**RAFFLES INSTITUTION**
**H2 Mathematics (9758)**
**2025 Year 6**

# Additional Practice Questions for Chapter S1A: Permutations and Combinations (Solutions)

**1** From a group of 5 women and 7 men, one of whom is Mr Lee, find how many committees of size 4 can be formed in which

**(i)** there are 2 men and 2 women,

**(ii)** there is at least 1 man and at least 1 woman,

**(iii)** there is at least 1 man and Mr Lee is in the committee.

**[ (i)** 210; **(ii)** 455; **(iii)**165]

**Solution**

**(i)** Number of committees with 2 men and 2 women = ${}^{7}C_{2} \times {}^{5}C_{2} = 210$.

**(ii)** Using the Complement Method, number of committees with at least 1 man and at least 1 woman = (Number of committees without restriction) – (Number of all men committees) – (Number of all women committees) = ${}^{12}C_{4} - {}^{7}C_{4} - {}^{5}C_{4} = 455$

Note : ${}^{5}C_{1} \times {}^{7}C_{1} \times {}^{10}C_{2}$ is not correct. There is double counting.

For eg, $W_1 \; M_1 \quad W_2 \; M_2$ and

$W_2 \; M_1 \quad W_1 \; M_2$

are the same selection but counted as 2 different ways in the above computation

[You may want to refer to Example 11 of lecture notes]

**(iii)** Since the committee must consist of at least one man and includes Mr Lee, we choose 3 people from the remaining 11 people. Number of ways = ${}^{11}C_{3} = 165$

**2** 4 boys, 4 girls and a teacher are to be seated at a round table. How many ways can they be arranged if

**(i)** there is no restriction?

**(ii)** the teacher is to be seated between any 2 girls?

**(iii)** none of the boys are to be seated together?

**[ (i)** 40320; **(ii)** 8640; **(iii)** 2880]

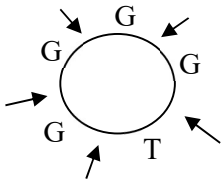
**Solution**

**(i)** Number of ways with no restriction = (9 – 1)! = 40320.

**(ii)** The teacher can be seated between any 2 girls in ${}^{4}C_{2} \times 2! = 12$ ways. Consider the teacher between any 2 girls as one unit:

Number of ways = $12 \times (7 - 1)! = 8640$.

**(iii)** We first seat the teacher and 4 girls. Number of ways = (5 – 1)! = 24.




Using the Slotting Method, number of ways such that none of the boys are seated together = $24 \times {}^5P_4 = 2880$

**3** How many 6-digit numbers

**(i)** are even?
**(ii)** begin and end with different digits?

**[(i)** 450000; **(ii)** 810000]

**Solution**

**(i)** Number of choices for the first digit = 9.
Number of choices for each of the 2nd, 3rd, 4th and 5th digit = 10.
For the 6-digit number to be even, number of choices for the last digit = 5.
Total number of 6-digit numbers that are even = $9 \times 10^4 \times 5 = 450000$.

**(ii)** Number of choices for the first digit = 9.
Since the first and last digits are different, number of choices for the last digit = (10 – 1) = 9.
Number of choices for each of the 2nd, 3rd, 4th and 5th digit = 10.
Total number of 6-digit numbers that begin and end with different digits
$= 9 \times 9 \times 10^4 = 810000$

**4** A rectangular table has 7 secured seats, 4 being on one side facing the window and 3 being on the opposite side. In how many ways can 7 people be seated at the table

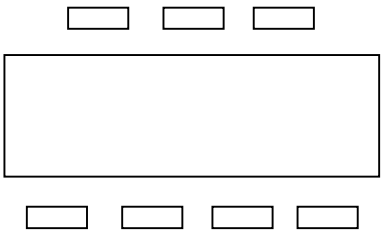
**(i)** if 3 people, X and Y and Z must sit on the side facing the window?

**(ii)** if 2 people, P and Q must sit on opposite sides?

**[ (i)** 576; **(ii)** 2880**]**

**Solution**

**WINDOW**



**(i)** if 3 people, X and Y and Z must sit on the side facing the window
We choose one more person to sit on the same side as X, Y, Z
Number of choices = ${}^4C_1 = 4$
The people in each row can be permuted within the row.

Total number of arrangements = 4 x 4! x 3! = 576

Altenative 1:
Number of ways to arrange 3 out of the remaining 4 people to sit on the side near the window = ${}^4P_3$
Number of ways to arrange X, Y, Z and the 4th person = 4!

Total number of arrangements = ${}^4P_3 \times 4! = 576$

Alternative 2:
Number of ways to choose 3 seats for X, Y, Z , and arrange them = ${}^4C_3 \times 3!$
Number of ways to arrange the remaining 4 people = 4!

Total number of arrangements = ${}^4C_3 \times 3! \times 4! = 576$

**(ii)** if 2 people, P and Q must sit on opposite sides

Case 1 : P is on the side with 4 seats, Q on the side with 3 seats.
Just like in **(i)**, we choose 3 people to sit with P (or 2 people to sit with Q), and permute within the rows.
Number of choices $= {}^5C_3 \times 4! \times 3!$

Case 2 : Q is on the side with 4 seats and P is on the side with 3 seats.
Number of choices is as above.

Hence, total number of choices $= {}^5C_3 \times 4! \times 3! \times 2 = 2880$

Alternative method:

Case 1: P sits on the side with 4 seats
Number of ways to choose a seat for P = ${}^4C_1$
Number of ways to choose a seat for Q = ${}^3C_1$
Number of ways to arrange the remaining 5 people = 5!

Number of ways $= {}^4C_1 \times {}^3C_1 \times 5!$

Case 2: Q sits on the side with 4 seats

Number of ways is as above.

Total number of ways $= {}^4C_1 \times {}^3C_1 \times 5! \times 2 = 2880$

**5 9205/1989/01/Q19(b)**

A school is asked to send a delegation of six pupils selected from six badminton players, six tennis players and five squash players. No pupil plays more than one game. The delegation is to consist of at least one, and not more than three, players drawn from each sport. Giving full details of your working, find the number of ways in which the delegation can be selected. **[**9450**]**

**Solution**

We use Systematic Listing and consider 7 cases.

| Number of Badminton players(6) | Number of Tennis players (6) | Number of Squash players (5) | Number of ways to select the 6 players |
|---|---|---|---|
| 1 | 2 | 3 | ${}^6C_1 \times {}^6C_2 \times {}^5C_3 = 900$ |
| 1 | 3 | 2 | ${}^6C_1 \times {}^6C_3 \times {}^5C_2 = 1200$ |
| 2 | 1 | 3 | ${}^6C_2 \times {}^6C_1 \times {}^5C_3 = 900$ |
| 2 | 3 | 1 | ${}^6C_2 \times {}^6C_3 \times {}^5C_1 = 1500$ |
| 3 | 1 | 2 | ${}^6C_3 \times {}^6C_1 \times {}^5C_2 = 1200$ |
| 3 | 2 | 1 | ${}^6C_3 \times {}^6C_2 \times {}^5C_1 = 1500$ |
| 2 | 2 | 2 | ${}^6C_2 \times {}^6C_2 \times {}^5C_2 = 2250$ |

Total number of ways = 900 + 1200 + 900 + 1500 + 1200 + 1500 + 2250 = 9450.

**6 AJC Prelim 9233/2003/01/Q7**

Find the number of distinct arrangements of the letters of the word '*THERMOMETER*'

**(i)** if at least 2 '*E*'s are together,

**(ii)** which must start and end with '*T*' or '*R*'.

**[(i)** 408240; **(ii)** 90720**]**

**Solution**

**(i)** There are 11 letters, with 3 'E's, 2 'T's, 2 'M's and 2 'R's, 1 'O', 1 "H"

Use the Complement Method:

Number of ways without restriction = $\frac{11!}{3!2!2!2!} = 831600.$

↑ T ↑ H ↑ R ↑ M ↑ O ↑ M ↑ T ↑ R ↑

Using the Slotting Method:

Number of ways to arrange the 2T, 2M, 2R, 1O, 1H = $\frac{8!}{2!2!2!} = 5040$

The 3 'E's can occupy 3 out of the remaining 9 slots in ${}^9C_3 = 84$ ways.

Number of ways such that the 3 'E's are always separated = 5040 × 84 = 423360.

Therefore, using the Complement Method, number of ways such that at least 2 'E's are together = 831600 – 423360 = 408240.

**(ii)** Case 1: The arrangement starts and ends with 'T'

Number of ways = $\frac{9!}{3!2!2!} = 15120$.

Case 2: The arrangement starts and ends with 'R'

A similar argument as Case 1 gives number of ways = 15120.

Case 3: The arrangement starts with 'T' and ends with 'R'

Number of ways = $\frac{9!}{3!2!} = 30240$.

Case 4: The arrangement starts with 'R' and ends with 'T'

A similar argument as Case 3 gives number of ways = 30240.

Total number of ways = $(15120 \times 2) + (30240 \times 2) = 90720$.

**7** How many 4-digit numbers greater than 5000 can be formed from the digits 0, 1, 2, 3, 4, 5 if

**(i)** no digit may be repeated?

**(ii)** any digit may be repeated?

**(iii)** only the digit 4 may be repeated?

[**(i)** 60; **(ii)** 215; **(iii)** 73]

**Solution**

**(i)** Since the first digit has to be the digit '5', we need to choose 3 digits from the digits 0,1,2,3,4. Since the order matters, number of ways = ${}^5P_3 = 60$.

**(ii)** Number of choices for the first digit = 1
Since any digit may be repeated, number of choices for each of the 2nd, 3rd and 4th digits= 6
Since we want a 4-digit number **greater** than 5000, we need to subtract one number, which is 5000 itself. Therefore number of ways = $(1 \times 6^3) - 1 = 215$.

**(iii)** There is only 1 choice for the first digit. We shall consider cases for the last 3 digits.

Case 1: The last 3 digits are all different

From **(i)**, number of ways = ${}^5P_3 = 60$.

Case 2: Two of the last 3 digits are the digit '4's
There are ${}^4C_1 = 4$ choices for the remaining digit (0, 1, 2 or 3). Since the order matters and there are 2 identical digits, number of ways = $4 \times \frac{3!}{2!} = 12$.

Case 3: The last 3 digits are all digit '4's

Number of ways = 1.

Total number of ways = 60 + 12 + 1 = 73

**8** A photographer is positioning 5 men and 4 women for a photo shoot. Find the number of ways the photographer can position them in a row

**(i)** if no two people of the same sex are to stand next to each other,
**(ii)** if the photographer positions the men in the order from shortest on the left to tallest on the right. (Assume that all the men are of different heights and are not necessarily standing together in a group.)

**[(i)** 2880; **(ii)** 3024**]**

**Solution**

**(i)** The only possible arrangement is M W M W M W M W M.
Number of ways to arrange the men = 5! = 120
Number of ways to arrange the women = 4! = 24
Thus, number of ways = 120 × 24 = 2880.

**(ii)** Number of ways to choose 4 out of the 9 positions for the 4 women and arrange the 4 women = ${}^{9}P_4 = 3024$.
Since the men are to stand in a fixed order (from shortest on the left to tallest on the right), number of ways to arrange the 5 men in the 5 remaining positions= 1
Therefore, number of ways = 3024 × 1 = 3024.

**9 SAJC Prelim 9740/2012/02/Q6**

A box contains 9 balls. Out of these 9 balls, there are 3 identical red balls, 2 identical yellow balls and 4 numbered green balls (each labelled with a different number from 1 to 4). 3 balls are to be picked out of the box, and the order in which they are picked out does not matter. Find the number of possible selections of 3 balls. **[3]**

**[**31**]**

**Solution**

3 identical red balls, 2 identical yellow balls, 4 numbered green balls
Equivalent to 6 types of distinct balls
Case 1: All three balls distinct

Number of possible selections of 3 balls is ${}^{6}C_3$
Case 2: Exactly two identical balls (AAB)

Number of possible selections = ${}^{2}C_1 \times {}^{5}C_1$ (2 types of balls available to choose the 2 identical balls from, then 5 types of balls remaining to choose the last ball)
Case 3: All three balls are identical (AAA).
Number of possible selections = 1

∴ Total number of possible selections is 20+10+1=31

**10** A painter is given a job to paint the doors of 5 adjacent rooms. Given that he has 5 colours of paint; red, blue, green, yellow and purple, find the number of ways he could accomplish his task if no adjacent doors are to be painted the same colour **and**

**(i)** he must use two different colours only,
**(ii)** he can use some or all of the colours.

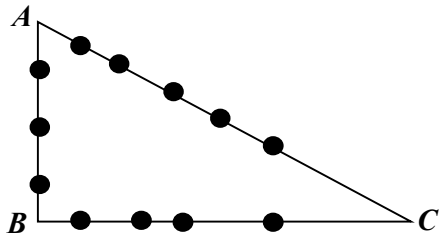
**[(i)** 20; **(ii)** 1280**]**

**Solution**

**(i)** Number of selections of 2 different colours = ${}^5C_2 = 10$. Since no adjacent doors are to be painted the same colour, there are only two cases, XYXYX and YXYXY. Therefore number of ways = $10 \times 2 = 20$.

**(ii)** Number of choices for the first door = 5
Since no adjacent doors are to be painted the same colour, number of choices for each of the $2^{nd}$, $3^{rd}$, $4^{th}$ and $5^{th}$ doors = 4
Total number of ways = $5\times4\times4\times4\times4 = 1280$

**11** The following diagram shows 12 **distinct** points on the sides of a triangle $ABC$.




**(i)** How many line segments are there joining any two points on different sides?

**(ii)** How many triangles can be formed by joining any one point on side $AB$, any one point on side $BC$ and any one point on side $AC$?

**[(i)** 47 ; **(ii)** 60**]**

**Solution**

**(i)** Number of ways to join a point on side $AB$ to any point on the other two sides $= 3\times9 = 27$
Number of ways to join a point on side $BC$ to a point on side $AC = 4\times5 = 20$.
Total number of ways to form a line segment = 27 + 20 = 47.

**(ii)** We need to choose a point on each of the sides, thus number of ways $= {}^3C_1 \times {}^4C_1 \times {}^5C_1 = 60$

**12** There are nine **different** waterslides at an amusement park. A person is allowed to go on each slide once only. However, he can choose to skip the slide or go on it.

**(i)** If John goes on at least two slides at the amusement park, how many ways can he select the slides to go on?

**(ii)** At another theme park, there are $n$ different slides. John visits the theme park frequently and selects 2 slides to go on during every visit. At each visit, he makes a different selection, and first realizes that this was no longer possible on the 29th visit. Determine algebraically, the value of $n$.

**(iii)** Give an example of a situation involving waterslides to which the expression $\frac{n!}{r!(n-r)!}$ is the solution.

**[(i)** 502; **(ii)** $n=8$**]**

**Solution**

**(i)** Consider the first slide. John can choose to go on the slide or skip the slide.
Therefore there are 2 choices. A similar argument follows for the other 8 slides.
Number of selections of all sizes $= 2\times2\times2\times2\times2\times2\times2\times2\times2 = 2^9 = 512$.
However in one of these selections, all the slides were skipped. Also, number of ways John can choose only one slide to go on = ${}^9C_1 = 9$.
Thus, using the Complement Method, number of ways John can choose at least two slides to go on = 512 – 1 – 9 = 502.

**(ii)** There are 28 different selections of 2 slides out of $n$ slides. Thus ${}^nC_2 = 28$.

$$ {}^nC_2 = \frac{n!}{2!(n-2)!} = 28$$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 28$$

$$n^2 - n - 56 = 0$$

$$(n-8)(n+7) = 0$$

Therefore $n = 8$ or $n = -7$ (Reject since $n$ is a positive integer).
OR Use GC Table of values with $y_1 = {}^xC_2$

**(iii)** The number of ways to choose $r$ slides to go on out of $n$ slides OR
The number of ways to choose $r$ slides to skip (i.e. not to go on) out of $n$ slides.

**13 MI Prelim 9740/2012/02/Q5b**

A group of 10 people consists of 9 men and 1 woman. Find the number of ways which the group can be seated at a round table with identical chairs if

**(i)** there is no restriction, **[1]**

**(ii)** 2 particular men, Caleb and James, do not want to sit beside the woman, but will like to sit together. **[3]**

The chairs at the table are replaced with 10 chairs of different colours.

**(iii)** Find the number of ways which the group can be seated at a round table if Caleb and James must still sit together, but they need not be separated from the woman. **[3]**

**[(i)** 362 880 **(ii)** 60480 **(iii)** 806 400**]**

**Solution**

**(i)** Number of ways $= (10-1)! = 362880$

**(ii)** Using slotting method, we arrange the 7 other men around the table first and then insert the woman and the unit of Caleb and James into 2 of the 7 spaces between the 7 men who are already seated.
Number of ways $= (7-1)! \times {}^{7}C_{2} \times 2! \times 2! = 60480$

**(iii)** Number of ways $= (9-1)! \times 10 \times 2! = 806400$

**14 TJC Prelim 9740/2012/02/Q5**

2 men and 5 women go to a restaurant. They choose an outdoor round table with 7 seats for their meal. Find the number of ways the group can be seated if

**(i)** the two men are not seated next to each other. **[2]**

**(ii)** one of the women, Mary, is to be seated between the two men. **[2]**

Before their orders arrive, they request to shift to a table in the 'non-smoking' section of the restaurant. They are then given a round table with 10 seats.

Find the number of ways they can be seated if

**(iii)** the empty seats are adjacent to each other. **[2]**

**(iv)** none of the empty seats are adjacent to each other and there must be more than 1 person between any two empty seats. **[2]**

**[(i)** 480 **(ii)** 48 **(iii)** 5040 **(iv)** 5040**]**

**Solution**

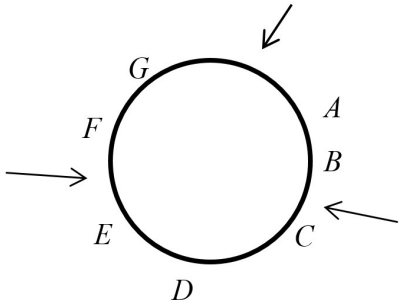
**(i)** Number of ways $= 6! - (5!)2 = 480$ or $(5-1)! \times {}^{5}P_{2} = 480$

**(ii)** Group Mary and the 2 men as one unit, number of ways $= (5-1)! \times 2 = 48$

**(iii)** Group 3 empty seats as one unit, number of ways $= (8-1)! = 5040$

**(iv)** **Method 1**

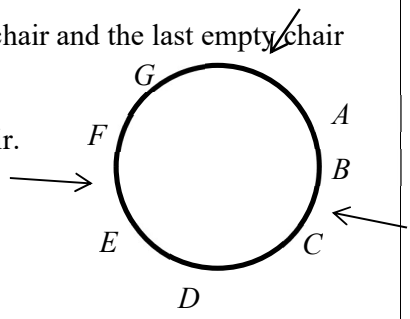
The condition of more than 1 person between empty chair leads to the following unique configuration where the "arrow is the empty chair".




There are $7!$ ways to seat 7 people in the distinct positions A, B, C, D, E, F, G.

**Method 2**

Seat the 7 people around the table in $(7-1)! = 6!$ ways. Insert the first empty chair in 7 possible slots, (only 1 such "2-3-2" configuration) followed by the next empty chair with 2 persons between the first and second empty chair and the last empty chair with 3 persons in between the second and last empty chair.




Number of ways $= (6!)7 = 7! = 5040$

**15 TJC Prelim 9233/2004/01/Q3 (modified)**

A soccer team consists of 3 goalkeepers, 8 defenders, 7 midfielders and 4 strikers. The coach has to select exactly 1 goalkeeper and 10 other players to start a match.

**(i)** In how many ways can he select the 11 players?

In each of the four sections (goalkeepers, defenders, midfielders and strikers), there is one STAR player. How many ways can the coach select his 11 players if

**(ii)** he must include all the STAR players?

**(iii)** he must include at least one STAR player?

**[(i)** 277134 **(ii)** 11440 **(iii)** 261118**]**

**Solution**

Team 3 G , 8 D, 7 M, 4 S - 22 players
Select 1 G, 10 other players

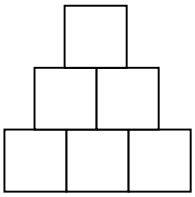
**(i)** From the 3 goalkeepers, we can only choose 1. From the remaining 19 players, we have to choose the remaining 10.
Number of ways = ${}^{3}C_{1} \times {}^{19}C_{10} = 277134$

**(ii)** After including the 4 STAR players, we have to choose 7 more players. Since the STAR goalkeeper is chosen, we do not need any more goalkeepers.
Number of other players left = 8 + 7 + 4 – 3 = 16
Number of ways = ${}^{16}C_{7} = 11440$

**(iii)** From (i), number of choices, without restriction = 277134
Number of ways where we do not include any STAR player = ${}^{2}C_{1} \times {}^{16}C_{10} = 16016$
Number of ways to include at least 1 STAR player = 277134 – 16016 = 261118

**16 MI Prelims 9740/2013/02/Q6**

Six identical boxes are arranged in 3 rows as shown in the following diagram.




Sandra is given 1 green, 2 blue and 3 red balls. The balls are identical except for their colour. She is to put one ball in each box. Find the number of ways she can do this when

**(i)** there is no restriction, **[2]**

**(ii)** the balls in the bottom row are of different colours, **[2]**

**(iii)** there are at least 2 red balls in the bottom row. **[3]**

**[(i)** 60 **(ii)** 18 **(iii)** 30**]**

**Solution**

**(i)** Number of ways with no restriction is $\frac{6!}{2!3!} = 60$

**(ii)** Number of ways such that the balls in the bottom row are of different colours is $\frac{3!}{2!}(3!) = 18$

**(iii)** Number of ways such that there are at least 2 red balls in the bottom row

$$= \underset{\text{3R bottom}}{\frac{3!}{2!}} + \underset{\text{2R+1B bottom}}{\frac{3!}{2!}(3!)} + \underset{\text{2R+1G bottom}}{\frac{3!}{2!}\cdot\frac{3!}{2!}} = 30$$

**17 RI Y6 CT2 9740/Q8**

Three couples who each have a child are to be seated at a round table with ten secured seats. Find the number of ways the nine people can be seated if

**(i)** they do not mind who they are sitting with, **[1]**

**(ii)** none of the children are to be seated on adjacent seats. **[3]**

Mr Bean, who knows the three families well, is invited to take a seat at the table. Find the number of ways to seat the ten people if each child is to be seated adjacent to both his or her parents. **[3]**

**[(i)** 362880 **(ii)** 151200 **;** 48**]**

**Solution**

**(i)** Considering the empty seat as a distinct object, this is an arrangement of 10 distinct objects in a circle.

$\therefore$ Number of ways $= (10-1)! = 362880$

**(ii)** Consider the 6 adults and empty seat as 7 distinct objects.

Number of ways of arranging them in a circle $= (7-1)!$

Number of ways to slot and arrange the 3 children in 3 of the 7 spaces between the 7 distinct objects $= {}^7P_3$ or ${}^7C_3 \times 3!$

$\therefore$ Number of ways in which none of the children are seated on adjacent seats $= (7-1)! \times {}^7P_3 = 151200$

Consider each couple and their child as a unit.

Number of ways of arranging 4 distinct units in a circle $= (4-1)!$

Each couple can arrange amongst themselves in 2! ways.

$\therefore$ Number of ways in which each child is seated between their parents $= (4-1)! \times (2!)^3 = 48$

**18** Seven cards each have a single digit written on them. The digits on the seven cards are 2, 2, 5, 7, 7, 7, 7 respectively.

Find the number of different 4-digit numbers that can be formed by placing four of the cards side by side. **[4]**

**[**39**]**

**Solution**

| | **4 cards selected** | **Number of 4-digit numbers that can be formed** |
|---|---|---|
| Case 1 | 7, 7, 7, 7 | 1 |
| Case 2 | 7, 7, 7, 2 | $\frac{4!}{3!}=4$ |
| Case 3 | 7, 7, 7, 5 | $\frac{4!}{3!}=4$ |
| Case 4 | 7, 7, 2, 2 | $\frac{4!}{2!2!}=6$ |
| Case 5 | 7, 7, 2, 5 | $\frac{4!}{2!}=12$ |
| Case 6 | 7, 5, 2, 2 | $\frac{4!}{2!}=12$ |

Number of different 4-digit numbers that can be formed $=1+4+4+6+12+12=39.$

**19 9205/2001/01/Q7**

In how many ways can a committee of 3 men and 3 women be chosen from a group of 7 men and 6 women? **[2]**

The oldest of the 7 men is *A* and the oldest of the 6 women is *B*. It is decided that the committee can include at most one of *A* and *B*. In how many ways can the committee now be chosen? **[3]**

**[**700 ; 550**]**

**Solution**

Number of ways to form a committee $= {}^7C_3 \times {}^6C_3 = 700.$

To form a committee which include at most one of *A* and *B:*

<u>Method 1: Consider complement</u>

Number of ways without restriction $= {}^7C_3 \times {}^6C_3 = 700$

Number of ways which include both *A* and *B* $= {}^6C_2 \times {}^5C_2 = 150$

Number of ways which include at most one of *A* and *B* $= 700 - 150 = 550.$

Method 2: Consider cases

Case 1: $A$ is included but not $B$
Number of ways $= {}^6C_2 \times {}^5C_3 = 150$

Case 2: $B$ is included but not $A$
Number of ways $= {}^6C_3 \times {}^5C_2 = 200$

Case 3: Both $A$ and $B$ are not included
Number of ways $= {}^6C_3 \times {}^5C_3 = 200$

Number of ways which include at most one of $A$ and $B$ $= 150 + 200 + 200 = 550.$

**20** Consider the eight letters in the word GEOMETRY.

**(a)** Find the number of different arrangements of all eight letters in which the vowels are always separated. **[3]**

**(b)** Four letters are selected from the eight letters.

**(i)** How many possible selections can be made if all the letters are distinct and the order of selection is not relevant? **[1]**

**(ii)** Find the number of 4-letter code-words that can be formed if there is no restriction on the letters selected. **[3]**

**[(a)** 7200 **(b)(i)** 35 **(b)(ii)** 1020**]**

**Solution**
8 letters GEOMETRY: 1G, 2 Es, 1O, 1M, 1T, 1R, 1Y

**(i)** There are 5 consonants G, M, T, R, Y and 3 vowels E, E, O.
Number of arrangements of the 5 consonants $= 5! = 120$
Number of ways to slot and permute the 3 vowels in 3 of the 6 spaces between the 5 consonants $= \dfrac{{}^6P_3}{2!} = 60$ (need to divide by $2!$ because 2 of the 3 vowels are identical)

Number of arrangements in which the 3 vowels are always separated $= 120 \times 60 = 7200.$

**(b) (i)** Since there are 7 distinct letters (G, E, O, M, T, R, Y),
number of possible selections $= {}^7C_4 = 35.$

**(ii)** Case 1: 4 letters selected are all distinct (includes no "E" or 1 "E")
Since there are 7 distinct letters (G, E, O, M, T, R, Y),
number of possible code-words $= {}^7P_4 = 840$

Case 2: 4 letters include both "E"s
Number of ways to select the other 2 letters $= {}^6C_2 = 15$

Number of ways to arrange the 2 "E"s and the 2 chosen letters $= \frac{4!}{2!} = 12$

Number of possible code-words $= 15 \times 12 = 180$

Number of 4-letter code-words $= 840 + 180 = 1020.$

**21 ACJC Prelim 9740/2013/02/Q7 modified**

Ten chefs, six males and four females, qualify for the final phase of a 'Top Chef' competition consisting of nine 'Elimination Challenges'. One chef is eliminated after every challenge and the last chef remaining is the winner of the competition.

**(i)** In how many ways can the eliminations be done? **[1]**

**(ii)** For the first challenge, the ten chefs are divided into two groups of five. In how many ways can the two groups be formed such that each group has at least one female chef? **[2]**

**(iii)** After the first challenge, a particular male chef was eliminated. At a photography session before the next challenge, nine chairs are arranged in two rows: four in front and five at the back. In how many ways can the remaining chefs be arranged so that the male chefs and the female chefs must alternate? **[3]**

**[(i)** 3628800 **(ii)** 120 **(iii)** 5760**]**

**Solution**

**(i)** Number of ways of eliminations is 10! =3 628 800

**(ii)** Method 1**:**By Complement
Required number
= (Number of ways to form 2 groups of 5 without restriction) – (Number of ways where one group has no female chefs)
$= \frac{{}^{10}C_5}{2} - {}^6C_5 = 120$

Method 2:
Case 1: one group has 1 female chef, the other has 3 female chefs.
Number of ways to form the groups is equivalent to number of ways to form the group of 1 female chef and 4 male chefs ${}^6C_4 \cdot {}^4C_1 = 60$
Case 2: both groups have exactly 2 female chefs.

Number of ways to form the groups is $\frac{{}^6C_3 \cdot {}^4C_2}{2} = 60$

(Note that there is a need to divide by "2" to avoid double counting as both

groups each consist of 2 female chefs and 3 male chefs)
Total number of ways = 60 + 60 = 120

**(iii)** Two cases for male and female chefs to alternate:

$\underline{M}\ \underline{F}\ \underline{M}\ \underline{F}\ \underline{M}$ or $\underline{M}\ \underline{F}\ \underline{M}\ \underline{F}\ \underline{M}$

$\underline{F}\ \underline{M}\ \underline{F}\ \underline{M}$ $\underline{M}\ \underline{F}\ \underline{M}\ \underline{F}$

Number of ways = $5! \times 4! \times 2 = 5760$

**22 NJC Common Test 9740/2012/02/Q4**

**(a)** A student is arranging 4 blue flags, 4 red flags, 1 green, 1 yellow and 1 purple flag in a line.

All the flags are identical except for the colour. Find the number of different possible ways to arrange the flags if

**(i)** the green, yellow and purple flags must be placed together; **[2]**

**(ii)** no blue flags are placed next to each other; **[2]**

**(iii)** a red flag at the beginning and another red flag at the end of the line. **[2]**

**(b)** Ten people at a company dinner consist of 7 guests and 3 hosts. The group is to be seated at a round table.

**(i)** Find the number of arrangements such that there are at least two guests seated between any two hosts.

**[3]**

**(ii)** Two particular guests could not make it to the dinner. Find the number of ways to arrange the remaining guests and hosts with 10 identical chairs. **[3]**

**[(a)(i)** 3780 **(a)(ii)** 14700 **(a)(iii)** 7560 **(b)(i)** 30240 **(b)(ii)**181440**]**

**Solution**

**(a)(i)** Consider the green, yellow and purple flags as 1 unit.
Number of ways to arrange the green, yellow and purple flags within the unit = 3!

Number of ways to arrange the unit with the rest of the flags = $\frac{9!}{4!4!}$

Thus number of possible arrangements for the flags = $3! \times \frac{9!}{4!4!} = 3780$

**(ii)** Number of ways to arrange the 4 red flags, 1 green, 1 yellow and 1 purple flag = $\frac{7!}{4!}$

Number of ways to slot in the blue flags = $\binom{8}{4}$

Number of possible arrangements such that no blue flags are placed next to another

$= \frac{7!}{4!} \times \binom{8}{4} = 14\,700$

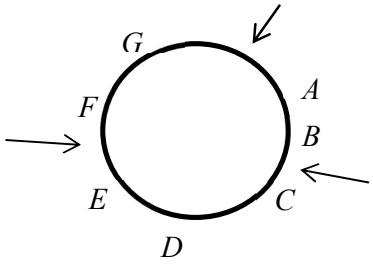
**(iii)** Number of possible arrangements such that a red flag at the beginning and another red flag at the end of the line is equivalent to arranging 4 blue flags, 2 red flags, 1 green, 1 yellow and 1 purple flag

$= \frac{(4+2+3)!}{4!2!} = 7560$

**(b)(i)** Seat the 7 guests around the table in $(7-1)! = 6!$ ways.

Insert the first host in 7 possible slots, followed by the next host with 2 guests between the first and second hosts and the last host with 3 guests in between the second and last hosts. Since the three hosts are distinct, there are $3!$ ways to permute them.

$\therefore$ Number of ways = $(6!) \times 7 \times 3! = 30240$




**(ii)** 5 guests, 3 hosts and 2 empty identical chairs

Suppose the 2 empty chairs are distinct.

Number of ways to arrange the 8 people and 2 empty chairs round a table = $9!$

Since the 2 chairs are identical, required number of ways is $\frac{9!}{2} = 181440$

**23 9740/2016/02/Q7**

The management board of a company consists of 6 men and 4 women. A chairperson, a secretary and a treasurer are chosen from the 10 members of the board. Find the number of ways the chairperson, the secretary and the treasurer can be chosen so that

**(i)** they are all women, **[1]**

**(ii)** at least one is a woman and at least one is a man. **[3]**

The 10 members of the board sit at random around a table. Find the probability that

**(iii)** the chairperson, the secretary and the treasurer sit in three adjacent places, **[3]**

**(iv)** the chairperson, the secretary and the treasurer are all separated from each other by at least one other person. **[3]**

**[(i)** 24 **(ii)** 576 **(iii)** $\frac{1}{12}$ **(iv)** $\frac{5}{12}$**]**

**Solution**

**(i)** Number of ways for the chairperson, secretary and treasurer to be all women is
$^4C_3 \times 3! = 24$

**(ii)** Number of ways so that at least one is a woman and at least one is a man is
$\left(\text{Number of ways to select 3 without restriction}\right)$
$-(\text{Number of ways where all 3 are women}) - (\text{Number of ways where all 3 are men})$
$= {}^{10}C_3 \times 3! - 24 - {}^6C_3 \times 3! = 576$

**(iii)** Group the chairperson, secretary and treasurer as one unit.
Number of ways is $7! \times 3!$
$\therefore$ Required probability is $\frac{7! \times 3!}{9!} = \frac{1}{12}$

**(iv)** Seat the remaining 7 people at the round table. Number of ways is $(7-1)! = 6!$

Select 3 of the 7 spaces in between these 7 people to seat the chairperson, secretary and treasurer in $^7C_3 \times 3!$ ways
$\therefore$ Required probability is $\frac{\left(^7C_3 \times 3!\right) \times 6!}{9!} = \frac{5}{12}$

****** THE END ******