Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2022

YEAR 5 IB DIPLOMA PROGRAMME

MATHEMATICS : ANALYSIS AND APPROACHES

HIGHER LEVEL

PAPER 1

Tuesday 20 September 2022

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics : Analysis and Approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**
- Questions with an asterisk (*) are common to both HL & SL papers

Section A (55 Marks)		**Section B (55 Marks)**	
Question	**Marks**	**Question**	**Marks**
1		**10**	
2			
3			
4		**11**	
5			
6			
7		**12**	
8			
9			
Subtotal		**Subtotal**	
TOTAL	**/ 110**		

This question paper consists of 14 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1.* [Maximum mark: 5]

Solve the following equation.

$$9^x - 12 \cdot 3^x + 27 = 0$$

2.* [Maximum mark: 6]

Given that $\log_3 x = 1 + \log_3 y = \lg\frac{2}{9} - \lg\frac{9}{125} + 2\lg\frac{9}{5}$, find the values of x and y.

3.* [Maximum mark: 6]

(a) Show that $\ln x$, $\ln x^2$ and $\ln x^3$ are 3 consecutive terms of an arithmetic progression. [2]

(b) Solve $\sum_{k=1}^{10} \ln x^k = 55$. [4]

4.* [Maximum mark: 7]

(a) Find the term independent of x in the expansion of $\left(px^2+\frac{1}{x}\right)^6$ in terms of p. [5]

(b) Given that the term independent of x is 60, find the values of p. [2]

5. [Maximum mark: 7]

(a) Factorise $x^3 - x^2 - 9x + 9$ completely. [3]

(b) Hence, solve the inequality $\dfrac{x^3 - x^2 - 9x + 9}{2 - x} \geq 0$. [4]

6. [Maximum mark: 4]

Express $\dfrac{x+1}{2x^2-3x-2}$ in partial fractions.

7. [Maximum mark: 7]

Consider the following system of equations where $k \in \mathbb{R}$.

$$\begin{aligned} x + y - z &= 3 \\ 2x + 4y + 2z &= 3 \\ x + y + \left(k^2 - 10\right)z &= k \end{aligned}$$

(a) Find the value of k for which the system of equations does not have any solution. [3]

(b) Find the general solution if the system has an infinite number of solutions. [4]

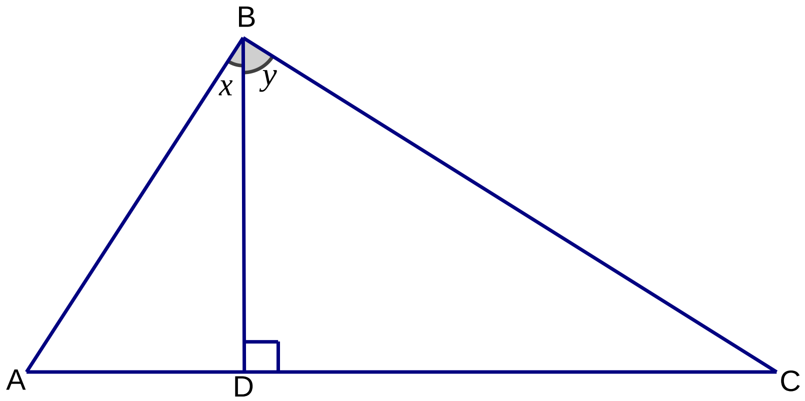
8. [Maximum mark: 6]

Two unit vectors **a** and **b** are given such that $|\mathbf{a}+2\mathbf{b}|=|\mathbf{b}-3\mathbf{a}|$.

(a) Show that $\mathbf{a}\cdot\mathbf{b}=\frac{1}{2}$. [4]

(b) Hence find the angle between **a** and **b** . [2]

9. [Maximum mark: 7]



The above diagram shows the triangle ABC such that BD is perpendicular to AC, $\angle ABD = x$ and $\angle CBD = y$.

(a) Given that $x = \arctan(a)$ and $\tan(x+y) = \dfrac{2a+b}{2-ab}$ where $a, b \in \mathbb{R}^+$, find the value of $\tan y$ in terms of b. [4]

(b) If $\tan x = \dfrac{2-b}{2+b}$, show that $\arctan\left(\dfrac{1}{a}\right) + \arctan\left(\dfrac{2}{b}\right) = \dfrac{3\pi}{4}$. [3]

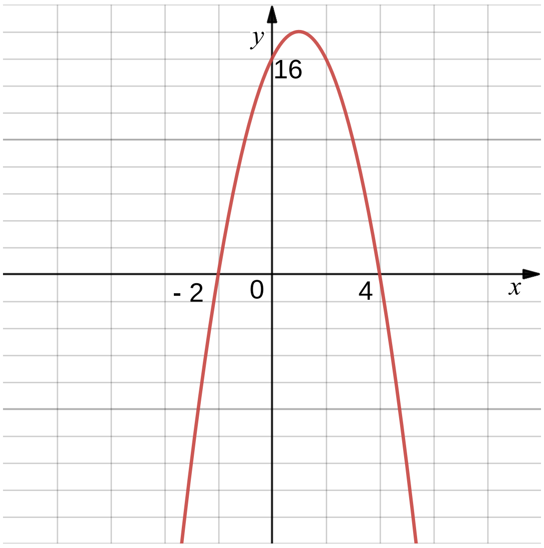
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

10.* [Maximum mark: 21]

(a)



(i) The diagram above shows the graph of a quadratic function f. Find an expression for f.

(ii) Express f in the form $a(x+b)^2+c$ where $a,b,c \in \mathbb{Z}$.

(iii) The graph of $y=3(x-4)^2-27$ can be obtained from the graph of f by a sequence of three transformations. Give a full description of each of the three transformations.

(iv) The point $M(a,b)$ lies on the graph of f. Write down the coordinates of the image of M under the transformations in (a)(iii). [10]

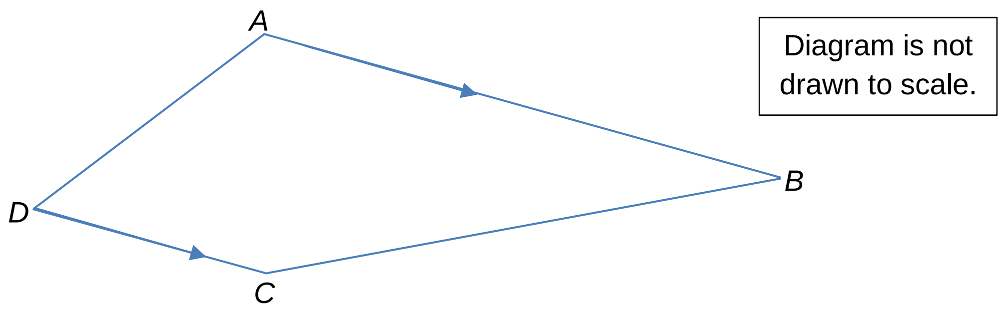
(b) Given that $g(x)=\dfrac{2x-4}{1-x}$, $x \neq 1$.

(i) Sketch the graph of g.

(ii) Hence, sketch on a different axis, the graph of $y=\left[g(x)\right]^2$. [7]

(c) By considering the graph of $y=\sin x$ and through a series of transformations, sketch the graph of $y=\dfrac{1}{2}\operatorname{cosec}\left(x-\dfrac{\pi}{2}\right)$ for $x \in \left]0,2\pi\right[$, including the coordinates of the turning point and the equations of the asymptotes. [4]

11. [Maximum mark: 18]



The points A, $B(-5,0,2)$, $C(a,a,1)$ and $D(2,-1,1)$ form a quadrilateral $ABCD$ such that $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are parallel.

The line joining A and D is parallel to the vector $\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$.

(a) Write down the vector equation of the line l_{AD} which passes through A and D. [1]

The cartesian equation of the line passing through C and D is $\dfrac{2-x}{2} = y+1$, $z=1$.

(b) Find the vector equation of the line l_{AB} which passes through A and B. [3]

(c) Hence find the coordinates of A. [3]

(d) Find the value of k such that $\overrightarrow{AB} \times \overrightarrow{AD} = k\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$. [4]

(e) Find the cartesian equation of the plane containing the points A, B and D. [2]

(f) Show that $a = 0$. [1]

(g) Find the area of the quadrilateral $ABCD$. [4]

12. [Maximum mark: 16]

Consider the complex number z such that $|z| = |z - \alpha i|$ where $\alpha > 0$.

(a) Show that imaginary part of z is $\frac{\alpha}{2}$. [2]

Let z_1 and z_2 be the two possible values of z such that $|z_1| = |z_2|$, $\text{Im}(z_1) = \text{Im}(z_2)$ and $0 < \arg(z_1) < \frac{\pi}{2}$. z_1 and z_2 can be represented on an Argand diagram by the points P and Q respectively.

(b) Sketch an Argand diagram showing the points P and Q. [2]

(c) Given that O is the origin, $\text{Re}(z_1) > \text{Im}(z_1)$, $|z| = 5$ and that the area of triangle POQ is 12 units^2,

(i) find z_1 in the form $a + ib$ where $a, b \in \mathbb{R}$;

(ii) write down z_2 in the form $a + ib$ where $a, b \in \mathbb{R}$. [5]

(d) Find the value of $\arg(z_1 - 1)$ and $\arg(z_2 + i)$ [2]

(e) Given that $\arg\left[\frac{(z_1 - 1)^3 (z_2 + i)^k}{-2i}\right] = \frac{3\pi}{2}$, find the least positive integer value of k. [5]

END OF PAPER