Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2020

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

TUESDAY 22 SEP 2020

2 hour

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

<table>
<tr><th colspan="2">Section A
(55 Marks)</th><th colspan="2">Section B
(55 Marks)</th></tr>
<tr><th>Question</th><th>Marks</th><th>Question</th><th>Marks</th></tr>
<tr><td>1</td><td></td><td rowspan="3">10</td><td rowspan="3"></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td><td rowspan="3">11</td><td rowspan="3"></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td><td rowspan="3">12</td><td rowspan="3"></td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>9</td><td></td></tr>
<tr><td>Subtotal</td><td></td><td>Subtotal</td><td></td></tr>
<tr><td>TOTAL</td><td colspan="3">/ 110</td></tr>
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COLEGIO DEL MUNDO • WORLD SCHOOL • ÉCOLE DU MONDE

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

The lengths of two sides of a triangle are 4 cm and 5 cm. Let θ be the angle between the two given sides. The area of the triangle is $\frac{5\sqrt{15}}{2}$ cm^2.

(a) Show that $\sin\theta = \frac{\sqrt{15}}{4}$ [2]

(b) Find the two possible values of the length of the third side. [5]

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2. *[Maximum mark: 6]

9, m, n are consecutive terms of a geometric progression and 9, 2m and 3n are consecutive terms of an arithmetic progression. Given that $m \neq n$, find the value of m and of n.

3. *[Maximum mark: 6]

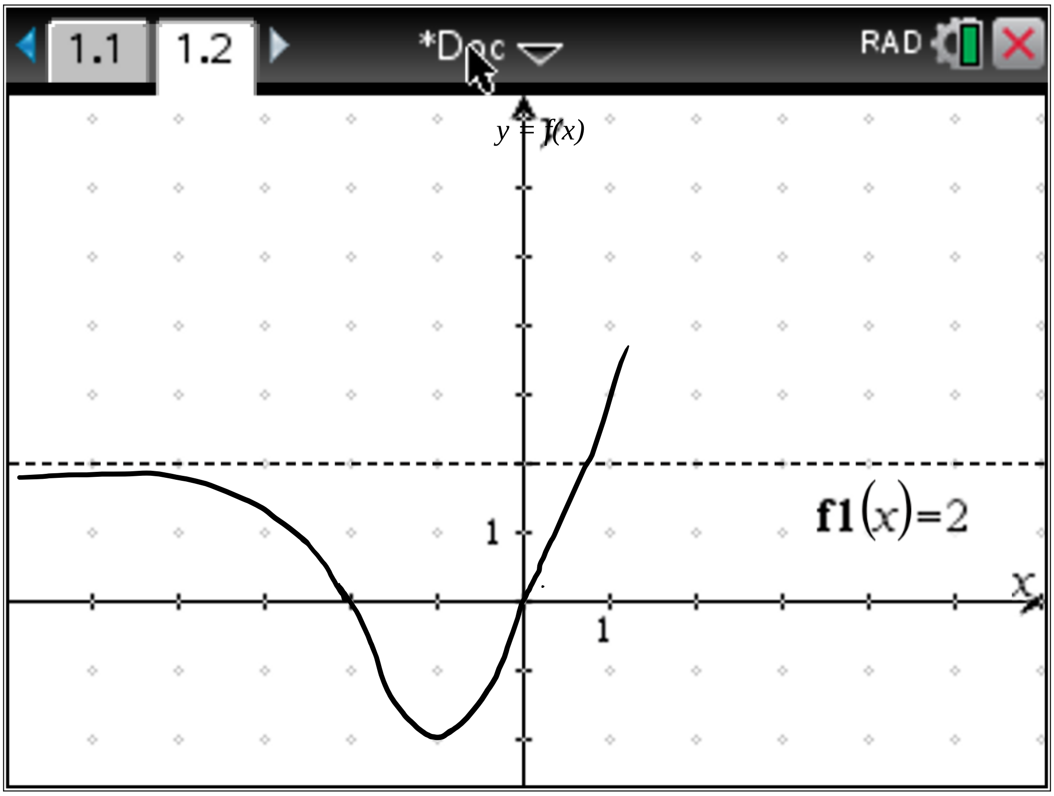
Given that $\sum_{k=3}^{22}[k + log_2(x)] = 170, x \in \mathbb{R}$, find the value of x.

4. [Maximum mark: 6]

$\alpha,\ \beta$ and γ are roots of the equation $z^3+7z^2+20z+50=0$. Without solving for $\alpha,\ \beta$ and γ find the polynomial of degree 3 whose roots are $\alpha-1,\ \beta-1$ and $\gamma-1$.

5. [Maximum mark: 7]

The following diagram shows the graph of $y = f(x)$ with x-intercepts at -2 and 0, minimum point (-1, -2) and asymptote $y = 2$



(a) Sketch the graph of $\dfrac{1}{f(x)}$ in the same diagram above indicating the x-intercepts, turning point and asymptote(s) clearly. [5]

(b) If the domain of another function $g(x)$ is $[0,8]$ and its range is $[4,11]$, write down the domain and range of $\dfrac{1}{g(x)}$ whenever possible. [2]

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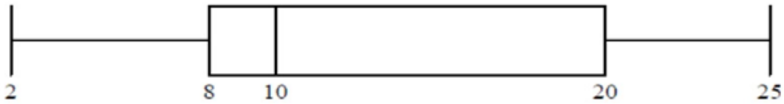
6. *[Maximum mark: 5]

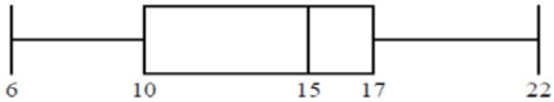
The box-and-whisker diagram below shows the amount of salt used in different types of snacks in 2 confectionery shops. Shop 1 has 240 snacks and Shop 2 has 320 snacks.

SHOP 1



Amount of salt per snack in grams

SHOP 2



Amount of salt per snack in grams

(i) State what the number 10 represents in Shop 1. [1]

(ii) Write down the approximate number of snacks whose amount of salt ranges from 2 to 20 grams in Shop 1. [2]

(iii) A health inspector visited the two shops to check the snacks in the shop. Snacks sold in the shop are required to have less than 10 grams of salt.
State, giving a reason, which shop has more snacks that **do not** meet the requirement. [2]

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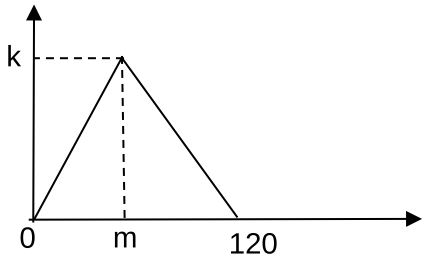
7. [Maximum mark: 7]

A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = \frac{1}{4}$ and $u_{n+1} = u_n - \frac{6}{(3n+5)(3n+8)}$, for $n \in Z^+$. Use the principle of mathematical induction to prove that for all $n \in Z^+$, $u_n = \frac{2}{3n+5}$.

8. [Maximum mark: 5]

The diagram shows the probability density function of a continuous random variable, X whose median is m.



(a) Find the value of k and of m. [4]

(b) Hence state the value of its mode. [1]

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9. [Maximum mark: 6]

Consider the following system of equations

$$2x+2y+6z=0$$
$$4x+5y+14z=4$$
$$2x-2y+\left(\alpha-2\right)z=\beta-12$$

(a) Find the values of α and β for which the system has an infinite number of solutions. [3]

(b) In the case where the number of solutions is infinite, find the general solution of the system of equations in parametric form. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. *[Maximum mark: 21]

a) A function f is defined by $f(x) = \dfrac{2x-1}{3}$, $a \le x \le \dfrac{5}{4}$.

(i) Given that the range of f is $-1 \le f(x) \le \dfrac{1}{2}$, find the value of a. [2]

(ii) Find $f^{-1}(x)$ and state its range. [3]

Another function g is defined by $g(x) = \dfrac{x}{x-5}$, $x \in \mathbb{R}$, $x \ne 5$.

(iii) Find $fg(x)$, given that it exists. [3]

(iv) Sketch the graph of $g(x)$ clearly showing the asymptotes and any intercepts. [3]

(v) Sketch on a separate clearly labeled diagram the graph of $y = \left(g(x)\right)^2$, showing any asymptotes, turning points and intercepts. [4]

b) (i) Sketch the graph of $h(x) = x^2 - 8x + 14,\ 0 \le x \le 8$, clearly showing the turning point of $h(x)$. [2]

(ii) Describe, in order, three transformations that transforms the graph of $h(x)$ into the graph of $p(x) = -(x-2)^2 + 1$, $-2 \le x \le 6$. [4]

11. [Maximum mark: 14]

Given the function $f(x) = \frac{2}{x^2+6x+8}$,

(i) express $f(x)$ in partial fractions. [3]

(ii) Hence, show that the first three terms in the expansion of $f(x)$ in descending powers of x can be expressed by $\frac{2}{x^2} - \frac{12}{x^3} + \frac{56}{x^4}$. [6]

(iii) State the range of values of x for which the expansion is valid. [2]

(iv) Using the expansion of $f(x)$, evaluate $\frac{2}{168}$ corrected to 4 decimal places. [3]

[Turn the page over

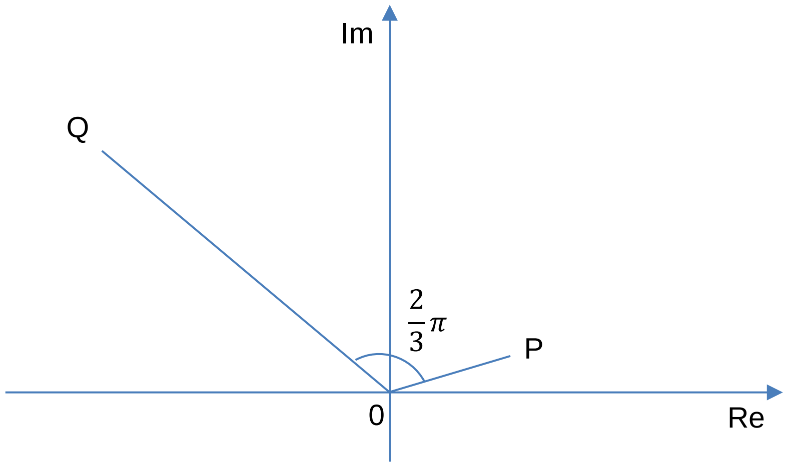
Do **not** write solutions on this page.

12. [Maximum mark: 20]

(a) $P(z)$ is a polynomial of degree four with real coefficients. Given that $2e^{i\frac{\pi}{3}}$ and $2ie^{i\frac{\pi}{3}}$ are roots of the equation. Express $P(z)$ as a product of 2 quadratic factors, $(z^2 + az + b)(z^2 + cz + d)$ such that a, b, c, d are real numbers. [7]

(b) The following diagram shows the complex plane with origin O, and the points P and Q represents the complex numbers z_1 and $z_1 z_2$ respectively.

OP is a third of the length of OQ and the angle POQ is $\frac{2}{3}\pi$.



[Diagram not drawn to scale]

Find z_2, giving your answer in the form $a + ib, a, b \in R$. [4]

(c) (i) Show that for complex numbers, z and w,

$$zw^* + z^*w = 2Re(zw^*).$$ [2]

(ii) Hence or otherwise, use proof by contradiction to prove the triangle inequality for complex numbers, that is, $|z + w| < |z|+|w|$. [7]

END OF PAPER