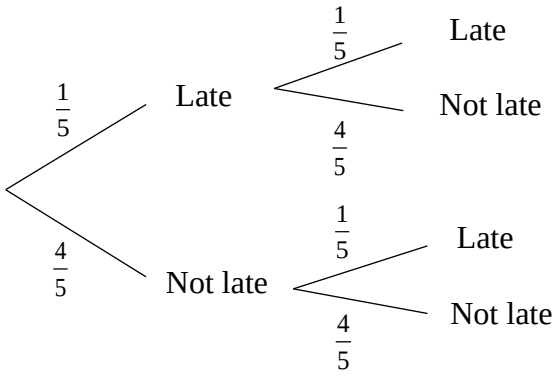
Broadrick Secondary School Prelims P2 2023
Elementary Mathematics - Secondary Four Express
Mark Scheme

Qn	Mark Scheme
1(a)	$= 3-(2x^2-8x+x-4)$ $= 3-2x^2+8x-x+4$ $= -2x^2+7x+7$
1(b)	$= \frac{1}{x+2}+\frac{2x-3}{(2x-3)(x+2)}$ $= \frac{1}{x+2}+\frac{1}{x+2}$ $= \frac{2}{x+2}$
1(ci)	$=(x-4)^2-4^2+25$ $= (x-4)^2+9$
1(cii)	$(x-4)^2$ is a square number, which makes it always ≥ 0. Hence adding 9 will always make the expression positive for any value of x.
2(a)	$\angle GAO = 90°$ (tan $\perp$ radius) reflex $\angle AOC = 126\times 2$ ($\angle$ at centre = 2 $\angle$s at circum.) $= 252°$ $\angle AOE = 252-180$ $= 72°$ $\angle OAE = \frac{180-72}{2}$ (base $\angle$s of isos. Δ) $= 54°$ $\angle EAG = 90-54$ $= 36°$
2(b)	$\angle EAF = \angle DCF$ ($\angle$s in same seg.) $\angle AFE = \angle CFD$ (vert. opp. $\angle$s) $\angle FEA = \angle FDC$ ($\angle$s in same seg.) $\therefore$ Triangle EAF and triangle DCF are similar. (AA)
3(ai)	$\overrightarrow{DB} = 3\boldsymbol{a}-\boldsymbol{b}$
3(aii)	$\overrightarrow{EC} = -\frac{3}{4}(3\boldsymbol{a}-\boldsymbol{b})+3\boldsymbol{a}$ $= -\frac{9}{4}\boldsymbol{a}+\frac{3}{4}\boldsymbol{b}+3\boldsymbol{a}$ $= \frac{3}{4}\boldsymbol{a}+\frac{3}{4}\boldsymbol{b}$

3(b)	$\overrightarrow{AD} = 4\boldsymbol{a} - (3\boldsymbol{a} - \boldsymbol{b})$ $= 4\boldsymbol{a} - 3\boldsymbol{a} + \boldsymbol{b}$ $= \boldsymbol{a} + \boldsymbol{b}$ $\overrightarrow{EC} = \frac{3}{4}(\boldsymbol{a} + \boldsymbol{b})$ $= \frac{3}{4}\overrightarrow{AD}$ Since $\overrightarrow{EC} = \frac{3}{4}\overrightarrow{AD}$, they are parallel vectors. Hence *EC* is parallel to *AD*.
3(ci)	$\frac{1}{3}$
3(cii)	$\frac{3}{7}$
4(a)	$\frac{100}{x}$
4(b)	$\frac{100}{x+0.64}$
4(c)	$\frac{100}{x} - \frac{100}{x+0.64} = 4.6$ $\frac{100(x+0.64) - 100x}{x(x+0.64)} = 4.6$ $64 = 4.6x(x+0.64)$ $4.6x^2 + 2.944x - 64 = 0$ $4600x^2 + 2944x - 64000 = 0$ $8(575x^2 + 368x - 8000) = 0$ $575x^2 + 368x - 8000 = 0$ (Shown)
4(d)	$575x^2 + 368x - 8000 = 0$ $x = \frac{-368 \pm \sqrt{368^2 - 4(575)(-8000)}}{2(575)}$ $x = \frac{-368 \pm \sqrt{18535424}}{1150}$ $x = 3.4237$ or $x = -4.0637$ (rej.) Cost to buy 20 litres of petrol in Jun 2022 = $(3.4237 + 0.64) \times 20$ = \$81.27 (to nearest cents) Jason is incorrect as it will cost \$81.27, which is more than \$80, to buy 20 litres of petrol in June 2022.
5(a, c, di, dii)	Refer to answer behind
5(b)	054º $(\pm 1°)$
5(e)	44 metres (± 1)

5(f)	$\tan\theta = \frac{9}{44}$ $\theta = \tan^{-1}\left(\frac{9}{44}\right)$ $\theta = 11.6°$ (to 1 d.p.)
6(a)	$\angle AED = 180 - 60$ (adj. $\angle$s on a st. line, equilateral Δ) = 120º $\angle ADE = 180 - 120 - 35$ ($\angle$ sum of Δ) = 25º $\frac{AE}{\sin 25°} = \frac{9}{\sin 35°}$ $AE = \frac{9\sin 25°}{\sin 35°}$ = 6.63131 = 6.63 cm (to 3SF)
6(b)	AC = 6.63131 + 9 + 5 = 20.631 cm $\cos ABC = \frac{17.5^2 + 7^2 - 20.631^2}{2(17.5)(7)}$ $\angle ABC = \cos^{-1}\left(\frac{17.5^2 + 7^2 - 20.631^2}{2(17.5)(7)}\right)$ = 106.696º = 106.7º (to 1 d.p.)
6(c)	Area of triangle ABC = $\frac{1}{2}\times 17.5\times 7\times \sin 106.696°$ = 58.7 cm² (to 3SF)
7(a)	$\frac{1}{3}\pi(5^2)h = \frac{3}{4}\times\frac{2}{3}\pi(5^3)$ $\frac{1}{3}h = \frac{1}{2}(5)$ $h = \frac{1}{2}(5)(3)$ $h = \frac{15}{2}$ = 7.5 cm (Shown)
7(b)	$\frac{\frac{1}{2}EF}{5} = \frac{4}{7.5}$ $\frac{1}{2}EF = \frac{20}{7.5} = \frac{8}{3}$ cm Volume of empty space = $\frac{1}{3}\pi\left(\frac{8}{3}\right)^2$ (4) $= \frac{256}{27}\pi$ cm³ Volume of sand = $\frac{1}{3}\pi(5^2)(7.5) - \frac{256}{27}\pi + \frac{2}{3}\pi(5^3)$ = 428.36 cm³ = 428 cm³ (to 3SF)

<table>
<tr><td>7(c)</td><td>$\left(\frac{20}{5}\right)^2 = \frac{16}{1}$
The material needed to make the larger ornament is 16 times more than the original amount.
Hence James is not correct.</td></tr>
<tr><td>8(a)</td><td>1.5</td></tr>
<tr><td>8(b)</td><td>Refer to answer behind</td></tr>
<tr><td>8(c)</td><td>(- 0.8, 1.525) and (0.825, 0.475) (± 0.025)</td></tr>
<tr><td>8(d)</td><td>$\frac{x^3}{2} - x - 1 + 2 = 2$
$\frac{x^3}{2} - x + 1 = 2$
Draw y = 2.
There is only 1 point of intersection between the line y = 2 and the curve $y = \frac{x^3}{2} - x + 1$.
Hence there is only 1 solution.</td></tr>
<tr><td>8(ei)</td><td>$y = -\frac{1}{2}x + 1$<table><tr><td>x</td><td>– 1</td><td>0</td><td>1</td></tr><tr><td>y</td><td>1.5</td><td>1</td><td>0.5</td></tr></table></td></tr>
<tr><td>8(eii)</td><td>$y = -\frac{1}{2}x + 1$ --- (1)
$y = \frac{x^3}{2} - x + 1$ --- (2)
Sub (1) into (2),
$\frac{x^3}{2} - x + 1 = -\frac{1}{2}x + 1$
$\frac{x^3}{2} - x + \frac{1}{2}x + 1 - 1 = 0$
$\frac{x^3}{2} - \frac{1}{2}x = 0$
$\frac{x^3 - x}{2} = 0$
$x^3 - x = 0$ (Shown)</td></tr>
<tr><td>9(ai)</td><td>$27.50</td></tr>
<tr><td>9(aii)</td><td>$33 – $22
= $11</td></tr>
<tr><td>9(aiii)</td><td>The interquartile range will remain the same since all students’ expenses increase by $5.</td></tr>
</table>

9(bi)	
9(bii)	P(late on at least 1 day) = $1-\frac{4}{5}\times\frac{4}{5}$ = $\frac{9}{25}$
9(biii)	P(late for sch) = $\frac{1}{2}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{4}$ = $\frac{9}{40}$
10(a)	For lowest possible cost of rental, 1. rent seventeen 10-pax round table for 170 pax and two 6-pax round table for the remaining 11 pax 2. rent exactly 181 chairs Cost = $17(25)+2(20)+181(2.4)$ = \$899.40
10(b)	Lowest cost for 9 required items, including beverage = 1+2.5+2.8+4.0+1.8+1.8+1.5+1.2+0.5 = \$17.10 Lowest cost for 10 items with 5% discount = (17.10 + 1.5) x 0.95 = \$17.67 Difference = 17.67 – 17.10 = \$0.57 Hence, the promotion price is higher by \$0.57. **Accept answers if students calculate with handling fee/GST/number of students etc.*

10(c)

Fixed cost (GST included):
1. Air Cooler Rental + GST = $450 x 1.08 = $486
2. Party Decoration Package = $2500
Sub-total = $2986

No. of pax	181	~199 (10%)	~208 (15%)
Cost per pax	$16.497	$15.005	$14.356

Fixed cost (per pax) (GST included):
3. Cost of food per pax, with 2% handling fee & GST
= $17.10 x 1.02 x 1.08 = $18.837

Variable cost:
4. Table and Chairs Rental

No. of pax	181	~199 (10%)	~208 (15%)
without GST	$17(25)+2(20)+181(2.4)$ = $\underline{\$899.40}$	$20(25)+199(2.4)$ = $977.60 $20(25)+201(2.2)$ = $\underline{\$942.20}$	$21(25)+208(2.2)$ = $\underline{\$982.60}$ $19(25)+3(20)+208(2.2)$ = $992.60
with 8% GST)	$971.352	$1017.576	$1072.008
Cost per pax	$5.367	$5.113	$5.154

No. of pax	181	~199 (10%)	~208 (15%)
Total cost per pax	$16.497+ $18.837 + $5.367 = $40.70 (to 2 d.p)	$15.005 + $18.837 + $5.113 = $38.96 (to 2 d.p)	$14.356 + $18.837 + $5.154 = $38.35 (to 2 d.p)

If each student pays $39, it will be enough to cover the costs incurred fully if there is an increase in attendance between 10 to 15%.
This is provided the school rents 201 chairs such that the unit cost of renting is reduced and the total cost of renting is lower than that of renting just 199 chairs.