exclusive, $\mathrm{P}(A) = \frac{2}{5}$, $\mathrm{P}(B \cap C) = \frac{1}{5}$ and $\mathrm{P}(A' \cap B' \cap C') = \frac{1}{10}$.

(iii) Find exactly the maximum and minimum possible values of $\mathrm{P}(A \cap B)$. **[4]**

[**(i)** $(1-a)(1-b)$ **(ii)** $1-a-c$ **(iii)** $\frac{1}{3}, \frac{2}{15}$]

Solution

(i)

$$\begin{aligned}\mathrm{P}(A' \cap B') &= 1 - \mathrm{P}(A \cup B) \\ &= 1 - \left[\mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)\right] \\ &= 1 - \left[a + b - ab\right] \text{ since } A \text{ and } B \text{ are independent} \\ &= (1-a) - b(1-a) \\ &= (1-a)(1-b) \\ &= \mathrm{P}(A')\mathrm{P}(B')\end{aligned}$$

$\therefore$ A' and B' are independent.

(ii) A and C are mutually exclusive

$\Rightarrow \mathrm{P}(A' \cap C') = 1 - \mathrm{P}(A) - \mathrm{P}(C) = 1 - a - c$

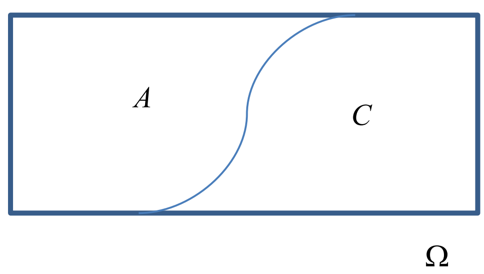
A' and C' are also mutually exclusive

$\Rightarrow \mathrm{P}(A' \cap C') = 0$

$\Rightarrow 1 - a - c = 0$

$\Rightarrow 1 - (a + c) = 0 \Rightarrow a + c = 1$

$\Rightarrow A \cup C = \Omega$



(iii)

$\mathrm{P}(A' \cap B' \cap C') = \frac{1}{10}$

$\Rightarrow 1 - \mathrm{P}(A \cup B \cup C) = \frac{1}{10}$

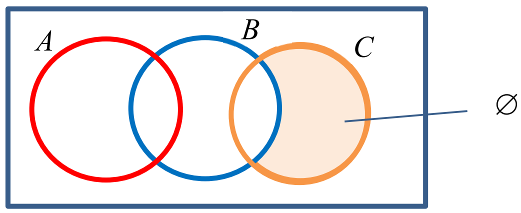
$\Rightarrow \mathrm{P}(A \cup B \cup C) = \frac{9}{10} \cdots (1)$

$\mathrm{P}(A \cap B) = ab$ ($\because A, B$ are independent)

$$= \frac{2}{5}b \quad \cdots(2)$$

<u>Method 1:</u>

Maximum value of $\mathrm{P}(A \cap B)$ occurs when b is maximum and c is minimum, ie when $C \subseteq B$.

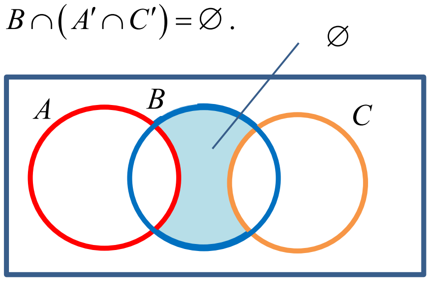


$$\mathrm{P}(A \cup B \cup C) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B) = \frac{2}{5} + b - \frac{2}{5}b = \frac{9}{10}$$

$$\Rightarrow b = \frac{1}{2}\left(\frac{5}{3}\right) = \frac{5}{6}$$

So maximum value of $\mathrm{P}(A \cap B) = \frac{2}{5}\left(\frac{5}{6}\right) = \frac{1}{3}$

Minimum value of $\mathrm{P}(A \cap B)$ occurs when b is minimum and c is maximum, ie when $B \cap \left(A' \cap C'\right) = \varnothing$.

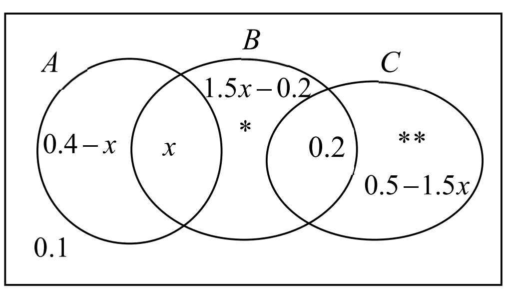


So $\mathrm{P}(B) = \mathrm{P}(A \cap B) + \mathrm{P}(B \cap C) \Rightarrow b = \frac{2}{5}b + \frac{1}{5} \Rightarrow b = \frac{1}{3}$

Minimum value of $\mathrm{P}(A \cap B) = \frac{2}{5}\left(\frac{1}{3}\right) = \frac{2}{15}$

<u>Method 2:</u>

Let $\mathrm{P}\left(A \cap B\right) = x$. From (2), $b = \frac{5}{2}x$.



(*): $\frac{5}{2}x - x - 0.2$

(**): $0.9 - [0.4 + \frac{5}{2}x - x]$

Since all the probabilities are nonnegative, we have

$x \geq 0$, $0.4 - x \geq 0$, $1.5x - 0.2 \geq 0$ and $0.5 - 1.5x \geq 0$.

Thus we have $x \geq 0$, $x \leq 0.4$, $x \geq \frac{2}{15}$ and $x \leq \frac{1}{3}$.

Combining all together we have $\frac{2}{15} \leq x \leq \frac{1}{3}$.

Minimum and maximum value of $\mathrm{P}(A \cap B)$ is $\frac{2}{15}$ and $\frac{1}{3}$ respectively.