Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2021

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 2

FRIDAY 24 SEP 2021

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

Section A (55 Marks)		Section B (55 Marks)	
Question	**Marks**	**Question**	**Marks**
1		10	
2			
3			
4		11	
5			
6			
7		12	
8			
9			
Subtotal		**Subtotal**	
TOTAL	/ 110		

This question paper consists of 12 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. *[Maximum mark: 6]

An infinite geometric sequence has the first term 1. The sum of the first three terms is 2. The terms in the sequence are all positive.

(a) Find the common ratio. [3]

(b) Find the sum of the infinite sequence. [3]

2. [Maximum mark: 6]

(a) Express $\dfrac{5}{(1+3x)(1-2x)}$ in partial fractions. [2]

(b) Hence expand $\dfrac{5}{(1+3x)(1-2x)}$ as a series of ascending powers of x up to and including the term in x^3. [4]

3. *[Maximum mark: 4]

The *moment magnitude* scale, M, is a measure of the strength of an earthquake, defined by $M = \frac{2}{3}\log_{10}(I) - 10.7$, where I is the intensity of the quake. The 2012 Aceh earthquake had a moment magnitude of 8.6. The 2004 Indian Ocean earthquake was 8 times as intense.

(a) Find the intensity of the Aceh earthquake [2]

(b) Find the magnitude of the Indian Ocean earthquake on the moment magnitude scale. Give your answer correct to 1 decimal place. [2]

4. [Maximum mark: 5]

(a) How many ways can the letters of the word "LOGARITHMS" be arranged such that the vowels are separated? [2]

(b) A bag contains 2 white pencils, 3 black pencils and 4 red pencils. In how many ways can 3 pencils be drawn from the bag, if at least one black pencil is to be included in the draw? [3]

5. *[Maximum mark: 8]

Two fair 8-sided dice are thrown and the number showing on each die is noted. The sum of these two numbers is S. Find the probability that

(a) S is at least '14', [2]

(b) less than two dice shows a '7', [3]

(c) less than two dice shows a '7', given that S is at least '14'. [3]

6. *[Maximum mark: 8]

It is speculated that there is an association between the amount of time spent online (in hours) and the amount of sleep hours per day. The following paired data are collected.

Time spent online (in hours), x	7.5	8	8	9	6	5	5	4	9	8.5
Sleep hours (in hours), y	6	5.5	5	6	7	7.5	8	9	5.5	4

(a) Find the Pearson's product moment correlation coefficient between the time spent online and sleep hours. Comment on your answer. [3]

(b) Find the equation of the regression line of y on x, where x is the time spent online and y is sleep hours. [2]

(c) Find the estimated sleep hours (to 1 decimal place) if the time spent online is 6.5 hours. [1]

(d) To find the estimated time spent online per day for a student who sleeps for 3.9 hours, give 2 reasons why the estimated time spent online using (b) might not be reliable. [2]

7. [Maximum mark: 5]

A continuous random variable X has a probability density function given by the function $f(x)$, where

$$f(x)=\begin{cases} kx^2 & , \ 0 \le x < 1 \\ k\,|\,2-x\,| & , \ 1 \le x \le 3 \\ 0 & , \ \text{otherwise.} \end{cases}$$

(a) Determine that $k=\dfrac{3}{4}$. [2]

(b) Hence find the median of X. [3]

8. [Maximum mark: 5]

A complex number $z = \dfrac{2i + k}{k - i}$ where k is a positive real number.

(a) Given that $\text{Re}(z) = 0.5$, find the value of k. [3]

(b) Determine $\arg(z)$, giving your answer in radians. [2]

9. [Maximum mark: 8]

The equation $x^3 + px^2 + qx + c = 0$, where p, q and $c \in \mathbb{R}$, has distinct roots α, β and γ.

(a) By expanding $(x-\alpha)(x-\beta)(x-\gamma)$, show that $q = \alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Show that $(\alpha+\beta+\gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma)$. [2]

It is given that $p = -2$, $q = 3$ and that one of the roots is di, where $d \in \mathbb{R}$ and $i = \sqrt{-1}$.

(c) Using the result from part (b), calculate the value of $\alpha^2 + \beta^2 + \gamma^2$. [1]

(d) Hence or otherwise, find the roots α, β and γ. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. *[Maximum Mark: 16]

(a) At a store, on average 3 out of 10 customers make large purchases. Due to entry restrictions, only a total of 50 customers are served at the store on any given day. Let X denote the number of customers who make large purchases on a given day.

(i) State the distribution of X. [1]

(ii) Find the probability that more than 20 customers make large purchases on a given day. [2]

(iii) In a period of 30 days, find the probability that on 6 of those days, there were more than 20 customers making large purchases per day. [3]

(iv) On a given day, find the value of n such that the probability of exactly n customers making large purchases, is more than 0.12. [3]

(b) Based on past results, the score of students for a mathematics test can be modelled by a normal distribution with mean of 60 and standard deviation of 14. The top 10% scorers will be placed in the Dean's list.

(i) Find the probability of a randomly selected student scoring more than 75. [2]

(ii) Find the minimum score (rounded to nearest whole number) for a randomly selected student to qualify for the Dean's List. [2]

(iii) A girl in the Dean's List achieved a score of 98. Assuming that the maximum score is 110, find the probability that another randomly selected student in the Dean's list would score higher than or same as her. [3]

11. [Maximum Mark: 15]

Given that $f\left(x\right)=\dfrac{x^2-x-2}{x-1}$, $x\in\mathbb{R}, x\neq 1$.

(a) Find the asymptote(s) of the graph of $f(x)$. [3]

(b) Sketch $f(x)$, indicating clearly the axial intercept(s) and asymptote(s) [3]

(c) Solve the inequality $\dfrac{x^2-x-2}{x-1}\geq 2$. [3]

(d) Hence or otherwise, solve $\dfrac{x^2+x-2}{-x-1}\geq 2$. [3]

(e) Deduce the range of k such that the graph of $f\left(\left|x\right|\right)$ intersects the graph of $y=x+k$ exactly once. [3]

12. [Maximum Mark: 24]

(a) A discrete random variable X has a probability distribution given in the following table.

x	0	1	2	3	4	5
$P(X = x)$	0.1	0.2	p	0.2	0.2	0.1

(i) Find $P(X = 2.5)$. [1]

(ii) If $E(X) = 2.5$, determine the value of p. [2]

(iii) Calculate $Var(X)$. [4]

(iv) Tabulate the cumulative distribution function of X. [3]

(b) Given that $z = \cos\theta + i\sin\theta$,

(i) Show that $z^n - \dfrac{1}{z^n} = 2i\sin n\theta, \text{ where } n \in \mathbb{Z}.$ [2]

(ii) Given that $z^4 + 4z^{-2} = z^{-4} + 4z^2$, hence show that $\sin 4\theta - 4\sin 2\theta = 0$ [3]

(iii) Determine the values of θ, where $-\pi < \theta \le \pi$. [5]

(iv) Hence or otherwise, state the roots of $z^4 + 4z^{-2} = z^{-4} + 4z^2$, in the form $x + iy$. [4]

END OF PAPER