Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2021

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS HIGHER LEVEL PAPER 2

SECTION A

Qn	Solution	Marks
1.		[6 marks]
(a)	Given that $a = 1$ and $a + ar + ar^2 = 2$, we have $r + r^2 = 1$ $r^2 + r - 1 = 0$ Solving the equation, we get $r = \frac{-1\pm\sqrt{5}}{2}$ Since all terms are positive, common ratio is positive. So $r = \frac{-1+\sqrt{5}}{2} (\approx 0.618)$	
	Many were unable to solve $r^3 - 2r + 1 = 0$ when they use sum to 3 terms.	
(b)	$S_\infty = \frac{a}{1-r} = \frac{1}{1-0.618}$ $= 2.618 \ldots \approx 2.62\ (3\ S.F.)$	
	Mostly ok except some wrote formula as $\frac{a}{r-1}$.	

Qn	Solution	Marks
2.		[6 marks]
(a)	$\frac{5}{(1+3x)(1-2x)} = \frac{A}{1+3x} + \frac{B}{1-2x}$ $5 = A(1-2x) + B(1+3x)$ Comparing: $A = 3,\ B = 2$ Alternative: By cover-up method, $\frac{5}{(1+3x)(1-2x)} = \frac{3}{1+3x} + \frac{2}{1-2x}$	
	Mostly well done except some who used $x = -\frac{1}{2}$ for substitution.	
(b)	$3(1+3x)^{-1} + 2(1-2x)^{-1}$ $3(1+3x)^{-1} = 3\left[1-3x+9x^2-27x^3+...\right]$ $2(1-2x)^{-1} = 2\left[1+2x+4x^2+8x^3+...\right]$ $\frac{5}{(1+3x)(1-2x)} = 3\left[1-3x+9x^2-27x^3+...\right] + 2\left[1+2x+4x^2+8x^3+...\right]$ $\approx 5-5x+35x^2-65x^3$	
	Some expanded $(1+2x)^{-1}$ instead when they used $2x$ instead of $-2x$ while others messed up in evaluating the binomial coefficients.	

Qn	Solution	Marks
3.		[4 marks]
(a)	For Aceh earthquake, $\frac{2}{3}\log_{10}(I) - 10.7 = 8.6$ $\log_{10} I = \frac{3}{2}(19.3)$ $I = 8.9125 \times 10^{28}$	
	Many left answer as $10^{28.95}$.	
(b)	For Indian Ocean earthquake, $M = \frac{2}{3}\log_{10}\left(8 \times 8.9125 \times 10^{28}\right) - 10.7$ $= \frac{2}{3}\log_{10}\left(7.1300 \times 10^{29}\right) - 10.7$ $= \frac{2}{3}(29.853) - 10.7$	
	$= 9.202 \ldots \approx 9.2$ (1.d.p.)	

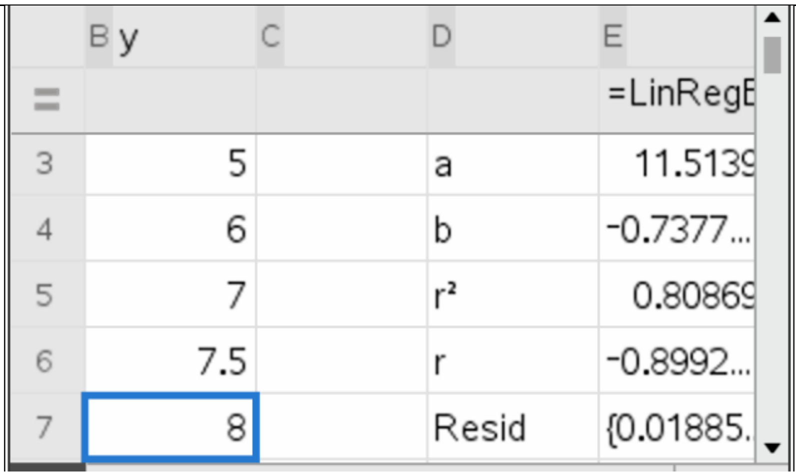
Qn	Solution	Marks
	Very well done except a few who were careless and left out -10.7 .	

Qn	Solution	Marks
4.		[5 marks]
(a)	${}^{8}P_{3} \times 7!$ $= 1693440$	
	Badly done. Every kind of erroneous thinking.	
(b)	Number of ways to select 3 black pencils $= {}^{3}C_{3}$ Number of ways to select 2 black and 1 non-black pencils $= {}^{3}C_{2} \times {}^{6}C_{1}$ Number of ways to select 1 black and 2 non-black pencils $= {}^{3}C_{1} \times {}^{6}C_{2}$ Total number of ways = $1+18+45=64$ Alternative: Complement Method Number of ways without restrictions $= {}^{9}C_{3} = 84$ Number of ways with no black pencils $= {}^{6}C_{3} = 20$ Required number of ways $= {}^{9}C_{3} - {}^{6}C_{3} = 64$	
	Many were able to think about cases of no black, one black and two black but made mistakes including using permutation instead of combination.	

Qn	Solution	Marks
5.		[8 marks]
(a)	$P(S \geq 14)$ $= P(S=14) + P(S=15) + P(S=16)$ $= \dfrac{3+2+1}{64} = \dfrac{3}{32}$	
	Many under counted the cases.	

Qn	Solution	Marks
(b)	$P\ (less \text{ than two '7'})$ $= \text{P(one '7')} + \text{P(no '7')}$ $= 2\left(\frac{1}{8}\right)\frac{7}{8} + \left(\frac{7}{8}\right)\left(\frac{7}{8}\right)$ $= \frac{14+49}{64} = \frac{63}{64}$ Alternative: $P\ (less \text{ than two '7'})$ $= 1 - P(two \text{ '7'})$ $= 1 - \left(\frac{1}{8}\right)^2$ $= \frac{63}{64}$	
	All kinds of errors.	
(c)	$P(\text{less than 2 dice shows } 7 \mid S \geq 14)$ $= \frac{P(\text{one } 7 \cap S \geq 14) + P(no\ 7 \cap S \geq 14)}{P(S \geq 14)}$ $= \frac{2/64 + 3/64}{6/64}$ $= \frac{5}{6}$	
	Badly done with many students not treating it as conditional probability.	

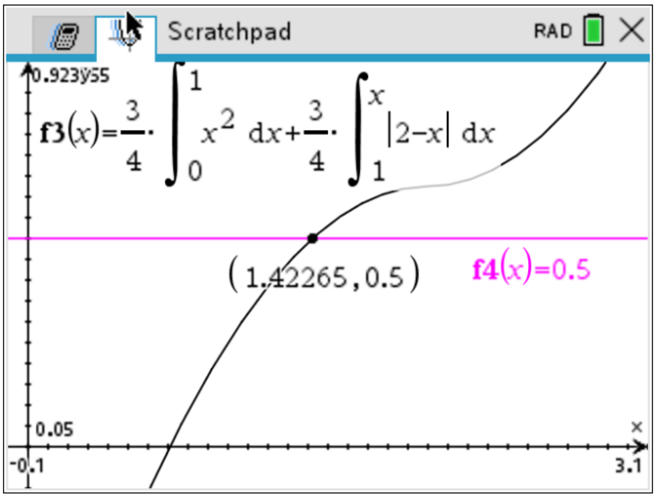
Qn	Solution	Marks
6.		[8 marks]



	B y	C	D	E
=				=LinRegE
3	5		a	11.5139
4	6		b	-0.7377...
5	7		r²	0.80869
6	7.5		r	-0.8992...
7	8		Resid	{0.01885.

(a)	$r = -0.8992....$ $= -0.899 \text{ (3 S.F.)}$ Strong negative linear correlation.	
	Well done although no table was drawn and some gave the value of r^2 instead of r.	
(b)	$y = 11.5 - 0.738x$	
	Mostly correct although several gave the wrong regression line.	
(c)	$y = 11.5 - 0.738(6.5)$ $= 6.703....$ $= 6.7 \text{ (1 d.p.)}$	
	Many did not correct to one decimal place.	
(d)	The line x on y should be used instead of y on x to estimate this value. Extrapolation is always unreliable.	
	Most could only give one reason.	

Qn	Solution	Marks
7.		[5 marks]
(a)	Since $f(x)$ is p.d.f, $\int_0^3 f(x)dx = 1$ $\therefore k\left[\int_0^1 x^2dx + \int_1^3 \lvert 2-x\rvert dx\right] = 1$ $k = \dfrac{1}{\int_0^1 x^2dx + \int_1^3 \lvert 2-x\rvert dx}$ $= \dfrac{1}{4/3}$ $= \dfrac{3}{4}$	
	Many did not realise they can use the GDC to evaluate $\int_1^3 \lvert 2-x\rvert\, dx$ and got stuck.	

Qn	Solution	Marks
	Testing the median of X: $\frac{3}{4}\int_0^1 x^2 dx = 0.25$ Since $0.25 < 0.5$, Median cannot be in first piece. Assume median in second piece: $\frac{3}{4}\int_0^1 x^2 dx + \frac{3}{4}\int_1^m \lvert 2-x \rvert dx = 0.5$  Median = 1.42265… (Median > 1) = 1.42 (3 S.F.)	
	Poorly done with many trying to solve manually. Yet others did not understand the concept of median.	

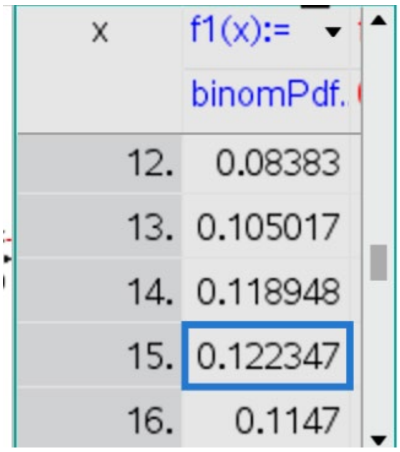
Qn	Solution	Marks
8.		[5 marks]
(a)	$z = \frac{(k+2i)(k+i)}{(k-i)(k+i)} = \frac{k^2+3ki-2}{k^2+1}$ $\frac{k^2-2}{k^2+1} = \frac{1}{2}$ $2k^2-4 = k^2+1$ $k^2 = 5$ $k = \pm\sqrt{5}$ $(\because -\sqrt{5} \text{ rejected})$ $k = \sqrt{5}$	
	Quite well done although some wrote k^2-1 or $k+1$ as the denominator for the LHS.	

Qn	Solution	Marks
(b)	$z=\dfrac{5+3\sqrt{5}i-2}{5+1}=\dfrac{\left(1+\sqrt{5}i\right)}{2}$ $\arg\left(z\right)=\tan^{-1}\left(\dfrac{\sqrt{5}}{1}\right)$ $=1.15026$ $=1.15\ (3\text{ S.F.})$	
	Those who got part (a) correct mostly got this part right as well. Many got the concept of arctan of (imaginary part/real part).	

Qn	Solution	Marks
9.		[8 marks]
a	$(x-\alpha)(x-\beta)(x-\gamma)=\left(x^2-(\alpha+\beta)x+\alpha\beta\right)(x-\gamma)$ $=x^3-\gamma x^2-(\alpha+\beta)x^2-(\alpha\gamma+\beta\gamma)x+\alpha\beta x-\alpha\beta\gamma$ $=x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\alpha\gamma)x-\alpha\beta\gamma.$ Hence q = $\alpha\beta+\beta\gamma+\alpha\gamma$	
	Well done although some made careless mistake in expanding.	
b	$(\alpha+\beta+\gamma)^2$ $=\alpha^2+\alpha\beta+\alpha\gamma+\alpha\beta+\beta^2+\beta\gamma+\alpha\gamma+\beta\gamma+\gamma^2$ $=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\alpha\gamma).$	
	Careless mistakes and some could not write the symbols properly and misread their own handwriting.	
c	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\alpha\gamma).$ $=(2)^2-2(3)=-2$	
	Mostly correct with several making careless substitution.	

Qn	Solution	Marks
d	Recognize di and $-di$ are both roots of the equation $\alpha + di + (-di) = 2 \Rightarrow \alpha = 2$ $\alpha^2 + (di)^2 + (-di)^2 = -2$ $2^2 - d^2 - d^2 = -2$ $2d^2 = 6$ $d = \pm\sqrt{3}$ The roots are $2, \sqrt{3}i, -\sqrt{3}i.$ Alternative: $q = \alpha\beta + \beta\gamma + \gamma\alpha = 3$ $2(di) + 2(-di) + d^2 = 3$ $d^2 = 3$ $d = \pm\sqrt{3}$ The conjugate roots are $\sqrt{3}i, -\sqrt{3}i$	
	Many could not do this part or have no time for this part.	

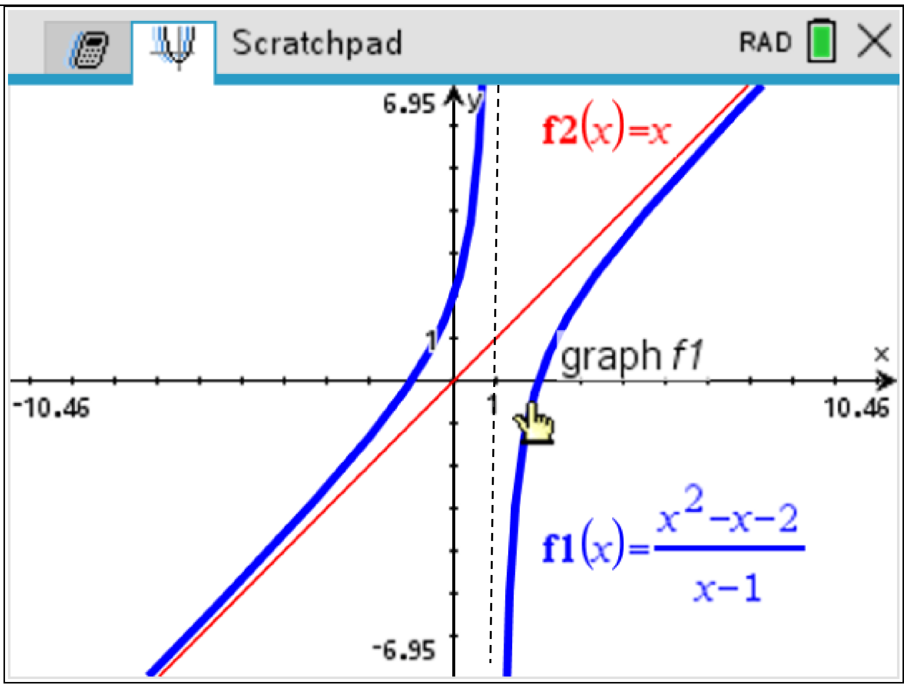
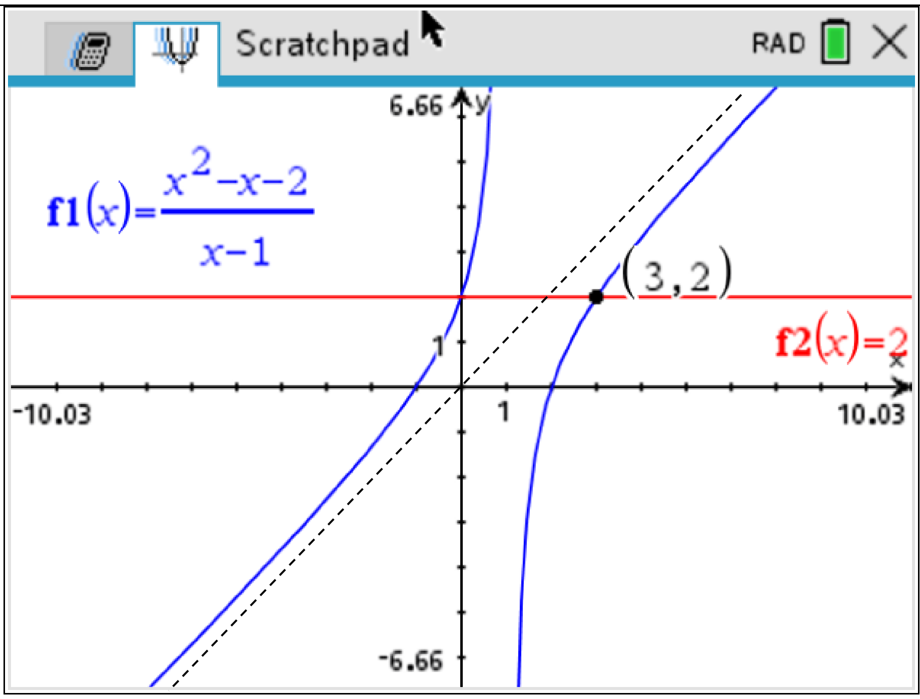
Mark Scheme for 2021 EOY P2 Section B

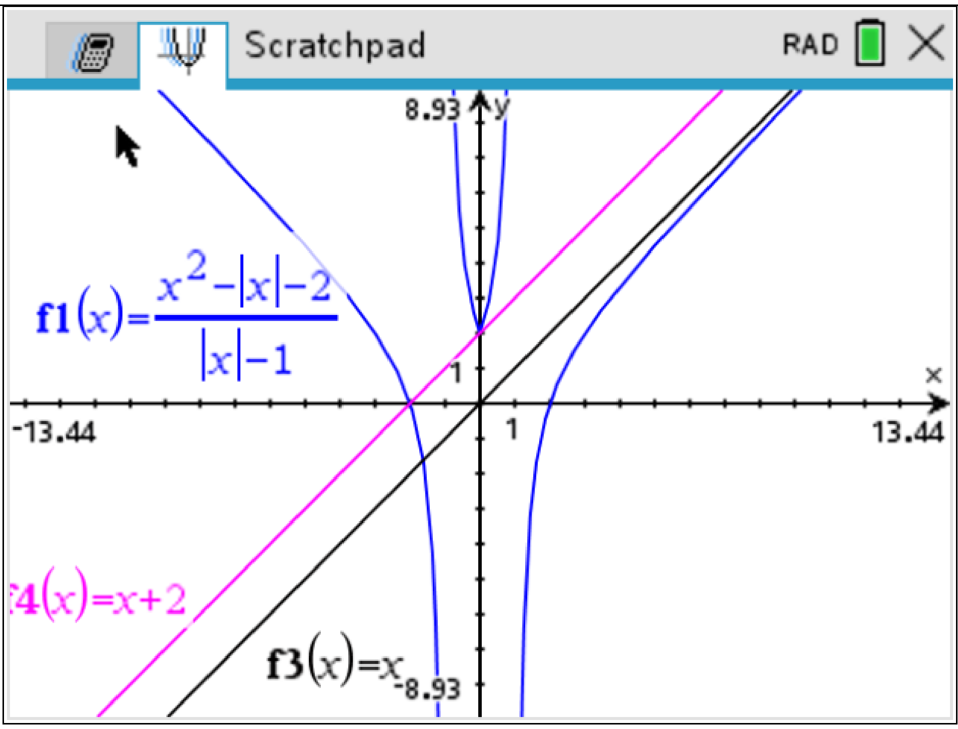
Qn	Solution	Marks
10(a)		[16 marks]
(a) i	$X \sim B(50, 0.3)$ or X follows a binomial distribution.	
	Several students used weird notations like X ~(50, 0.3) or P ~ X(50,0.3) or are unsure how to state the distribution.	
(a) ii	$P(X > 20)$ $= binomCDF(50, 0.3, 21, 50)$ $= 0.04776...$ $= 0.0478$ (3 S.F.)	
	Significant numbers of students use $binomCDF(50, 0.3, 20, 50)$ including the 20 by mistake.	
(a) iii	$Y \sim B(30, 0.0478)$ $P(Y = 6)$ $= binomPdf(30, 0.0478, 6)$ $= 0.002178...$ $= 0.00218$ (3 S.F.)	
	Generally not well done if part (ii) is wrong. Most could see that it is binomial here but may have made careless mistakes in keying in numbers.	
(a) iv	$P(X = n) > 0.12$ $f(x) = binomPdf(50, 0.3, x)$  x | f1(x):= binomPdf. 12. | 0.08383 13. | 0.105017 14. | 0.118948 15. | 0.122347 16. | 0.1147 N = 15 Alternative:	

	${}^{50}C_n(0.3^n)(0.7^{50-n}) > 0.12$ Using GDC, n = 15.	
	Badly done. Some tried using inverse binomial gdc function which is for questions with the form $P(X \le n) > 0.12$. Hence tend to get wrong answers. A number use brute force to list out all the possibilities. Students should use the graph method, then ctrl-t for table approach which is easier.	
10(b)		
(b) i	$S \sim N(60,14^2)$ $P(S > 75)$ $= normCDF(75,10\wedge 99,60,14)$ $= 0.1419...$ $= 0.142$ (3 S.F.)	
	Some wrongly assumed that the maximum mark is only 100 and got the wrong probability. Some forgot that Normal dist is a CRV and put 76 for lower bound wrongly. Some even used normPDF instead of normCDF. Some mixed up the standard deviation with the variance.	
(b) ii	$P(S > a) = 0.1$ $P(S < a) = 0.9$ $a = invnorm(0.9,60,14)$ $a = 77.94...$ $a \approx 78$	
	A sizeable number of students rounded 77.9 to 80 instead of 78! They mistook what is the <u>nearest whole number.</u>	
(b) iii	$P(S \ge 98 \mid S \ge 78)$ $= \dfrac{P(S \ge 98 \cap S \ge 78)}{P(S \ge 78)}$ $= \dfrac{P(98 \le S \le 110)}{P(S \ge 78)}$ $= \dfrac{normcdf(98,110,60,14)}{0.1}$ $= 0.031434...$ $= 0.0314$ (3 S.F.) $\quad [accept\ 0.0317 \text{ see below}]$ Alternative:	

	$P(S \geq 98 \mid S \geq 78)$ $= \dfrac{P(S \geq 98 \cap S \geq 78)}{P(S \geq 78)}$ $= \dfrac{P(98 \leq S \leq 110)}{P(78 \leq S \leq 110)}$ $= \dfrac{normcdf(98,110,60,14)}{normcdf(78,110,60,14)}$ $= 0.03172...$ $= 0.0317$ (3 S.F.)	
	Many students did not interpret the conditional probability for a student “in the Dean’s List”. Hence, they did not divide by the probability of a student in the dean’s list.	

	Solution	**Marks**
11.		[15 marks]
(a)	$f(x) = \dfrac{x^2 - x - 2}{x - 1}$ $= x - \dfrac{2}{x-1}$ Oblique Asymp: $y = x$ Vertical Asymp: $x = 1$	
	Badly done. Some students could only identify the vertical asymptote. It seems that many were not aware of the long division approach to simplify the equation first before sketching. A few did not write the equations for the asymptotes correctly; some wrote asymptotes as $y \neq x$ etc.	

(b)		
	Badly done. Many students did not sketch the oblique asymptote or their graph did not tend towards the asymptotes. Some even have graphs that cut the asymptotes Several students did not label the key points.	
(c)	 $0 \le x < 1$ or $x \ge 3$	

	Badly done. Those who use graphical approach were generally able to identify the range, except those with careless mistakes with the inequality signs. Many students opted for the algebraic method and make tons of mistakes in simplifying the inequalities correctly.	
(d)	Since $\frac{x^2-x-2}{x-1} \geq 2$ Replace x with $-x$: $\therefore \frac{x^2+x-2}{-x-1} \geq 2$ $0 \leq -x < 1$ or $-x > 3$ $\Rightarrow -1 < x \leq 0$ or $x \leq -3$	
	Badly done. Those who use substitution method or graphical approach were generally able to identify the range, except those with careless mistakes with the inequality signs. Many students opted for the algebraic method and make tons of mistakes in simplifying the inequalities correctly.	
(e)	 There is only one intersection if the line is between y = x and y = x+2 Hence $0 \leq k < 2$	

<table>
<tr><td></td><td>Badly done. Those who use graphical approach were generally able to identify the range, except those with careless mistakes with the inequality signs.

Many students opted for the algebraic method and make tons of mistakes in simplifying the inequalities correctly.</td><td></td></tr>
</table>

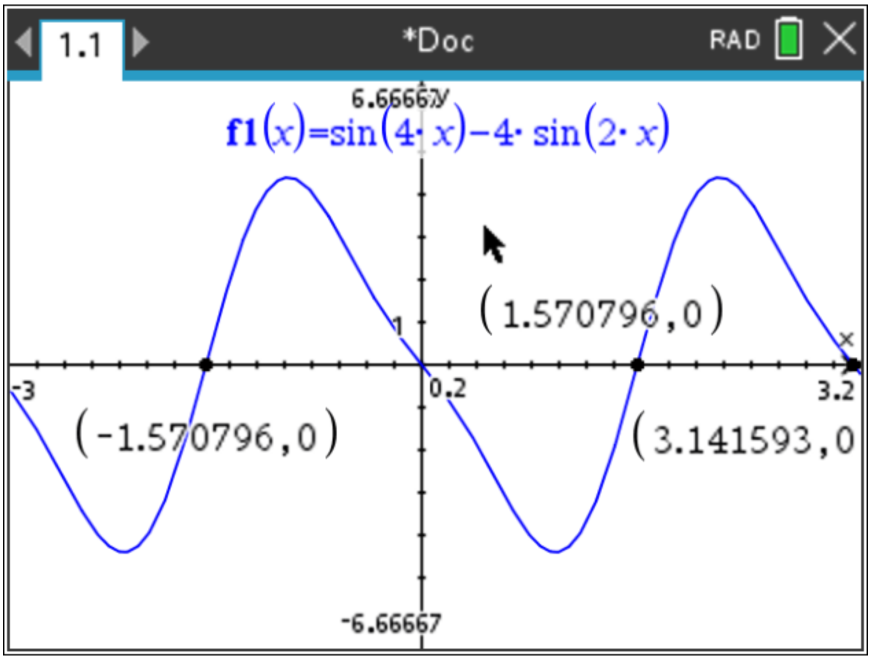
<table>
<tr><th>Qn</th><th>Solution</th><th>Marks</th></tr>
<tr><td>12</td><td></td><td>[24 marks]</td></tr>
<tr><td>(a) i</td><td>$P(X=2.5)=0$</td><td></td></tr>
<tr><td></td><td>Surprising numbers of students could not do this question or gave answer other than 0.</td><td></td></tr>
<tr><td>(a) ii</td><td>$E(X)=0+1\times 0.2+2p+3(0.2)+4(0.2)+5\times 0.1=2.5$
$2.5=2.1+2p+3$
$2p=0.4$
$p=0.2$

<u>Alternative:</u>

By symmetry, 2.5 is in the middle p = 0.2</td><td></td></tr>
<tr><td></td><td>Well done.</td><td></td></tr>
<tr><td>(a) iii</td><td>$E(X^2)=0+1(0.2)+4(0.2)+9(0.2)+16(0.2)+25(0.1)$
$=8.5$
$Var(X)=E(X^2)-(E(X))^2=8.5-2.5^2$
$=2.25$</td><td></td></tr>
<tr><td></td><td>Generally ok. A few made careless mistakes in calculating variance.</td><td></td></tr>
<tr><td>(a) iv</td><td>
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$P(X\le x)$</td><td>0.1</td><td>0.3</td><td>0.5</td><td>0.7</td><td>0.9</td><td>1</td></tr>
</table>

Alternative:

$$P(X\le x)=\begin{cases} 0 \quad , & x<0 \\ 0.1+0.2x, & x=0,1,2,3,4 \\ 1 \quad , & x\ge 5 \end{cases}$$
</td><td></td></tr>
</table>

	Badly done. Many students could not do this question. They did not use a table or misinterpret the question and work out something unrelated.	
(b) i	$z^n - \frac{1}{z^n} = \cos n\theta + i\sin n\theta - \frac{1}{\cos n\theta + i\sin n\theta}$ $= \cos n\theta + i\sin n\theta - (\cos n\theta - i\sin n\theta)$ $= 2i\sin n\theta$	
	Generally ok	
(b) ii	$z^4 + 4z^{-2} = z^{-4} + 4z^2$ $z^4 - z^{-4} - 4z^2 + 4z^{-2} = 0$ $(z^4 - z^{-4}) - 4(z^2 - z^{-2}) = 0$ $2i\sin 4\theta - 8i\sin 2\theta = 0$ $\sin 4\theta - 4\sin 2\theta = 0$	
	Generally ok	
(b) iii	$2\sin 2\theta\cos 2\theta - 4\sin 2\theta = 0$ $2\sin 2\theta(\cos 2\theta - 2) = 0$ $\sin 2\theta = 0 \quad or \quad \cos 2\theta = 2$ (rejected) $2\theta = -\pi, 0, \pi, 2\pi$ $\theta = \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi$ Alternative: 	

	$\theta = \frac{-\pi}{2}, 0, \frac{\pi}{2}, \pi$	
	Badly done. Many could work out or use convoluted trigo algebra to arrive at wrong answers. Common mistakes include cancelling out $\sin 2\theta$ from both sides of the equation $\sin 2\theta \cos 2\theta = 2\sin 2\theta$, resulting in no solutions. Those who use graphical approach tend to get this correct.	
(b) iv	$\theta = -\frac{\pi}{2}, \quad z_1 = -i$ $\theta = 0, \quad z_2 = \cos 0 + i\sin 0 = 1,$ $\theta = \frac{\pi}{2}, \quad z_3 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i,$ $\theta = \pi, \quad z_4 = -1.$	
	Badly done, especially when earlier part iii is wrong. Some students despite having the correct angles, were not able to find the correct complex numbers.	