Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2017

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

11th October 2017

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- No calculators are allowed for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		9	
2			
3		10	
4			
5			
6		11	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

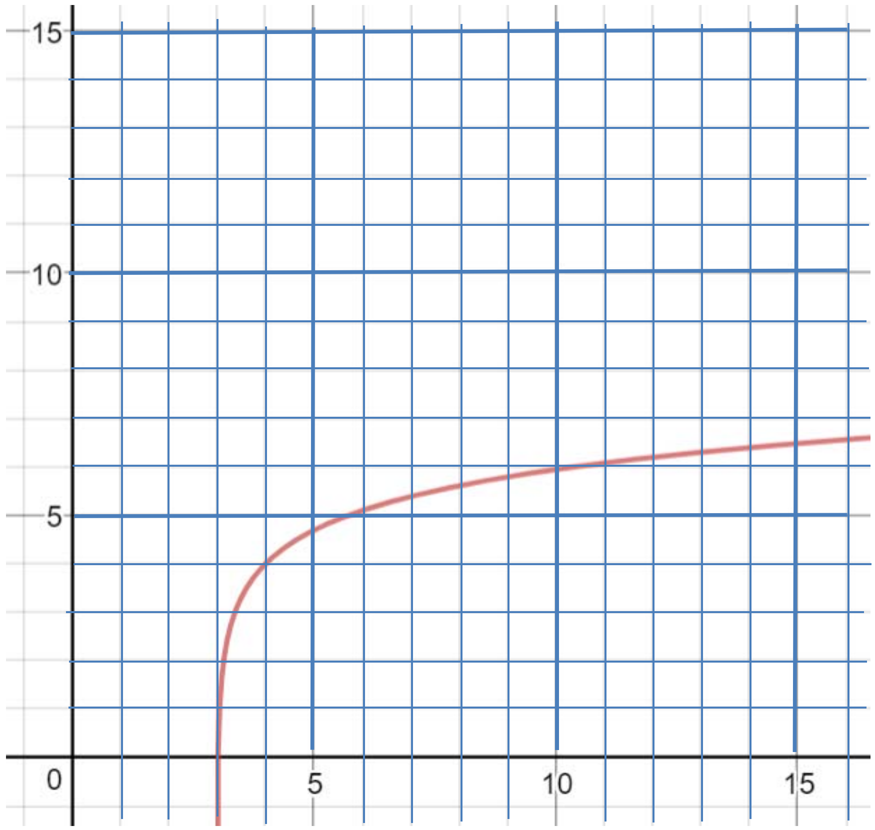
1. [Maximum mark: 6]

(a) Find the term independent of x in the expansion of $\left(x^2+\frac{2}{x}\right)^6$. [3]

(b) Prove that $\binom{n}{r}\times\frac{n-r}{r+1}=\binom{n}{r+1}$. [3]

2. [Maximum mark: 5]

The diagram shows the graph of f .



(a) On the same diagram, sketch the graph of f^{-1} . [4]

(b) Hence find the point of intersection of $f(x)$ and $f^{-1}(x)$. [1]

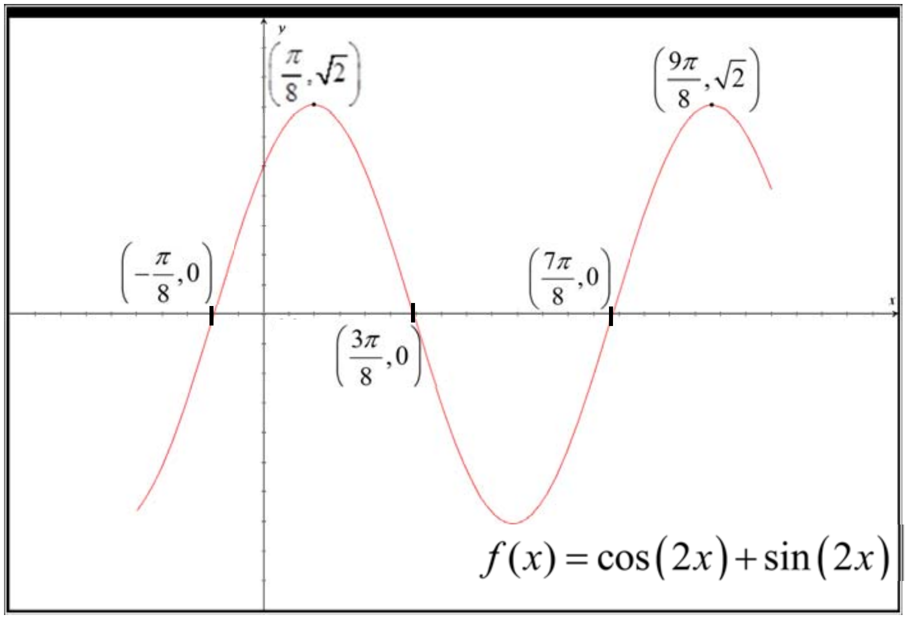
3. [Maximum mark: 5]

The cubic polynomial $f(x)$ and quadratic polynomial $p(x)$ are related by the following equation: $f(x) = p(x)\cdot(x-1)+4$.

The roots of $f(x)$ are at $x = 2$ and $x = 3$. Find $f(x) = x^3 + Ax^2 + Bx + C$, where A, B and C are constants to be determined.

4. [Maximum mark: 5]

The graph of function $f(x) = \cos(2x) + \sin(2x)$ for $-1 \le x \le 4$ is shown below.



(a) The function f can also be expressed as $f(x) = a\sin(2(x+b))$ where $a \in \mathbb{R}$ and $0 \le b \le 2$. Find the value of a and b. [2]

(b) Find $f'(x)$, hence show that $\cos(2x) - \sin(2x) = \sqrt{2}\cos\left(2\left(x + \frac{\pi}{8}\right)\right)$. [3]

5. [Maximum mark: 5]

Determine the relationship between the following lines. Explain your answer clearly

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 8 \\ 3 \end{pmatrix} + k \begin{pmatrix} 2 \\ 5 \\ 4 \end{pmatrix}, k \in \mathbb{R}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 9 \end{pmatrix} + h \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}, h \in \mathbb{R}.$$

6. [Maximum mark: 5]

Use mathematical induction to prove that $7-4^n$ is divisible by 3 for $n \in \mathbb{Z}^+$.

7. [Maximum mark: 7]

The functions f and g are defined by $f(x) = x^2 + 1$, $x \in \mathbb{R}$ and $g(x) = -\sqrt{x-1}$, $x \in \mathbb{R}, x \geq 1$.

(a) State the ranges for f and g. [2]

(b) Give a reason why fg exists. [2]

(c) Find the composite function fg and state its domain. [3]

8. [Maximum mark: 12]

(a) Given the equation of the curve $x^2 - 2xy + 2y^2 = 4$, find $\dfrac{dy}{dx}$ in terms of x and y.

Hence find the value(s) of x when the tangent is parallel to the x-axis. [8]

(b) Differentiate $\arctan\sqrt{x}$ with respect to x. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 22]

Consider the function $f(x)=\dfrac{ax+b}{cx+d}$, $x\in\mathbb{R}$, $x\neq-\dfrac{d}{c}$ for non-zero constants a, b, c and d.

(a) (i) Find an expression for $f'(x)$.

(ii) Given that $ad>bc$, show that $f(x)$ is an increasing function for all values of

$x\in\mathbb{R}$, $x\neq-\dfrac{d}{c}$. [6]

(b) Given $a=2$, $b=0$, $c=3$ and $d=-2$, find $f^{-1}(x)$ for $x\in\mathbb{R}, x\neq\dfrac{2}{3}$.

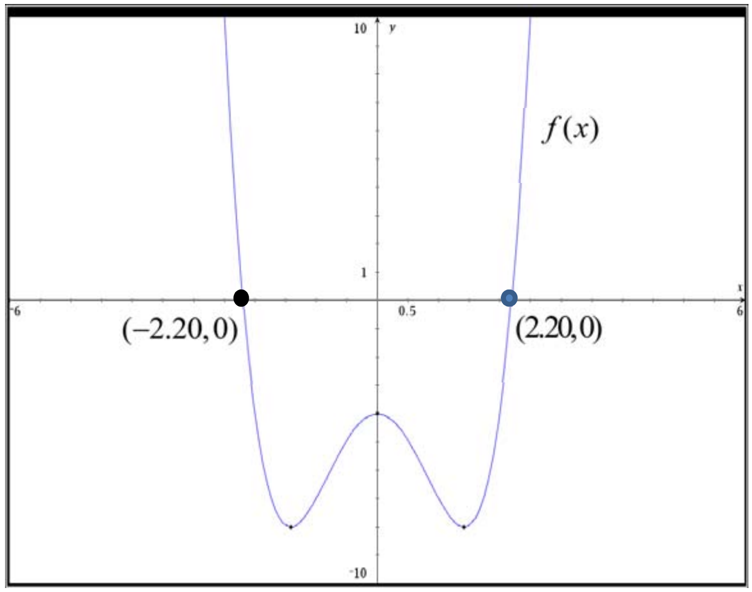
(i) Hence or otherwise show that $f^{2}(x)=x$ and find $f^{201}(x)$.

(ii) With the given values of a,b,c and d, solve the inequality $f(x)\leq x$. [13]

(c) The function h is defined by $h(x)=p\sin x+qx+r$, $x\in\mathbb{R}$, where p,q and r are real constants. Given that h is an odd function, find the value of r. [3]

10. [Maximum mark: 15]

The graph of a polynomial function $f(x) = x^4 - 4x^2 - 4$ of degree four is shown below. $f(x)$ has roots at $(-2.20, 0)$ and $(2.20, 0)$.



(a) Determine the interval(s) on the domain of f where f is concave downwards. [4]

(b) The graph of $f(x)$ is translated by h units in the positive y-direction. Find all possible value(s) of h such that the translated function have four real and distinct roots. [6]

(c) Sketch the graph of $\dfrac{1}{f(x)}$ for the function $f(x)$ shown above. State clearly the coordinates of any asymptotes, intercepts and turning points on the graph. [5]

[Turn the page over]

11. [Maximum mark: 13]

Let $p(x) = 8x^3 - 8x^2 - 6x + 10, x \in \mathbb{C}$.

(a) The roots of the equation $p(x) = 0$ are α , β and γ . Find the values of

(i) $\alpha + \beta + \gamma$,

(ii) $\alpha\beta\gamma$,

(iii) $\alpha\beta + \alpha\gamma + \beta\gamma$. [3]

(b) Find all three roots of $p(x) = 0$.

Show that the roots form a triangle with an area of 1 unit^2 when plotted on an Argand diagram. [5]

(c) A new polynomial is defined by $q(x) = p(x-2)$.

(i) Find the value of the sum of roots of the equation $q(x) = 0$.

(ii) Find the value of the product of roots of the equation $q(x) = 0$. [5]

END OF PAPER