Raffles Girls' School
(SECONDARY)

Established in 1879

Name: ____________________

Class: 4 / ______ Register No: ______

MATHEMATICS 2
YEAR FOUR

Pen-and-Paper Assessment 1

Friday **5 March 2021** **1 hour 30 minutes**

INSTRUCTIONS TO CANDIDATES

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagram or graphs.
Do not use staples, paper clips, glue or correction tape.

Answer **all** the questions in the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

Omission of essential working will result in a loss of marks.

You are reminded of the need for clear presentation in your answers.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **60** and the weighting is **20%**.

For examiners' use

Question	Marks
1	/4
2	/5
3	/5
4	/6
5	/8
6	/9
7	/11
8	/12
PAU	
Total Marks	60

Parent's / guardian's Name: ____________________

Signature: Date:

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a curve is $y = 3x^2 + kx$ and the equation of a line is $y = -3x - k$, where k is a constant. Explain why there is only one value of k for which the line is a tangent to the curve. [4]

2 Prove the following identities:

(a) $\operatorname{cosec} 2x \equiv \tan x + \cot 2x$ [3]

(b) $\dfrac{\sin x - \sin y}{\sin x + \sin y} \equiv \cot \dfrac{x+y}{2} \tan \dfrac{x-y}{2}$ [2]

3 Given that $\sec A = m$ where $270° \le A \le 360°$, express each of the following in terms of m.

(a) $\tan(180° - 2A)$ [3]

(b) $\cos\dfrac{A}{2}$ [2]

4 Express $3\sin x+\sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ where $R>0$ and $0<\alpha<\dfrac{\pi}{2}$. Given that $0\le x\le 2\pi$, find the smallest value of x for which $\left(3\sin x+\sqrt{3}\cos x\right)^2-2$ has a minimum value.

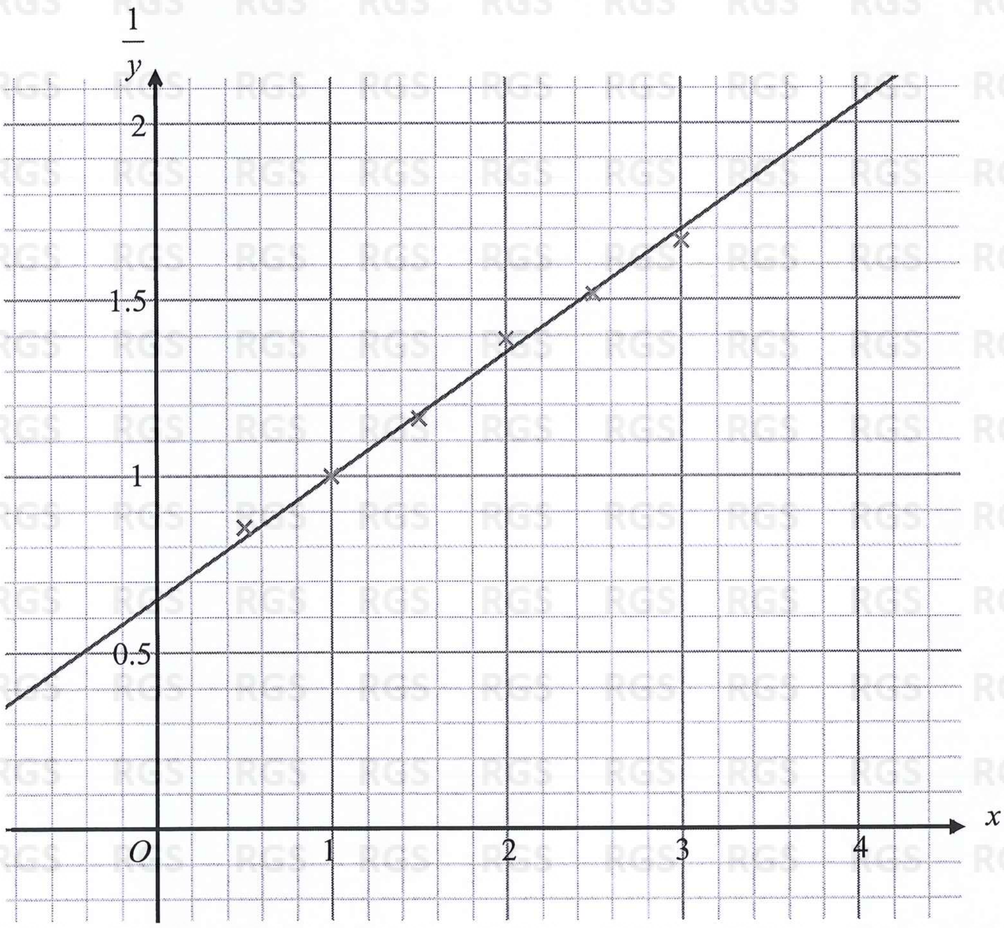
[6]

[THIS PAGE IS INTENTIONALLY LEFT BLANK FOR QUESTION 4]

5 The table below shows the experimental values of two variables x and y.

x	0.5	1.0	1.5	2.0	2.5	3.0
y	1.17	1.00	0.86	0.72	0.66	0.60

It is known that x and y are related by the equation of the form $y = \frac{a}{x+b}$, where a and b are constants. A straight line graph of $\frac{1}{y}$ against x is plotted to represent the above data.

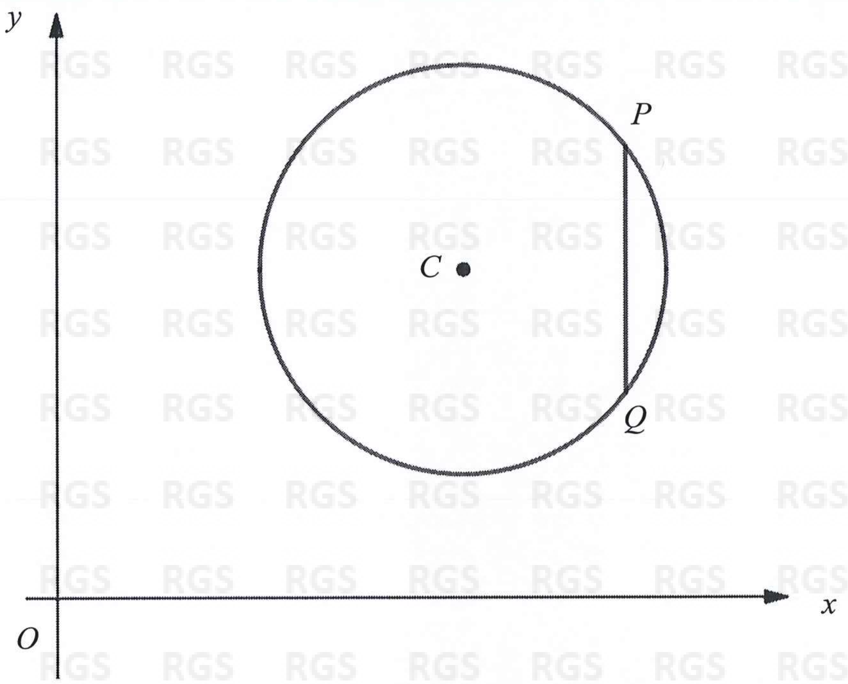


(i) Use the graph to estimate the value of a and b. [5]

(ii) By drawing another straight line on your graph, estimate the values of x and y that also satisfy $2xy + 5 = 4y$. [3]

[THIS PAGE IS INTENTIONALLY LEFT BLANK FOR QUESTION 5]

6



In the diagram above, the equation of a circle with centre C is $x^2+y^2-20x-16y+139=0$. The circle passes through the points P and Q such that PQ is parallel to the y-axis and is to the right of C. It is given that $PQ = 6$ units.

(i) By completing the squares, express the equation of the circle in standard form. State the radius and coordinates of the centre of the circle. [2]

(ii) Find the coordinates of P. [3]

(iii) Give 2 reasons to explain why the line l with equation $4x+6y=21$, is not the perpendicular bisector of PC. Show your working fully to support your reasons. [4]

[THIS PAGE IS INTENTIONALLY LEFT BLANK FOR QUESTION 6]

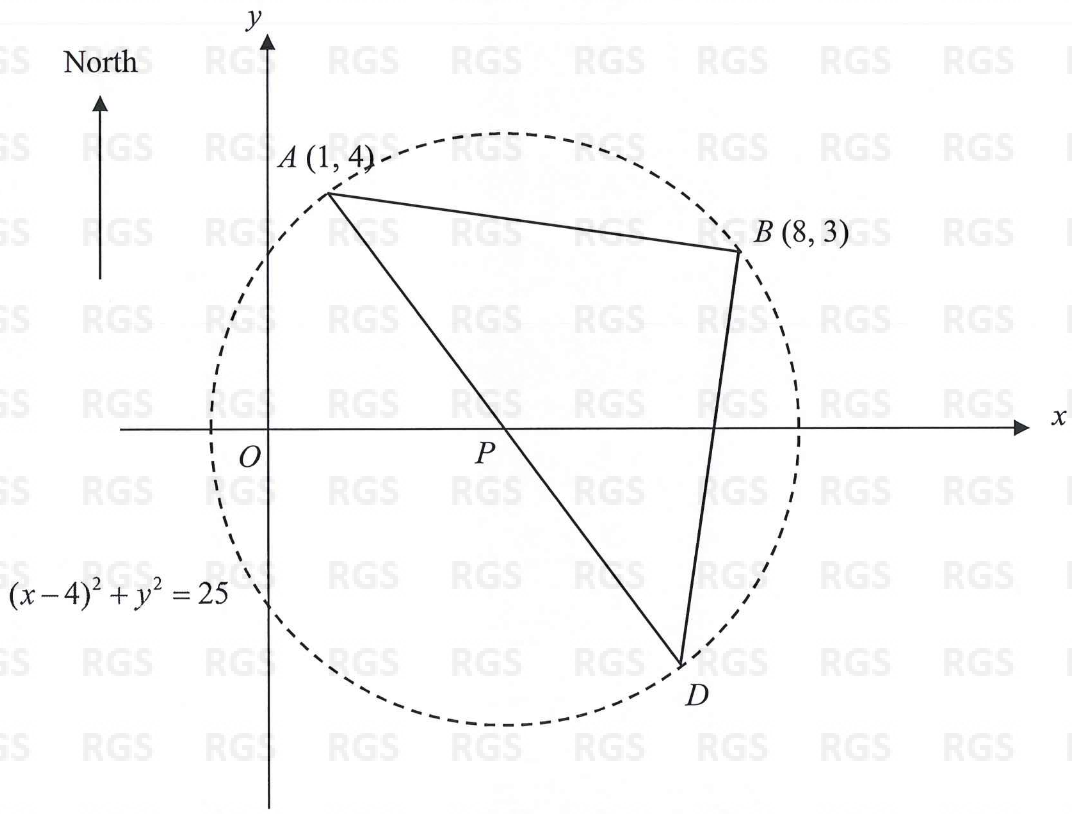
7 **(a)** Solve $\sin 2x = \sin x$ for $0 \le x \le \pi$. [4]

(b) **(i)** Show that $2 - \tan\theta = \dfrac{3(\cos 2\theta + 1)}{2\sin^2\theta}$ can be expressed as $\tan^3\theta - 2\tan^2\theta + 3 = 0$. [2]

(ii) Hence solve the equation $2 - \tan 2x = \dfrac{3(\cos 4x + 1)}{2\sin^2 2x}$ for $0° \le x \le 180°$. [5]

[THIS PAGE IS INTENTIONALLY LEFT BLANK FOR QUESTION 7]

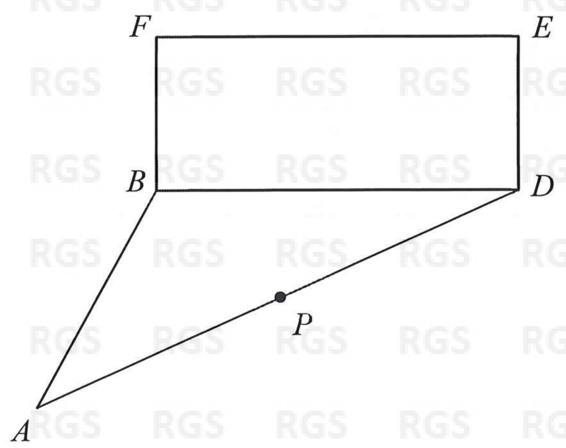
8 In the diagram, a garden is enclosed by a circle with equation $(x-4)^2+y^2=25$. Point P is the centre of the circle. Points $A(1, 4)$, $B(8, 3)$ and D lie on the circumference of the garden such that AD is the diameter.



(i) It is given that North is in the direction of the positive y-axis. Find the bearing of D from A. [3]

$BDEF$ is a 3-metre high vertical rectangular wall in the garden.

(ii) Find the largest angle of depression of A from the edge EF. Explain why this angle of depression is the largest. [3]



The garden is being enlarged to include the area enclosed by a second circle that passes through points B and D. The second circle has a radius of $5\sqrt{5}$ m and its centre is on the other side of the wall from P.

(iii) Find the equation of the second circle. [6]

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