Anglo-Chinese School

(Independent)

FINAL EXAMINATION 2022
YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS : ANALYSIS AND APPROACHES

HIGHER LEVEL

PAPER 2

FRIDAY 23 SEP 2022

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL and SL papers

Section A (55 Marks)		**Section B (55 Marks)**	
Question	**Marks**	**Question**	**Marks**
1		**10**	
2			
3			
4		**11**	
5			
6			
7		**12**	
8			
9			
Subtotal		**Subtotal**	
TOTAL	**/ 110**		

This question paper consists of 13 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1.* [Maximum mark: 6]

In an infinite geometric sequence, $u_1 = a, u_2 = a^2 - 2$.

(a) Given that $a = \frac{9}{5}$, show that the sum to infinity of the sequence exists. [2]

(b) Find the least value of n such that $S_\infty - S_n < \frac{1}{100}$. [4]

2.* [Maximum mark: 4]

The time T hours taken for a machine to cool down to temperature x°C after it has been turned off can be modelled by $T = 5ln\left(\frac{13}{x-k}\right)$, where $T \geq 0$ and k is a positive constant. The machine has an initial temperature of 36°C when it is turned off.

(a) Show that $k = 23$. [1]

(b) Sketch the graph of the machine's temperature for $T \geq 0$, clearly stating the coordinates of any axial intercept and the equation of any asymptote. [3]

3.* [Maximum mark: 6]

On 1st January 2022, Sam invested \$600 in an account that pays a nominal annual interest rate of 1.2%, compounded monthly. The amount of money in Sam's account at the end of each year follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer correct to 4 significant figures. [2]

(b) Sam makes no further deposits or withdrawals from the account. Find the year in which the amount of money in Sam's account becomes more than \$1000. [4]

4.* [Maximum mark: 5]

The functions f and g are defined by $f(x) = \frac{3-2x}{x+1}$, $x \in R, x \neq -1$ and $g(x) = x - a$, where $x \in R, a \in R$.

(a) Find the range of f. [2]

(b) Given that $(f \circ g)(x)$ has an asymptote at $x = -4$ for all $x \in R$, determine the value of a. [3]

5. [Maximum mark: 7]

Consider the quartic equation $z^4 - 3z^3 + az^2 + bz + c = 0,\ z \in C$.

Two of the roots of this equation are 2 and $1 + 2i$. Find the possible values of a, b and c where $a, b, c \in R$.

6. [Maximum mark: 7]

Find the roots of the equation $(w - 2i)^3 = 8i, w \in C$. Give your answers in Cartesian form.

7. [Maximum mark: 7]

The following diagram shows two circles of radius 1 unit, with centres at O and A respectively. The circles intersects at B.

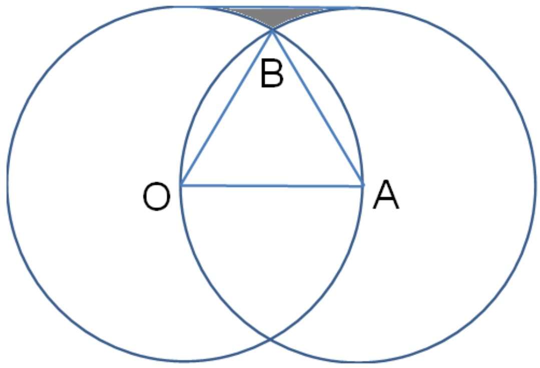


Diagram not drawn to scale.

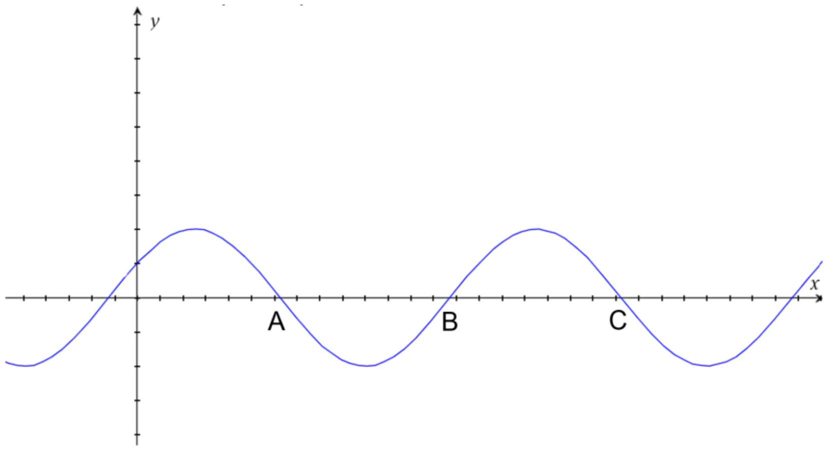
(a) Find the value of angle AOB, giving your answers in radians. [1]

(b) Find the area of the shaded region. Give your answer correct to 3 significant figures. [6]

8. [Maximum mark: 5]

Consider the graph of the function $f(x) = \cos(px - q), x \in R$ where p and q are positive constants.

The graph of f intersects the x-axis at point A, point B and point C. This is shown in the following diagram. The coordinates of B are $(1.1, 0)$ and the coordinates of C are $(1.7, 0)$.



(a) Find the coordinates of A. [1]
(b) Find the value of p. [2]
(c) Find the smallest positive value of q. [2]

9. [Maximum mark: 8]

Consider a triangle with interior angles A, B, C, and with sides of lengths a, b, c as shown in the following diagram.

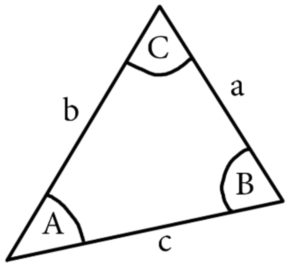


Diagram not drawn to scale.

(a) Show that $\frac{a}{c} = \frac{sinA}{sinC}$. [2]

(b) Hence show that $\frac{a}{c} sin2C + \frac{c}{a} sin2A = 2sin(\pi - B)$. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10. [Maximum Mark: 16]

The complex number z is given by $z = a - 3i$, where a is a real number.

(a) Find the possible values of a if $\frac{z^2}{z^*}$ is purely imaginary. [6]

For the rest of the question, it is further given that $a = 3\sqrt{3}$.

(b) (i) Find the smallest integer value of n such that $|z^n| \geq 1000$. [4]

(ii) On a single Argand diagram, plot and label the points A and B representing the complex numbers z and z^* respectively. [2]

(iii) The complex numbers z and z^* satisfies the equation $z^* = wz$. Using the Argand diagram in 10bii), find w in the form $re^{i\alpha}$ where $-\pi < \alpha \leq \pi$. [4]

11. [Maximum Mark: 16]

The function f is defined as $f(x) = x^4 - x^3 - 3x^2 - x + 4$, $x \in R$.

(a) Find the solutions of $f(x) > 0$. Give your answer correct to 3 significant figures. [3]

(b) **The domain of f is now restricted to $\in [a, 1]$.**

(i) Find the smallest value of a for which f has an inverse. Give your answer correct to 3 significant figures. [2]

(ii) For this value of a, sketch the curves $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, clearly stating the coordinates of the end points of each curve and the coordinates of any point(s) of intersection of the curves. [4]

(iii) Find the value of $f^{-1}(3.4)$. Give your answer correct to 3 significant figures. [2]

(c) A function h has an oblique asymptote at $y = x - 1$, vertical asymptotes at $x = -2$ and $x = 2$, and passes through the point with coordinates $(0, -2)$. Find the equation of h in the form $a(x) + \frac{k}{b(x)}$, where $k \in R$ and $a(x), b(x)$ are functions to be determined. [5]

12. [Maximum Mark: 23]

The following diagram shows a cuboid whose vertices are O, A, B, C, D, E, F and G.

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are shown on the diagram, where $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$

and $\boldsymbol{c} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix}$.

P is a point on the diagonal OE that divides OE in the ratio $3:1$.

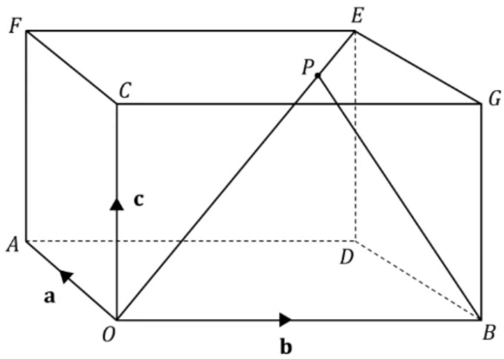


Diagram not drawn to scale.

(a) Find $\overrightarrow{OP}$ and $\overrightarrow{BP}$. [4]

(b) Write down a vector equation of the line that passes through B and P. [2]

(c) Show that if the line segment BP is extended, it will intersect FE. [7]

(d) Show that FE is divided in the ratio $2:1$ by the point of intersection with the extended line segment of BP. [4]

(e) Show that the area of the triangle OBP is $\frac{3}{8}|\boldsymbol{b} \times (\boldsymbol{a} + \boldsymbol{c})|$.

Hence or otherwise find the value of the area of triangle OBP, giving your answer correct to 3 significant figures. [6]

END OF PAPER