Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2016

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

10 October 2016

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL information booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**

Section A (60 Marks)		Section B (60 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**10**	
2			
3			
4		**11**	
5			
6			
7		**12**	
8			
9			
Subtotal		**Subtotal**	
TOTAL	**/ 120**		

WORLD SCHOOL ib

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A polynomial $x^4 + ax^2 + bx + 8$ has factor $x^2 - 3x + 2$. Find the values of *a* and *b*. [6]

2. [Maximum mark: 8]

(a) If $\cos(x - \alpha) = k\cos(x + \alpha)$, express $\tan x$ in terms of *k* and $\tan\alpha$. [4]

(b) Hence find the value(s) of *x*, where 0°≤ x <360°, when $k = \frac{1}{2}$ and $\alpha = 70°$. [4]

3. [Maximum mark: 8]

(a) Describe a sequence of **two** geometric transformations which maps the function

$g(x) = 3^{2x+1}$ to $y = \frac{1}{g(x)}$. [4]

(b) If $\frac{1}{g(x)}$ intersects $y = -x^2 + k$ at only one point, find the value of *k* corrected to 3 decimal places. [4]

4. [Maximum mark: 6]

Solve the simultaneous equations $iz + 2w = 0$ and $z - w^* = 3$, giving z and w in the form of $a + ib$, where $a, b \in \mathbf{R}$. [6]

5. [Maximum mark: 8]

A function has equation $y = \frac{1}{x} + \frac{2}{x^2}, x \neq 0$.

(a) Using differentiation or otherwise, find the coordinate of the non-stationary inflexion point of the curve. [3]

(b) Sketch the curve, stating the equations of any asymptotes and the coordinates of any turning points and points of intersections with the axes. [3]

(c) Hence find the solution set of the inequality $-\frac{2}{9}x^2 + x + \frac{20}{9} > \frac{1}{x} + \frac{2}{x^2}$. [2]

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6. [Maximum mark: 5]

(a) Two boats depart from the same pier at right angles to each other. The faster boat travels at 8km/h and the slower boat travels at 6km/h. Show that the rate at which the distance between the boats changes is independent of time. [3]

(b) A cruise ship departs from the same pier according to $x(t) = 3t - 1, y(t) = 2t + 3$, where the distance units are km and the time units are hours. Find the speed of the cruise ship. [2]

7. [Maximum mark: 5]

The plane π has equation $3x - y - 2z = -28$.
P is a point on the plane with coordinates $(-4, 6, 5)$.
A line perpendicular to the plane passes through P, find the exact value of the shortest distance from the line to the origin.

8. [Maximum mark: 8]

Given that $z_1 = 2$ and $z_2 = 1 + i\sqrt{3}$ are roots of the cubic equation $z^3 + az^2 + bz + c = 0$ where *a, b, c* ϵ R,

(i) write down the third root, z_3, of the equation; [1]

(ii) find the values of *a, b* and *c*. [4]

(iii) Explain why $z^4 - 6z^3 + 16z^2 - 24z + 16 = 0$ has only **three distinct** roots, z_1, z_2 and z_3. [3]

9. [Maximum mark: 6]

A group of 6 men and 6 women are assigned to sit in three distinct rows of four seats each.

(i) Find the number of ways in which the 12 people can be arranged if there is no restriction. [2]

(ii) Find the number of ways to arrange the 12 people if each row has at least one woman. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 21]

(a) Find the values of *h* for which the following system of equations has
(i) no solution,
(ii) an infinite number of solutions.

$$\pi_1: x + 2y - 2z = -2$$
$$\pi_2: -x + 2y - z = 5$$
$$\pi_3: -4y + 3z = h$$

[4]

(b) Given that the system of equations can be solved, find the line of intersection of the planes which has direction vector $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$. [5]

(c) Show that the line $\frac{x-4}{2} = z + 2,\ \ y = -5$ lies on the plane π_1. [5]

(d) The z-axis meets the plane π_2 at the point P.
(i) Find the coordinates of P. [2]
(ii) Hence find the coordinates of F, the foot of the perpendicular from P to π_1. [5]

11. [Maximum mark: 23]

A function is defined by $4y^2 + 8y + x^2 = 1$.

(a) Show that $\frac{dy}{dx} = \frac{-x}{4y+4}$. [3]

(b) Find the equation of the normal to the curve at the point $\left(\frac{1}{4}, -\frac{1}{8}\right)$. [4]

(c) Find the distance between the two points on the curve where each tangent is parallel to the line $y = \frac{1}{4}x$. [7]

(d) By completing the square or otherwise, show that the curve can be expressed in the form $(y-h)^2 + \frac{x^2-a}{k} = 1$, where *a*, *h* and *k* are integers to be determined. [4]

(e) Sketch the curve, stating the coordinates of any stationary point(s) and point(s) of intersection(s) with the axes. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 16]

(a) Differentiate the following with respect to *x*:

(i) $y = \arccos x + 2\sqrt{1 - x^2}$, $x \in [-1, 1]$ [3]

(ii) $y = \sin^2\left(\frac{1}{x}\right)$ [4]

(b) A ladder is sliding down along a vertical wall. The ladder is 10 meters long and the top is slipping at the constant rate of 10 m/s.

(i) How fast is the bottom of the ladder moving along the ground when the bottom is 6 meters from the wall? [4]

(ii) If A is the area under the ladder formed by the ladder, wall and ground, how far is the bottom of the ladder away from the wall when A is maximum? [3]

(iii) State the range of values of the horizontal distance of the bottom of the ladder from the wall when A is decreasing. [2]

END OF PAPER