Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2019

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

24 September 2019

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**8**	
2			
3		**9**	
4			
5			
6		**10**	
7			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

This question paper consists of 10 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Find the value of k if

$$\sum_{r=0}^{\infty} \frac{k}{2^r} = 5\,.$$

.

2. [Maximum mark: 5]

Consider the equation $(h-1)x^2 - xh + (h+1) = 0$, where h is a real constant.

Find the set of values of h for which this equation has real roots.

3. [Maximum mark: 8]

The sum of the first *n* terms of a sequence $\{u_n\}$ is given by $S_n = n^2 + r^2 n - \frac{3}{4}n$, where $n \in Z^+$ and $r \in R$

(a) Write down the value of u_1 in terms of r. [1]

(b) Prove that $\{u_n\}$ is an arithmetic sequence, stating clearly its common difference. [7]

4. [Maximum mark: 8]

Use Mathematical Induction to prove that $(1+x)^n \geq 1 + nx$ for $\{n: n \in Z^+\}$ where $x > 1$.

5. [Maximum mark: 8]

(a) Express the binomial coefficient $\binom{n+1}{n-2}$ as a polynomial in n. [3]

(b) Hence find the coefficient of x^{5-n} in the expansion of $\left(x+\frac{1}{x}\right)^{n+1}$ in terms of n. [5]

6. [Maximum mark: 7]

(a) Show that $log_{r^2} x = \frac{1}{2} \log_r x$. [2]

(b) Hence express $y = log_3 x + log_9 x$ in terms of $ln(x)$ and describe the transformation that will map $y = ln(x)$ into $y = log_3 x + log_9 x$. [5]

7. [Maximum mark: 9]

Consider the equations of the lines:

$$l_1:\ \mathbf{r}=\begin{pmatrix}-11\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\1\end{pmatrix},\ \lambda\in\mathbb{R} \text{ and } l_2:\ \frac{x+2}{3}=y-1=1-z.$$

(a) Determine whether or not l_1 and l_2 intersect. [6]

(b) Find a vector that is perpendicular to the two lines. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

8. [Maximum mark: 16]

i) (a) Expand and simplify $(x-1)(x^5+x^4+x^3+x^2+x+1)$. [2]

(b) Given that *w* is a root of the equation $z^6-1=0$ which does not lie on the real axis in the Argand diagram, show that $w^5+w^4+w^3+w^2+w+1=0$. [2]

(c) Find the roots of the equation $z^6=1$, giving your answers exactly in the form $re^{i\theta}$, where $-\pi<\theta\leq\pi$. [3]

ii) Consider the complex numbers $z_1=1-\sqrt{3}i$ and $z_2=2-2i$.

(a) Find the exact value of the argument of $\frac{z_1}{z_2}$. [3]

(b) Express $\frac{z_1}{z_2}$ in the form $a+ib$, where $a,b\in R$. [3]

(c) Using your results, find the exact value of $tan\frac{-\pi}{12}$, giving your answer in the form $\sqrt{a}-b$, $a,b\in Z^+$. [3]

9. [Maximum mark: 16]

(a) The curve *C* is defined by equation $2x^2-y^2+xy=4$.

(i) Find $\frac{dy}{dx}$ in terms of *x* and *y*. [3]

(ii) Explain clearly whether *C* has any turning point/s. [3]

(iii) Find the equation of the tangent to the curve *C* at the point $P(\sqrt{2},0)$. [2]

(iv) The tangent to the curve *C* at the point $P(\sqrt{2},0)$ meets the y-axis at the point *A*. Find the area of triangle *PAO*. [2]

(b) Consider the function f defined by $f(x)=e^x cosx$, $0\leq x<\pi$.

(i) Find an expression for $f'(x)$. [2]

(ii) Express $f''(x)$ in the form $Ae^x sinx$, where A is a constant. [2]

(iii) Find the coordinates of the point of inflexion/s of the graph of f. (The proof of inflexion is not required.) [2]

[Turn the page over]

10. [Maximum mark: 18]

(a) The functions f and g are defined by

$$f : x \mapsto -(x-2)^2 + 4, \quad x \in \mathbb{R}, \ x \le 2,$$

$$g : x \mapsto \frac{1}{(x-7)^2 - 4}, \quad x \in \mathbb{R}, \ x \le k, \ x \ne 5.$$

(i) Sketch the graphs of f, f^{-1} and ff^{-1} on the same axes, showing the relationship between the three graphs and their domains. [4]

(ii) If $f(a) = f^{-1}(a)$, show that $a^2 - 3a = 0$. [3]

(iii) State the maximum value of k such that the inverse of g exists. [1]

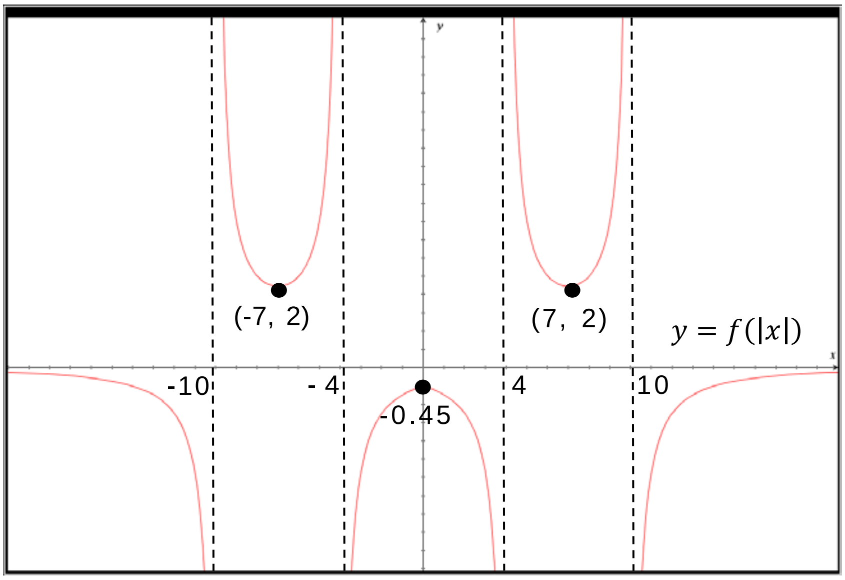
(iv) Hence find the inverse of g, stating its domain. [4]

(b) A rational function is defined by $f(x) = \frac{a}{(x-b)(x-c)}$ where the parameters $b, c > 0$ and a, b and $c \in Z$.

The following diagram represents the graph of $y = f(|x|)$.

$f(|x|)$ has a maximum point (0, -0.45) and minimum points (-7, 2) and (7, 2).

The horizontal asymptote of $f(|x|)$ is $y = 0$ and the vertical asymptotes of $f(|x|)$ are $x = -10$, $x = -4$, $x = 4$ and $x = 10$.



(i) Using the information on the graph, find the values of *a*, *b* and *c*. [3]

(ii) Sketch the graph of $\frac{1}{f(|x|)}$. [3]

END OF PAPER