FMS(S) Sec 4 Express Add. Math Preliminary Exam 2024 Paper 2

1	$e^{\frac{1}{2}x} = -\frac{3}{2}$ (rej. as -ve) *or* $x = 2\ln 2 = 1.39$ (3 s.f) Therefore, the equation has only one solution where x=1.39
2	$a = \frac{5}{6}, b = \frac{7}{6}$
3b	$\int \frac{\ln 2x}{x^3}\,\mathrm{d}x = -\frac{1}{2}\left(\frac{\ln 2x}{x^2} + \frac{1}{2x^2}\right) + C_3$
4a	$-5 < a < 3$
4b	$a = -2$
5a	$\frac{1}{4}$
5b	$4\sin\theta - 4\sin^2\theta$
5c	$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$
6i	$y = -\frac{8}{x} - 2x + 15$
6ii	$x = 2$ or $x = -2$
6iii	$\frac{d^2y}{dx^2} = -\frac{16}{x^3}$ At $x = 2$, $\frac{d^2y}{dx^2} = -\frac{16}{2^3} = -2 < 0$ $\therefore$ Maximum point at $x = 2$ At $x = -2$, $\frac{d^2y}{dx^2} = -\frac{16}{(-2)^3} = -\frac{16}{-8} = 2 > 0$ $\therefore$ Minimum point at $x = -2$
7a	$x^{18} + 9\,mx^{15} + 36m^2x^{12} + \ldots$
7bii	$2 \times 84 + 126 = 294$
8b	$P = 500e^{-0.1t}$
8c	$\ln 100 = 4.6$ When $\ln P = 4.6$ $t = 16$ (nearest year)
9b	$9\cos\theta + 4\sin\theta = \sqrt{97}\cos(\theta - 24.0°)$
9c	$\theta = 76.427° \approx 76.4°$
9d	Maximum value of d = $\sqrt{97}$ and occurs when $\theta = 24.0°$

10a	$v = 6\cos 4t$ When $t = 0, v = 6$ Initial velocity of the particle is 6m/s.
10b	$t = 0.870$ (3 s.f)
10c	$s = \frac{3}{2}\sin 4t + c$ When t = 0, s = 0, c = 0 $s = \frac{3}{2}\sin 4t$ $s = -0.432$ Displacement = -0.432m (3 sf)
10d	Total distance travelled = $(1.5 \times 2) + 1.5$ = 4.5m
11a	Grad EF = $\frac{2-(-6)}{-1-1} = -4$ Grad DF = $\frac{4-2}{7-(-1)} = \frac{1}{4}$ Since Grad $DF \times$ Grad $EF = -4 \times \frac{1}{4} = -1$ $\therefore DF \perp EF$ $\angle DFE = 90°$ (Right angle in semi-circle) $\therefore DE$ is the diameter of C_1 Centre of $C_1 = \left(\frac{1+7}{2}, \frac{-6+4}{2}\right)$ = $(4, -1)$
11b	$x^2 + y^2 - 8x + 2y - 17 = 0$
11c	$(x+8)^2 + (y+1)^2 = 34$
11d	$\therefore$ (3 , 4) lies within C_1 only