Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2018

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

8 October 2018

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- No calculators are allowed for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL information booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		9	
2			
3		10	
4			
5			
6		11	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

WORLD SCHOOL ib

This question paper consists of 11 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Find the term independent of *x* in the expansion of $\left(\frac{1}{\sqrt{x}} - 1\right)^4 (1-x)^2$. [5]

2. [Maximum mark: 5]

(i) Show that $\frac{1}{\sqrt{4r-3}+\sqrt{4r+1}} = \frac{\sqrt{4r+1}-\sqrt{4r-3}}{4}$ [3]

(ii) Hence find the exact value of $\sum_{r=1}^{100} \frac{1}{\sqrt{4r-3}+\sqrt{4r+1}}$. [2]

3. [Maximum mark: 5]

Consider the geometric series $1 + tan\theta + tan^2\theta + tan^3\theta + \ldots.$ such that $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$. Find the values of θ for which the sum to infinity $S_\infty < \frac{3+\sqrt{3}}{2}$.

4. [Maximum mark: 6]

The equation of a curve is given by $4x^3 + xy^2 = 5xy$. The curve meets the line $y = x$ at the origin and the point P. Find the equation of the tangent to the curve at P. [6]

5. [Maximum mark: 9]

(a) There are 4 married couples, 1 single man and 1 single woman. Find the number of ways these 10 people could be arranged in a row if the couples are with their spouse. [3]

(b) By mathematical induction, prove that the formula for sum to n terms of a Geometric Progression whose first term is *a* and common ratio, *r* is given by

$$S_n = \frac{a(r^n - 1)}{r - 1}$$ [6]

6. [Maximum mark: 8]

The roots of the equation $2x^3 + x^2 - 5x + 3 = 0$ are α, β and γ. Find

(i) $\alpha^2 + \beta^2 + \gamma^2$

(ii) a cubic equation, with numerical coefficients, which has roots $\alpha - 1, \beta - 1$ and $\gamma - 1$.

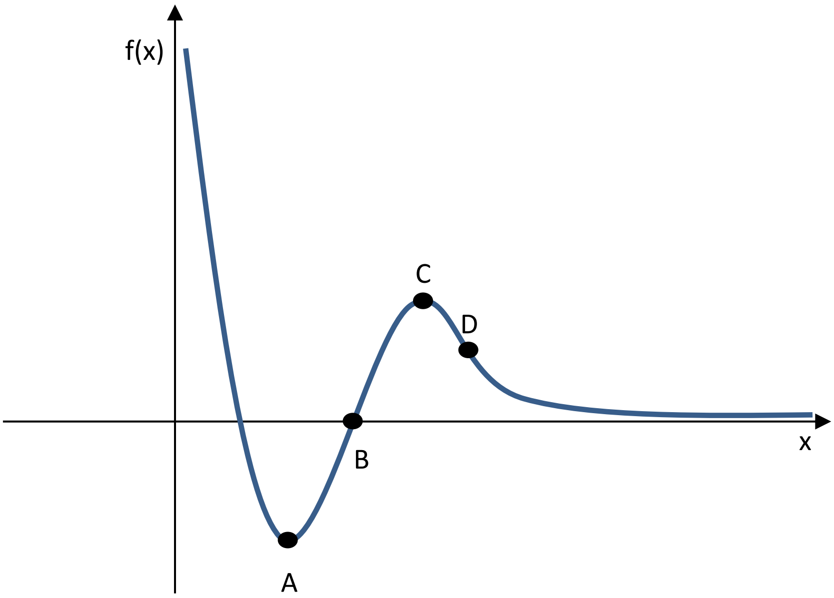
7. [Maximum mark: 6]

(i) Express $\frac{\pi}{12}$ as a difference of two angles and hence prove that $\arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \frac{\pi}{12}$. [4]

(ii) Write down the value of x for which $\arccos(x) = \frac{5\pi}{12}$. [2]

8. [Maximum mark: 6]

The diagram below shows the graph of a function y = f(x), with x and y axes as the asymptotes. Point A is the local minimum and C the local maximum. B and D are non-stationary points of inflection. Sketch the graph of the derivative (or gradient) function, $y = f'(x)$, in the space provided below. Label points A, B, C, D and any asymptotes clearly on your graph.

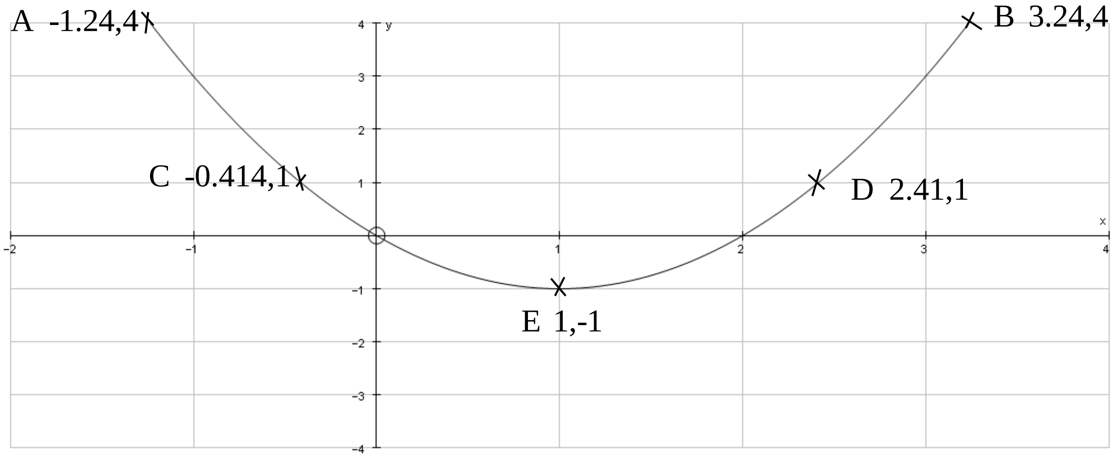


Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 18]



(a)

(i) The diagram shows the function $g(x) = x(x-2)$. By writing $g(x)$ in the form $(x-a)^2 + b$ or otherwise, describe the geometrical transformation which maps $f(x) = x^2$ onto $g(x)$. [5]

(ii) Sketch the graph of $\frac{1}{g(x)}$, stating clearly the coordinates of the images of A, B, C, D and E. [4]

(b) The function h is defined by $h: x \to (x-1)^2 + 2, x \in R, x \geq 1$. The function k is defined by $k: x \to ax + b$, $x \in R$ where $x \geq -2$, *a* and *b* are constants.

If $hk: x \to 4x^2 + 16x + 18, x \in R, x \geq -2$,

(i) Explain whether $k: x \to -2x - 3,\ x \geq -2$ is possible. [2]

(ii) Find the value of *a* and of *b*. [4]

(c) The function *p* is defined by $p: x \to \frac{\mathbf{4}}{(\mathbf{2x-3})^{\mathbf{2}}+\mathbf{1}}$ for $x \in R, x \leq \frac{3}{2}$, find $p^{-1}(x)$ and write down the domain of $p^{-1}(x)$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

(a) Two skew lines are defined by

$$l_1: \boldsymbol{r} = \begin{pmatrix} -3 \\ -4 \\ 6 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}, t \in R \quad \& \quad l_2: \boldsymbol{r} = \begin{pmatrix} 4 \\ -7 \\ -3 \end{pmatrix} + s\begin{pmatrix} -3 \\ 4 \\ -1 \end{pmatrix}, s \in R$$

The lines lie on parallel planes.

(i) Find a vector, **n,** perpendicular to both l_1 and l_2. [2]

(ii) Find the shortest distance between l_1 and l_2. [4]

(b) The position vector of a point P at time *t* is given by

$$\boldsymbol{p} = (1+t)\boldsymbol{i} + (2-2t)\boldsymbol{j} + (3t-1)\boldsymbol{k}, \; t \geq \mathbf{0.}$$

(i) Find the initial position of P. [2]

(ii) Show that P moves along the line L with Cartesian equation

$$\mathrm{x} - 1 = \frac{\mathbf{2-y}}{\mathbf{2}} = \frac{\mathbf{z+1}}{\mathbf{3}}$$ [2]

(iii) (a) Find the value of *t* when P lies on the plane Π with equation $2\mathrm{x} + \mathrm{y} + \mathrm{z} = 6$.

(b) State the coordinates of P at this time.

(c) Hence find the total distance travelled by P before it meets the plane. [6]

11. [Maximum mark: 16]

(a) Given $1, w, w^2, w^3, w^4$ and w^5 are solutions of the complex equation $\mathrm{z}^6 = 1$, find in terms of w, all solutions of *z* satisfying $(1-\mathrm{z})^6 - \mathrm{z}^6 = 0.$ [3]

(b)

(i) Using de Moivre's theorem, show that

$$\left(\sqrt{3} - i\right)^n = 2^n\left(cos\frac{1}{6}n\pi - isin\frac{1}{6}n\pi\right)$$ where *n* is an integer. [4]

(ii) Determine the values of positive integers *n* such that $\left(\sqrt{3} - i\right)^n$ is real and negative. [4]

(iii) If $\sqrt{\mathbf{3}} - \boldsymbol{i}$ is a root of the equation $z^9 + 16(1+i)z^3 + a + ib = 0$**,** find the values of the real constants *a* and *b*. [5]

END OF PAPER