Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2019

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

26th September 2019

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

Section A (50 Marks)		Section B (50 Marks)	
Question	**Marks**	**Question**	**Marks**
1		**9**	
2			
3		**10**	
4			
5			
6		**11**	
7			
8			
Subtotal		**Subtotal**	
TOTAL	**/ 100**		

This question paper consists of 11 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Consider the ΔABC with $\measuredangle BAC = 18°$, $AC = 6$ units and $BC = 3$ units. Find the possible values of the length of AB.

2. [Maximum mark: 6]

(a) Given that $z^* = \dfrac{2-2i}{-1+\sqrt{3}i}$, find the values of $|z|$ and $\arg(z)$. [4]

(b) Hence, find the value of $\arg(-z)$. [2]

3. [Maximum mark: 4]

Consider the system of equations:

$$x - y + 2z = -1$$
$$2x + y - z = 3$$
$$2x - 2y + kz = 0$$

(a) Given $k = 2$, find the solution of the system. [2]

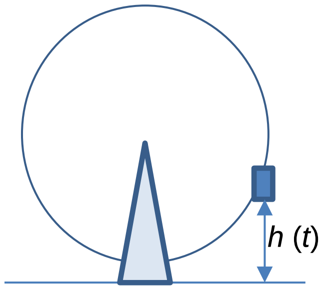
(b) Comment on the system, with clear explanation, when $k = 4$. [2]

4. [Maximum mark: 7]

The height of the base of a cabin of the Ferris Wheel is given

by $h(t) = 114 - 113\cos(\frac{t}{5}\pi), 0 \le t \le k$, where k is an integer,

t is in minutes and h is in metres above the ground in one cycle.



(a) State the value of k. [1]

(b) Find the maximum and the minimum height of the base of this cabin. [2]

(c) Find the duration when the height is at least 200m. [4]

5. [Maximum mark: 5]

There are eight boys, six girls and five adults in a party. To play a game they need to form a team of five players. How many ways can they form the team, if

(a) there are no restrictions, [2]

(b) there is only one adult in the team. [3]

.

6. [Maximum mark: 9]

Given $P = R\cos y$ and $Q = R\sin y$ where P, Q and $y \in \mathbb{R}$, show that

(a) $R^2 = P^2 + Q^2$ [2]

(b) $y = \arctan\dfrac{Q}{P}$ [1]

(c) $P\cos x + Q\sin x \equiv R\cos(x - y)$ [2]

(d) Hence solve $3\cos x + 4\sin x = 5$ for $0 \le x \le 5$. [4]

7. [Maximum mark: 7]

The function f is defined by $f(x) = x\sqrt{4-x}$.

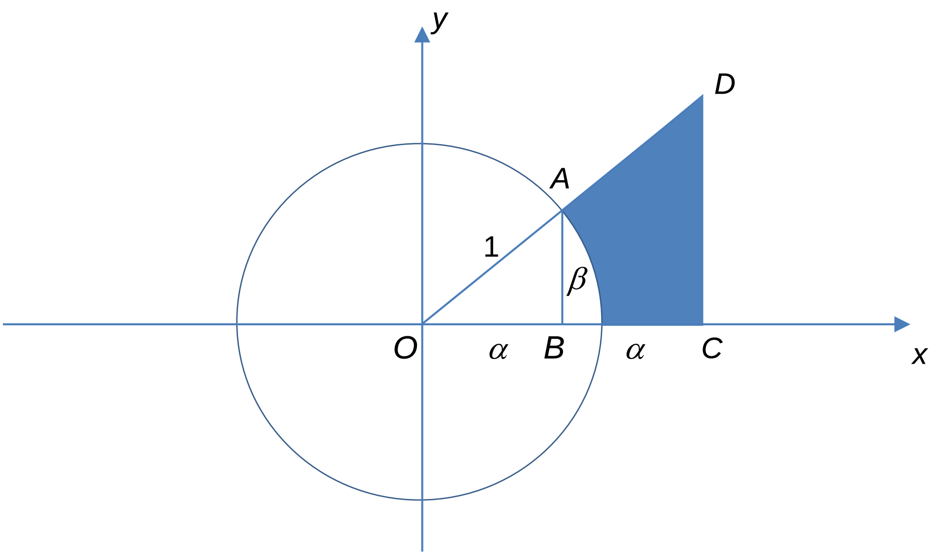
(a) State the largest possible domain for f . [1]

(b) Sketch the graph of $y = f(x)$ and $y = f'(x)$ on the same axes. Showing clearly all the intercepts and turning point(s). [3]

(c) Hence solve $f'(x) > f(x)$. [3]

8. [Maximum mark: 7]

The radius of the circle in the diagram below is 1 unit. D lies on the extension of OA. Points B and C are on the x-axis. Given that $OB = BC = \alpha$ units, $AB = \beta$ units and both AB and DC are perpendicular to the x-axis.



(a) Show that the area of the shaded region is $2\alpha\beta - \frac{1}{2}\arcsin\beta$ sq units. [4]

(b) If $\alpha = \beta$, show that the area is $1 - \frac{\pi}{8}$ sq units. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 12]

Let f be a function defined by $f(x) = \cos x - \arcsin\left(\left|\frac{x}{5}\right|\right)$ for $-5 < x < 5$ and $x \neq 0$.

(a) Explain clearly whether f is an even or odd function. [2]

(b) Sketch the graphs of $y = f'(x)$ and $y = f''(x)$ on the same axes, showing clearly all the x–intercepts. [6]

(c) Hence, state the x coordinates of the points of inflection. [2]

(d) State the values of x for which f is concave up. [2]

10. [Maximum mark: 21]

The points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ a \\ -1 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} -1 \\ 2 \\ b \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \text{ and } \overrightarrow{OD} = \begin{pmatrix} 11 \\ 1 \\ 1 \end{pmatrix}$$

(a) Find the values of a and b if $\overrightarrow{OA}$ is perpendicular to the plane OBC. [4]

(b) Find the Cartesian equation of the plane, π_1, that passes through D and is parallel to the plane OBC. [4]

(c) Find the coordinates of the point of intersection between π_1 and the line passing through AC. [5]

(d) Hence, or otherwise, find the distance between A and π_1. [5]

(e) Given another plane with equation, $\pi_2 : x - 2y + z = 5$, find the Cartesian equation of the line of intersection of π_1 and π_2. [3]

11. [Maximum mark: 17]

The equation of a curve is $y = \dfrac{3x^2 + x - 12}{(x+1)(e^x + 1)}$ where $x \neq k$.

(a) State the value of k. [1]

(b) Sketch the curve with the intercepts and asymptotes stated clearly. [4]

(c) Hence, by drawing an additional straight line solve the equation

$\dfrac{3x^2 - x - 14}{2(x+1)} - e^x = 0$. [5]

(d) (i) Solve the inequality $\dfrac{3x^2 + x - 12}{(x+1)(e^x + 1)} > 0$. [2]

(ii) Hence solve the inequality $\dfrac{3(\tan\theta)^2 + \tan\theta - 12}{(\tan\theta + 1)(e^{\tan\theta} + 1)} > 0$ for $0 \leq \theta \leq \pi$. [5]

END OF PAPER