



Additional Practice Questions for Chapter S4: Sampling (Solutions)

1 SAJC Prelim 9233/2005/02/Q28

A manufacturer of candles claimed that it produced birthday candles with a mean burning time of 6 minutes. A random sample of 150 birthday candles was tested and the burning times, X minutes, were summarized by $\sum (x - 5) = 120$ and $\sum (x - 5)^2 = 638$.

Calculate the unbiased estimates for the mean μ and variance σ^2 . **[3]**

Solution

Let $y = x - 5$. Then $\sum y = 120$, $\sum y^2 = 638$.

$$\bar{y} = \frac{1}{n} \sum y = \frac{120}{150} = 0.8$$

Now $\bar{y} = \bar{x} - 5$

$$\therefore \bar{x} = \bar{y} + 5 = 5.8$$

$$\begin{aligned} s_x^2 = s_y^2 &= \frac{1}{n-1} \left[\sum y^2 - \frac{(\sum y)^2}{n} \right] \\ &= \frac{1}{149} \left(638 - \frac{120^2}{150} \right) \\ &= 3.64 \text{ (3 s.f.)} \end{aligned}$$

Unbiased estimate for μ and σ^2 are 5.8 and 3.64 (3 s.f.)

2 ACJC Prelim 9233/2005/02/Q27

Two firms A and B manufacture similar components with mean breaking strengths of 6 units and 5.5 units, and standard deviations 0.4 units and 0.25 units respectively. It is given that both distributions are normal. If random samples of 100 components from firm A and 50 from firm B are tested, find the probability that the mean breaking strength of the sample from firm A minus that from firm B will be between 0.45 and 0.55 units.

[3]**Solution**

Let X and Y be the breaking strengths of a random component manufactured by firms A and B respectively.

Then $X \sim N(6, 0.4^2)$ and $Y \sim N(5.5, 0.25^2)$.

$$\text{i.e. } \bar{X} \sim N\left(6, \frac{0.4^2}{100}\right) \quad \text{and} \quad \bar{Y} \sim N\left(5.5, \frac{0.25^2}{50}\right).$$

$$\bar{X} - \bar{Y} \sim N\left(0.5, \frac{0.4^2}{100} + \frac{0.25^2}{50}\right)$$

$$\text{i.e. } \bar{X} - \bar{Y} \sim N(0.5, 0.00285)$$

$$P(0.45 < \bar{X} - \bar{Y} < 0.55) = 0.651 \quad (3 \text{ s.f.})$$

3 HCI Prelim 9233/2005/02/Q24

The life, in hours, of a randomly chosen light bulb produced by a manufacturer is normally distributed with mean 1100 and standard deviation 70. How large a sample is required such that the probability that the mean life in the sample shall exceed 1120 is not more than 5%?

[5]

Do you need to use Central Limit Theorem in your working? Explain.

[1]**Solution**

Let X be the life, in hours, of a randomly chosen light bulb. $X \sim N(1100, 70^2)$.

Let the sample size be n . Then $\bar{X} \sim N\left(1100, \frac{70^2}{n}\right)$

$$P(\bar{X} > 1120) \leq 0.05$$

$$P\left(Z > \frac{1120 - 1100}{70/\sqrt{n}}\right) \leq 0.05$$

$$P\left(Z > \frac{2\sqrt{n}}{7}\right) \leq 0.05$$

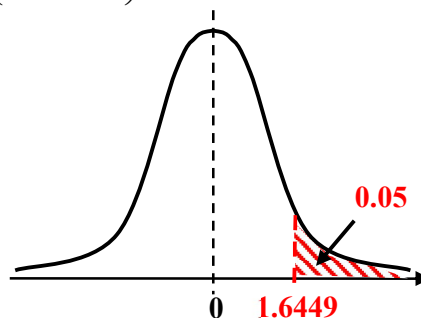
$$\text{From GC, } P(Z > 1.6449) = 0.05$$

$$\frac{2\sqrt{n}}{7} \geq 1.6449$$

$$\Rightarrow n \geq 33.14$$

$$\therefore \text{Least } n = 34$$

It is not necessary to use the Central Limit Theorem since X is normally distributed.



4 AJC Prelim 9233/2003/02/Q29 Or (part)

The times taken by two runners A and B in a 400 metre race are independent and may be assumed to be normally distributed. The times (in seconds) taken by A has mean 46 and standard deviation 0.5 and the times taken by B has mean 46.2 and standard deviation 0.8.

Let $\bar{X}$ denote the average time for A to run the 400 metre track on 10 different occasions and $\bar{Y}$ denotes the average time for B to run the same track on 16 different occasions. Find the value of t such that

$$P\left(\left|\bar{X} - \bar{Y} + 0.2\right| > t\right) = 0.1. \quad [4]$$

Solution

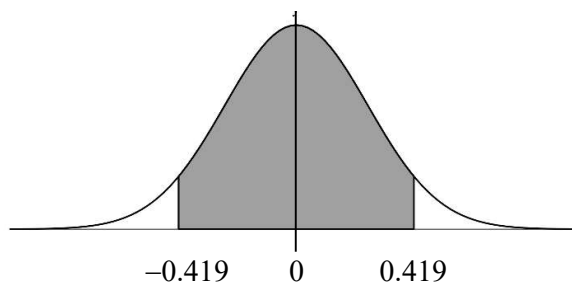
$$\text{Since } \bar{X} \sim N\left(46, \frac{0.5^2}{10}\right) \text{ and } \bar{Y} \sim N\left(46.2, \frac{0.8^2}{16}\right),$$

$$\bar{X} - \bar{Y} \sim N(-0.2, 0.065).$$

$$P\left(\left|\bar{X} - \bar{Y} + 0.2\right| > t\right) = 0.1$$

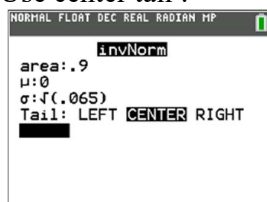
$$\text{Since } \bar{X} - \bar{Y} + 0.2 \sim N(0, 0.065),$$

$$\therefore t = 0.419 \text{ (3 s.f.)}$$



GC screenshots for reference:

Use center tail :



OR : Use right tail



5 TJC Prelim 9233/96/02/Q9

In a certain examination with a very large entry, the marks obtained by the male candidates were found to follow a normal distribution with a mean of 54 and a standard deviation of 16. $\bar{X}$ denotes the mean mark scored by a sample of 4 male candidates.

- (i) State the sampling distribution of $\bar{X}$.
- (ii) Find the probability that $\bar{X}$ will exceed 70.
- (iii) Given that there is a probability of 0.95 that $\bar{X}$ differs from its mean mark by less than c , find the value of c .

In the same examination the marks obtained by the female candidates were found to follow an independent normal distribution with a mean of 59 and a standard deviation of 20.

- (iv) Find the probability that the total marks obtained by a randomly chosen male candidate and a randomly chosen female candidate exceed 120.

It is given that $\bar{Y}$ denotes the mean mark scored by a random sample of 5 female candidates.

- (v) State the sampling distribution of $\bar{Y} - \bar{X}$.
- (vi) Find $P(\bar{Y} > \bar{X})$.

Solution

Let X be the mark obtained by a male candidate in a certain exam.

Then $X \sim N(54, 16^2)$

$$(i) \quad \bar{X} \sim N\left(54, \frac{16^2}{4}\right) \quad \text{i.e.} \quad \bar{X} \sim N(54, 8^2)$$

$$(ii) \quad P(\bar{X} > 70) = 0.0228 \quad (3 \text{ s.f.})$$

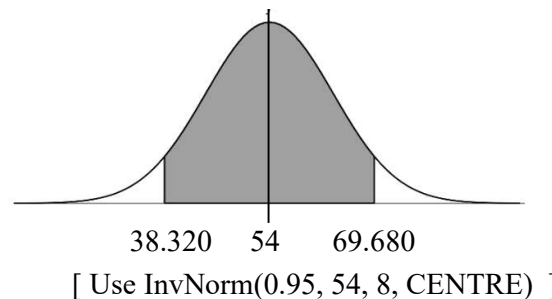
$$(iii) \quad P(|\bar{X} - 54| < c) = 0.95$$

$$P(-c < \bar{X} - 54 < c) = 0.95$$

$$P(54 - c < \bar{X} < 54 + c) = 0.95$$

$$\text{From GC, } P(38.320 \leq \bar{X} \leq 69.680) = 0.95$$

$$\begin{aligned} \therefore 54 + c &= 69.680 & (5 \text{ s.f.}) \\ c &= 15.7 & (3 \text{ s.f.}) \end{aligned}$$



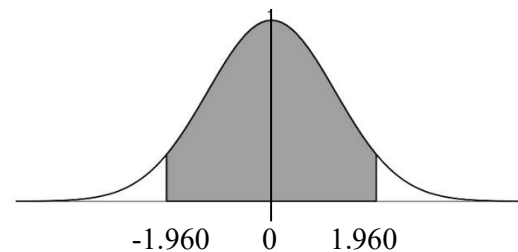
$$\text{OR: } P(|\bar{X} - 54| < c) = 0.95$$

$$P\left(\left|\frac{\bar{X} - 54}{8}\right| < \frac{c}{8}\right) = 0.95$$

$$P\left(|Z| < \frac{c}{8}\right) = 0.95$$

$$P\left(-\frac{c}{8} < Z < \frac{c}{8}\right) = 0.95$$

$$\text{From GC: } P(-1.960 < Z < 1.960) = 0.95$$



$$[\text{ Use InvNorm}(0.95, 0, 1, \text{CENTRE})]$$

$$\frac{c}{8} = 1.960$$

$$c = 15.7 \text{ (3 s.f.)}$$

Let Y be the marks obtained by a female candidate in the same exam.

$$Y \sim N(59, 20^2)$$

$$(iv) \quad P(X + Y > 120) = 0.392 \text{ (3 s.f.)} \quad \text{where } X + Y \sim N(113, 656)$$

$$(v) \quad \bar{Y} \sim N\left(59, \frac{20^2}{5}\right) \quad \text{i.e. } \bar{Y} \sim N(59, 80)$$

$$\text{Then } \bar{Y} - \bar{X} \sim N(5, 144)$$

$$(vi) \quad P(\bar{Y} > \bar{X}) \\ = P(\bar{Y} - \bar{X} > 0) = 0.662 \text{ (3 s.f.)}$$

6 CJC Prelim 9740/2012/02/Q11

A sample of n observations was taken from a population of mean μ and variance 10. Find the least sample size required such that the probability of the sample mean lying between $\mu - 0.5$ and $\mu + 0.5$ is more than 0.89. State an assumption that you have to make in order to proceed with this calculation. [6]

Solution

Let X be the random variable.

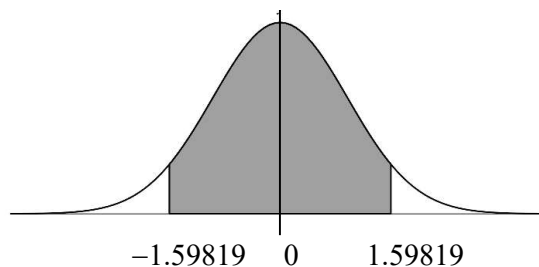
Assume that n is sufficiently large for Central Limit Theorem to hold.

Then, $\bar{X} \sim N\left(\mu, \frac{10}{n}\right)$ approximately

$$P(\mu - 0.5 < \bar{X} < \mu + 0.5) > 0.89$$

$$P\left(\frac{\mu - 0.5 - \mu}{\sqrt{\frac{10}{n}}} < Z < \frac{\mu + 0.5 - \mu}{\sqrt{\frac{10}{n}}}\right) > 0.89$$

$$P\left(-0.5\sqrt{\frac{n}{10}} < Z < 0.5\sqrt{\frac{n}{10}}\right) > 0.89$$



$$\text{From GC, } P(-1.59819 < Z < 1.59819) = 0.89$$

From the diagram,

$$0.5\sqrt{\frac{n}{10}} > 1.59819$$

$$\sqrt{\frac{n}{10}} > 3.19638$$

$$\frac{n}{10} > 10.21685$$

$$n > 102.1685$$

Least $n = 103$

7 9233/2002/02/Q30 OR (modified)

A random variable X has expectation 1 and variance $1/3$.

The random variable S is the sum of 80 independent observations of X .

Find an approximate value for $P(75 < S < 90)$.

[3]**Solution**

Given $E(X) = 1$, $\text{Var}(X) = \frac{1}{3}$ and $S = X_1 + X_2 + \dots + X_{80}$

Since $n = 80$ is large, by Central Limit Theorem,

$$S \sim N\left(80(1), 80\left(\frac{1}{3}\right)\right) \text{ approximately.}$$

$$\text{i.e. } S \sim N\left(80, \frac{80}{3}\right)$$

$$P(75 < S < 90) = 0.807 \quad (3 \text{ s.f.})$$

OR :

Since $n = 80$ is large, by Central Limit Theorem,

$$\bar{X} \sim N\left(1, \frac{1/3}{80}\right) \text{ approximately.} \quad \text{i.e. } \bar{X} \sim N\left(1, \frac{1}{240}\right)$$

$$P(75 < S < 90)$$

$$= P\left(\frac{75}{80} < \bar{X} < \frac{90}{80}\right) = 0.807 \quad (3 \text{ s.f.})$$

8 DHS Prelim 9758/2021/02/Q7 (modified)

Mike plays a game at a fun-fair. Each game costs \$3 to play. The probability distribution of the amount of money \$X, that Mike wins from a game is given by

$$P(X = x) = \begin{cases} kx & \text{for } x = 1, 2, 3, \\ \frac{1}{3}k(x-1) & \text{for } x = 4, 5, 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (i) Show that $k = \frac{1}{10}$. [1]
- (ii) Find the expectation and variance of his net gain for one game. [3]
- (iii) Assuming that each game is independent of one another, estimate the probability that his total net gain in 30 games is more than \$20. [3]

Solution	
(i)	$\sum P(X = x) = 1$ $k + 2k + 3k + \frac{3k}{3} + \frac{4k}{3} + \frac{5k}{3} = 1$ $k = \frac{1}{10} \text{ (shown)}$
(ii)	$E(X) = \frac{1}{10} + 2\left(\frac{2}{10}\right) + 3\left(\frac{3}{10}\right) + 4\left(\frac{1}{10}\right) + 5\left(\frac{2}{15}\right) + 6\left(\frac{1}{6}\right) = \frac{52}{15}$ $\text{Expected net gain} = E(X - 3) = E(X) - 3 = \frac{52}{15} - 3 = \frac{7}{15}$ $\text{Variance of net gain} = \text{Var}(X - 3) = \text{Var}(X)$ $E(X^2) = \frac{1}{10} + 2^2\left(\frac{2}{10}\right) + 3^2\left(\frac{3}{10}\right) + 4^2\left(\frac{1}{10}\right) + 5^2\left(\frac{2}{15}\right) + 6^2\left(\frac{1}{6}\right) = \frac{218}{15}$ $\text{Hence, } \text{Var}(X) = \frac{218}{15} - \left(\frac{52}{15}\right)^2 = \frac{566}{225}$
(iii)	Let $Y = X - 3$ denote the net gain of one game. Since $n = 30$ is large, by Central Limit Theorem, $Y_1 + Y_2 + \dots + Y_{30} \sim N\left(30\left(\frac{7}{15}\right), 30\left(\frac{566}{225}\right)\right) \text{ approximately.}$ $P(Y_1 + Y_2 + \dots + Y_{30} > 20) = 0.245 \text{ (3 s.f.)}$

9 RVHS Prelim 9758/2021/02/Q9 (modified)

During each round of practice, John, does 10 multiple-choice questions and his score X , is the number of questions he answered correctly. On average, he has an 85% chance of answering each multiple-choice question correctly.

- (i) State in the context of the question, one assumption needed to model X by a binomial distribution. [1]

On a particular day, John does 4 rounds of practice.

- (ii) Find the probability that the total score for John in the 4 rounds of practice is more than 36. [2]

- (iii) Find the probability that John obtains a score of at most 9 in each of his first 2 rounds of practice. [3]

To qualify for an award, John will need a mean score of at least 8.8 for all his 50 rounds of practice.

- (iv) Estimate, using an approximation, the probability that John will qualify for the award. [3]

(i)	The event that John manages to answer a multiple-choice question correctly is independent of him answering any other multiple-choice questions correctly. OR: The probability of John answering a question correctly is a constant at 0.85.
(ii)	Let Y be the total score of John in the first 4 rounds. Then $Y \sim B(40, 0.85)$ So $P(Y > 36) = 1 - P(Y \leq 36)$ $= 1 - 0.86983$ $= 0.130$ (3 s.f.)
(iii)	$X \sim B(10, 0.85)$ Required probability $= P(X_1 \leq 9) \times P(X_2 \leq 9)$ $= (0.80312)^2$ $= 0.645$ (3 s.f.)
(iv)	For $X \sim B(10, 0.85)$, $E(X) = np = 10 \times 0.85 = 8.5$ $\text{Var}(X) = np(1-p) = 10 \times 0.85 \times (1-0.85) = 1.275$ Since sample size, $n = 50$ is large, $\bar{X} \sim N\left(8.5, \frac{1.275}{50}\right)$ approximately by Central Limit Theorem. $P(\bar{X} \geq 8.8) = 0.0301$ (3 s.f.)

10 SAJC Prelim 9758/2021/02/Q11 (modified)

- (a) Anne, a Bubble Tea (BBT) seller, intends to increase the sales of BBT using a drink vending machine which delivers BBT into a cup when cash payment is made into the machine. The volume of BBT dispensed is normally distributed with mean 210 ml and standard deviation 5 ml. The capacity of a cup is 220 ml and the nominal amount of BBT in a cup is stated as 212 ml.
- (i) Find the probability that a cup overflows when BBT is dispensed into the cup. [1]
- (ii) A customer bought five cups of BBT from the vending machine. Find the probability that at most one cup of BBT will overflow. [2]
- (iii) Anne received complaints from some customers that there is a high proportion of cups with less than the nominal amount of BBT. It is assumed that the standard deviation of the volume of BBT dispensed is fixed while the mean volume of BBT dispensed could be adjusted. Find the range of the mean volume of BBT dispensed such that not more than 10% of the cups will contain less than the nominal amount of BBT. [3]
- (iv) Another customer bought n cups of BBT. Find the approximate probability that the total volume of BBT dispensed exceeds $211n$ ml as n becomes very large. [2]
- (v) Anne wishes to gather feedback about her BBT. She decided to interview 50 customers who bought BBT from the vending machine during lunch time. Give a reason as to whether Anne would obtain a random sample of customers. [1]
- (b) On another occasion, Anne deployed a staff to operate a BBT counter at a wedding reception. The average volume of BBT per serving is 200 ml and the standard deviation of the volume of BBT is given to be 10 ml. Estimate the probability that the mean volume of 60 servings of BBT prepared by the staff is less than 198 ml. [3]

(a) (i)	Let X be the volume of BBT dispensed, in ml. $X \sim N(210, 5^2)$ $P(\text{cup overflows}) = P(X > 220) = 0.02275 = 0.0228$ (3 s.f.)
(ii)	Let Y be the number of BBT cups, out of 5, which overflows. $Y \sim B(5, 0.02275)$ $P(Y \leq 1) = 0.995$ (3 s.f.) OR: $[P(X < 220)]^5 + \binom{5}{1} [P(X < 220)]^4 P(X > 220) = 0.995$ (3 s.f.)

(iii)	Let the mean volume of BBT dispensed be μ ml. $W \sim N(\mu, 5^2)$ $P(W < 212) \leq 0.10$ $P(Z < \frac{212 - \mu}{5}) \leq 0.10$ From the GC, $P(Z < -1.2816) = 0.10$ From the diagram, $\frac{212 - \mu}{5} \leq -1.2816$ $\mu \geq 218$ Range of mean volume is $218 \leq \mu \leq 220$ (3 s.f.)
(iv)	$X_1 + X_2 + \dots + X_n \sim N(210n, 25n)$ $P(X_1 + X_2 + \dots + X_n > 211n)$ $= P(Z > \frac{211n - 210n}{5\sqrt{n}})$ $= P(Z > \frac{1}{5}\sqrt{n})$ Since n becomes very large, $P(Z > \frac{1}{5}\sqrt{n}) \rightarrow 0$.
(v)	The BBT seller would not obtain a random sample as she only interviewed customers who bought her BBT during lunch time. She would not be able to get feedback from customers who made the purchase during other hours, hence the not all the customers have an equal chance of being selected. The sample is not randomly selected as result.
(b)	Let T be the volume of bubble tea served at the wedding reception, in ml. Since the sample size, $n = 60$ is large, $\bar{T} \sim N(200, \frac{100}{60})$ approximately by Central Limit Theorem $P(\bar{T} < 198) = 0.0607$ (3 s.f.)

11 JPJC Prelim 9758/2020/02/Q10

[In this question, you should state clearly the values of the parameters of any distribution you use.]

Mr Lim is a tuition teacher who charges his students according to the duration of different topics taught during the lesson. The duration, in minutes, of Pure Math and Statistics sessions in a single lesson are modelled as having independent normal distributions with means and standard deviations as shown in the table below.

	Mean	Standard deviation
Pure Math	55.4	3.6
Statistics	36.2	2.3

- (i) Find the probability that the total duration of two randomly chosen Pure Math sessions differs from three times the duration of a randomly chosen Statistics session by at least 3 minutes. [4]
- (ii) The probability of the average duration of 5 Statistics sessions exceeding r minutes is at least 0.3. Find the range of values of r , giving your answer correct to 2 decimal places. [3]

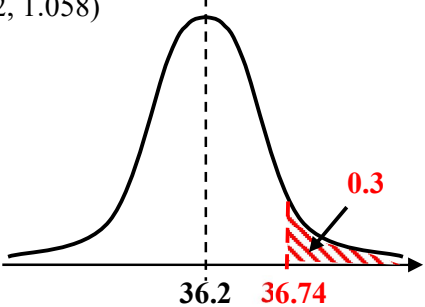
Students pay \$2.50 and \$1.50 for each minute of Pure Math and Statistics sessions during a lesson respectively. In addition, students pay a fixed charge of \$24 per lesson. Assume that one lesson of Mr Lim consists of a Pure Math and a Statistics session.

- (iii) (a) Calculate the mean and variance of the total cost paid in a randomly chosen lesson. [2]
- (b) Find the probability that out of 15 randomly chosen lessons, a student pays less than \$220 in at most 5 lessons. [3]

Another tuition teacher, Miss Lee, charges her students according to the duration of the lesson. The duration, in minutes, in a single lesson is normally distributed with mean 88.5 and standard deviation 3.8.

- (iv) 20 lessons from Mr Lim and 16 lessons from Miss Lee are randomly selected. Find the probability that the mean duration of these lessons is between 90 and 91 minutes. [3]

(i)	Let X and Y be the duration of Pure Math and Statistics taught in a single lesson. Then $X \sim N(55.4, 3.6^2)$ and $Y \sim N(36.2, 2.3^2)$ Let $T = X_1 + X_2 - 3Y$ $E(T) = 2E(X) - 3E(Y) = 2(55.4) - 3(36.2) = 2.2$ $\text{Var}(T) = 2\text{Var}(X) + 3^2 E(Y) = 2(3.6^2) + 9(2.3^2) = 73.53$ $T \sim N(2.2, 73.53)$ $P(T \geq 3)$ $= 1 - P(T < 3)$ $= 1 - P(-3 < T < 3)$ $= 0.735 \text{ (3 s.f.)}$
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(ii)	$\bar{Y} \sim N\left(36.2, \frac{2.3^2}{5}\right)$ i.e. $\bar{Y} \sim N(36.2, 1.058)$ $P(\bar{Y} > r) \geq 0.3$ From GC, $P(\bar{Y} > 36.74) = 0.3$ From the diagram, $0 \leq r \leq 36.74$ (2 d.p.) 
(iii) (a)	Let T be the total cost paid by a student for a randomly chosen lesson. where $T = 2.5X + 1.5Y + 24$ $E(T) = 2.5E(X) + 1.5E(Y) + 24 = (2.5)(55.4) + (1.5)(36.2) + 24 = 216.8$ $\text{Var}(T) = (2.5)^2 \text{Var}(X) + (1.5)^2 \text{Var}(Y) = (2.5)^2(3.6^2) + (1.5)^2(2.3^2) = 92.9025$
(iii) (b)	$T \sim N(216.8, 92.9025)$ $P(T < 220) = 0.63005$ (5 s.f.) Let L be the no. of lessons, out of 15 that cost less than \$220. $L \sim B(15, 0.63005)$ $P(L \leq 5) = 0.0190$ (3 s.f.)
(iv)	Let W and V be the duration of Mr Lim and Miss Lee's class in a lesson respectively. $W \sim N(91.6, 18.25)$ and $V \sim N(88.5, 3.8^2)$ Let $M = \frac{W_1 + \dots + W_{20} + V_1 + \dots + V_{16}}{36}$ $E(M) = \frac{1}{36} [20E(W) + 16E(V)] = \frac{1}{36} [20(91.6) + 16(88.5)] = \frac{812}{9}$ $\text{Var}(M) = \left(\frac{1}{36}\right)^2 [20\text{Var}(W) + 16\text{Var}(V)] = \left(\frac{1}{36}\right)^2 [20(18.25) + 16(3.8^2)] = 0.45990$ $M \sim N\left(\frac{812}{9}, 0.45990\right)$ $P(90 < M < 91) = 0.503$ (3 s.f.) OR: Let W and V be the duration of Mr Lim and Miss Lee's class in a lesson respectively. $W \sim N(91.6, 18.25)$ and $V \sim N(88.5, 3.8^2)$ Let $S = W_1 + \dots + W_{20} + V_1 + \dots + V_{16} \sim N(3248, 596.04)$ $P(90 < \frac{W_1 + \dots + W_{20} + V_1 + \dots + V_{16}}{36} < 91)$ $= P(90 \times 36 < S < 91 \times 36)$ $= P(3240 < S < 3276) = 0.503$ (3 s.f.)

12 MI Prelim 9758/2020/02/Q11(modified)

With the rising number of students owning a mobile device, Nurturing Primary School decides to conduct a survey on the amount of screen time the students spend on their mobile devices each day. There are 8 classes for each level from Primary One to Primary Six.

- (a) As Mrs Neha only teaches the Primary Six cohort, she decides to conduct the survey for all the 8 classes of Primary Six students. Explain whether these 8 classes of Primary Six students form a sample or a population. [1]
- (b) For the data collected to be more representative of the entire student population, Mrs Neha decides to conduct the survey for 100 students from Primary One to Primary Six. She randomly surveyed 100 students who stayed back in school for CCA activities on a particular day. Explain whether this sample is a random sample. [1]

Solution

(a) The 8 classes of Primary Six students form a sample as they are observations taken from the entire Nurturing Primary School population consisting of 64 classes, and classes from Primary 1 to 5 are not included in the sample.

(b) No, this sample is not a random sample because not all students have an equal probability of being selected, as students who do not stay back for CCA on that day do not have a chance of being selected.

The End